

Solution:

Problem 5 of the 22nd W. J. Blundon Contest, which appeared in Crux at [2006:355]. We present the official solution that appeared at [2007:133], modified by the editor.

First note that

$$2x = 2005 - 5y = 5(401 - y) .$$

This implies that $2x$ is divisible by 5 and hence x is divisible by 5. Therefore, there exists some positive integer n so that $x = 5n$.

It follows that

$$2n = 401 - y .$$

Since y is a positive integer we have

$$2n \leq 400 \implies n \leq 200 .$$

Moreover, for each $1 \leq n \leq 200$

$$y = 401 - 2n$$

is a positive integer. Therefore, each $1 \leq n \leq 200$ gives us a pair

$$(x, y) = (5n, 401 - 2n) .$$

There are exactly 200 such pairs.