

- P1.** Let $a, b \geq 2$ be relatively prime integers. Let S consist of the points in the plane with integer coordinates lying strictly inside the triangle with vertices $(0, 0)$, $(a, 0)$, $(0, b)$.

Determine, with proof,

$$\sum_{(x,y) \in S} (a - 2x)(b - 2y)$$

in terms of a and b .

Remark. Here the summation denotes that we sum the value $(a - 2x)(b - 2y)$ over all points (x, y) in S .

- P2.** There are n types of coins in Wario's gold mine. Each coin of the i th type is worth d_i cents, where d_1, \dots, d_n are distinct positive integers. A positive integer D is denoted *lucky* if the following holds: For each positive integer k , any collection of coins (containing any number of coins of each type) with a total value of exactly kD cents can be split into k groups, each worth D cents.

Does a lucky number necessarily exist?

- P3.** Turbo the snail plays a game on a board with $2n$ rows and $2n$ columns. There are $2n^2$ monsters who first choose to occupy $2n^2$ distinct cells, with Turbo's knowledge. After this, Turbo chooses any cell and labels it 1. Starting from this cell, Turbo then walks through all other $4n^2 - 1$ cells exactly once, labelling them in order with $2, 3, \dots, 4n^2$. Turbo only moves between cells which share an edge, and never returns to a cell.

The final score is the sum of the labels of the cells with monsters. The monsters are trying to place themselves to maximize the score, while Turbo is trying to minimize the score based on the monsters' positions.

Find, in terms of n , the largest score which the monsters can guarantee.

- P4.** A sphere with center I is inscribed in a tetrahedron $ABCD$. Suppose that the angle between any two faces of $ABCD$ is acute. Moreover, suppose that

$$\frac{\text{vol}(IABC)}{BC} = \frac{\text{vol}(IACD)}{CD} = \frac{\text{vol}(IADB)}{DB}.$$

Show that AI is perpendicular to the plane BCD .

Remark. Here, $\text{vol}(IABC)$ denotes the volume of tetrahedron $IABC$, and similarly for $IACD$ and $IADB$.

- P5.** For each $n \geq 1$, determine the maximum integer c_n for which there exists a polynomial f of degree n with rational coefficients, an irrational number a , and c_n distinct rational numbers a_1, a_2, \dots, a_{c_n} such that $f(a + a_i)$ is a rational number for all $1 \leq i \leq c_n$.