Solution:

Problem 6 of the 2003 Maritimes Mathematics Competition, which appeared in Crux Mathematicorum [2003:354]. We present the official solution which appeared at [2004:324], slightly modified.

We first prove that $n \leq 3$.

Let us assume to the contrary that exists positive integers m, n with $n \geq 4$ satisfying the given equation.

Since $n \geq 4$, the number $n!$ is a multiple of 4. Therefore

$$m^2 - 3 = 11 \cdot n! + 2000$$

is a multiple of 4. Thus, there exists some integer k so that

$$m^2 - 3 = 4k\,.$$

This implies that m is odd, and hence there exists an integer l such that $m = 2l+1$, which gives

$$4k = m^2 - 3 = (2l+1)^2 - 3 = 4l^2 + 4l - 2 \Longrightarrow 2 = 4(l^2 + l = k)\,,$$

a contradiction.

Since we got a contradiction, our assumption that $n \geq 4$ is wrong. Therefore, $n \leq 3$.

Now, we test the three cases $n = 1, n = 2$ and $n = 3$.

When $n = 1$ we have

$$m^2 = 2003 + 11 = 2014$$

is not a perfect square, since 2014 is even and not divisible by 4.

When $n = 2$ we have

$$m^2 = 2003 + 22 = 2025 = 25 \cdot 81 = (5 \cdot 9)^2 = 45^2\,.$$

is a perfect square.

When $n = 3$ we have

$$m^2 = 2003 + 33 = 2036$$

is not a perfect square, since

$$\begin{aligned} 45^2 &= 2025 < 2036 \\ 46^2 &= (45+1)^2 = 45^2 + 90 + 1 > 2036\,. \end{aligned}$$

It follows that the only pair is

$$(m, n) = (45, 2)\,.$$