2025 Canada Jay Mathematical Competition

Official Solutions

A competition of the Canadian Mathematical Society.

Part A: 5 marks each

1. What is the units (ones) digit of the product

$$1 \times 3 \times 5 \times 9 \times 11 \times 13 \times 15 \times 17 \times 19 \times 21 \times 23\,?$$

(A) 3 **(B)** 5 **(C)** 7 **(D)** 8

Solution: Since the numbers are all odd, the product must be odd: and since one factor is 5, the units digit in the product is a 5.

Answer: (B)

2. Manaswi wants to watch a 32 minute video but she has to leave for school in 20 minutes. What's the slowest playback speed listed below that will let her finish the video before she has to leave?

(A) 1.25x **(B)** 1.5x **(C)** 1.75x **(D)** 2x

Solution: Manaswi needs to watch half of the movie (16 minutes) in 10 minutes. Therefore, the speed needs to be at least 1.6 x.

Answer: (C)

3. An extra large apple pie is cut into 12 equal slices. First, Angus eats $\frac{1}{3}$ of the pie. Then, Betty eats 3 slices of pie. Finally, Carl eats 20% of the remaining pie. How many slices are left over for Doris?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Solution: Angus eats $\frac{1}{3} \times 12 = 4$ slices. After Betty eats her pieces, there are $12-4-3 = 5$ slices left. Carl eats $20\% = \frac{20}{100}$ of 5 slices, meaning that Carl eats 1 slice. There are $5 - 1 = 4$ slices left for Doris.

Answer: **(D)**

4. Alice, Benjamin, and Chris have 30 candies between them. Alice gives 6 of her candies to Benjamin and 2 of her candies to Chris. Benjamin gives 5 of his candies to Chris. At this point each of them has the same number of candies.
How many candies did Benjamin start with?

(A) 3 **(B)** 9 **(C)** 10 **(D)** 15

Solution: Let us work backwards.

At the end, each friend has $30 \div 3 = 10$ candies. Before giving 5 candies to Chris, Benjamin had 15 candies. Before receiving 6 candies from Alice, Benjamin had 9 candies.

Therefore, Benjamin started with 9 candies.

Answer: **(B)**

5. Pineview Elementary School is mailing out newsletters. They assemble two teams of student volunteers to help prepare them.

- Team A starts at noon and works for 2 hours at a constant rate of 900 flyers per hour.
- Team B starts when team A finishes and works for 4 hours at a constant rate of 600 flyers per hour.

At what time were half of the flyers prepared?

(A) 2:00 pm
(B) 2:30 pm
(C) 3:00 pm
(D) 3:30 pm

Solution: Crew A prepares $2 \times 900 = 1800$ flyers, while crew B prepares $4 \times 600 = 2400$. Thus, there were $1800 + 2400 = 4200$ flyers prepared in total.

Half the total number of flyers is $4200 \div 2 = 2100$.

When crew A finished, there are 1800 flyers, therefore half of the flyers were prepared when crew B finished 300 flyers.

Since crew B prepares 600 flyers per hour, it will take them $300 \div 600 = \frac{1}{2}$ hour to reach the halfway point.

Since crew B started at 2:00 pm, the halfway point is reached at $2:30$ pm.

Answer: $\boxed{\textbf{(B)}}$

Part B: 5 marks each

1. How many 2-digit numbers have the property that when we add 18 to them the 2 digits get interchanged (switch positions)?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

Solution: Let a, b be the two digits of such a number. Then, the number is $10a + b$.

We have

$$\begin{aligned} 10a + b + 18 &= 10b + a \quad \Longrightarrow \\ 9a + 18 &= 9b \quad \Longrightarrow \\ b &= a + 2\,. \end{aligned}$$

Thus, there are 7 such numbers:

$$13, 24, 35, 46, 57, 68, 79\,.$$

Answer: **(C)**

2. If we know:

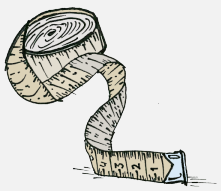
(I) Jane is taller than Sandy.

(II) Sandy is not as tall as Ed.

(III) Alicia is taller than Sandy, but shorter than Ed.

(IV) Ron is not as tall as Alicia, but taller than Sandy.

(V) Sara is not as tall as Jane, but taller than Ed.



Find the tallest and smallest of the group

(A) Jane and Sandy
(B) Jane and Ron
(C) Sara and Ed
(D) Ron and Sandy
(E) Ed and Sara

Solution: Statement IV tells us that Sandy is shorter than Ron who, in turn, is shorter than Alicia.

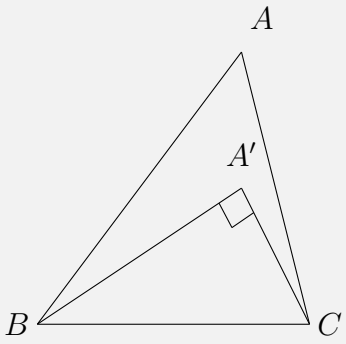
Statement II tells us, in addition, that Alicia is shorter than Ed.

Statement V tells us that Ed is shorter than Sara who, in turn, is shorter than Jane.

Therefore, the ranking of the students, from shortest to tallest, is Sandy, Ron, Alicia, Ed, Sara, and Jane.

Answer: **(A)**

3. Given a triangle ABC, we move point A toward side BC while keeping points B and C fixed. In this process, the measure of $\angle BAC$ increases. Suppose that when A moves to a new position A', the angle $\angle BAC$ increases from $40°$ to $90°$, so that $\angle BA'C = 90°$. If it also given that $\angle ABA' = \angle ACA'$, find the measure of $\angle ABA'$.



(not to scale)

(A) $20°$ **(B)** $25°$ **(C)** $40°$ **(D)** $50°$ **(E)** $90°$

Solution: Since the sum of the interior angles of a triangle is $180°$, an increase in $\angle BAC$ will result in a decrease in the sum of the other two angles. $\angle ABA' + \angle ACA' = 90° - 40° = 50°$. So, $\angle ABA' = 25°$.

Answer: **(B)**

4. Two positive integers have a greatest common divisor of 3 and a product of 108.
How many different values can the smallest of the two integers take?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Solution: Let $x < y$ be the two numbers. Then, there exist relatively prime numbers a, b so that

$$x = 3a$$
$$y = 3b\,.$$

Since $xy = 108$ we get

$$ab = 12\,.$$

12 can be written as a product in three different ways: $1 \times 12, 2 \times 6$ and 3×4, but only two of those are products of relatively prime numbers.

Thus, the smallest of the two integers can only be $3 \cdot 1 = 3$ or $3 \cdot 3 = 9$.

So the answer is 2.

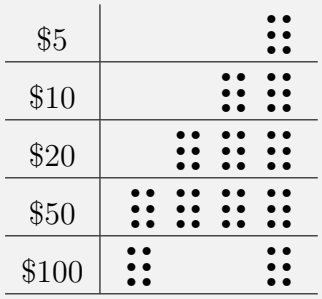
Alternate Solution 108 can be written as a product of two numbers the following ways:

$$1 \cdot 108$$
$$2 \cdot 54$$
$$3 \cdot 36$$
$$4 \cdot 27$$
$$6 \cdot 18$$
$$9 \cdot 12\,.$$

From these, only $3 \cdot 36$ and $9 \cdot 12$ have a gcd of 3.

Answer: **(B)**

5. Canadian banknotes have Braille markings in the corner so people can identify the value of a bill by touch alone. A group of 6 dots is called a Braille block. The \$5 bill has one Braille block, \$10 has two, \$20 has three, \$50 has four, and \$100 has two blocks that are far apart, as shown.



Brooke has some amount of bills and says, "the total number of Braille blocks on all the bills I'm holding is 6." Which of the following cannot be the total value of all the bills that Brooke is holding?

(A) \$30 **(B)** \$35 **(C)** \$55 **(D)** \$125 **(E)** \$150

Solution: \$55 cannot be the value of the bills Brooke holds.

Indeed, if Brooke holds a \$50 bill, then she has at least \$60 in total. Otherwise, she can have at most \$20 for each 3 groups of Braille blocks and hence at most \$40.

The other answers are possible:

$\$30 = 6 \times \$5 =$ 

$\$35 = \$5 + \$10 + \$20 =$ 

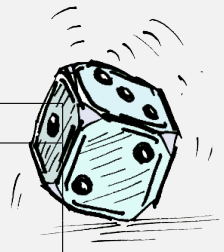
$\$125 = \$5 + \$100 + \$20 =$ 

$\$150 = \$100 + \$50 =$ 

Answer: (C)

Part C: 6 marks each

1. Yuchen played a game with the letters $\boxed{\text{b}}$, $\boxed{\text{r}}$, $\boxed{\text{d}}$ and the vowels (a,e,i,o,u) to make up words. He tossed a fair die and followed these rules:



Dice Roll	Rule
⚀	write down the letter "a" and toss the die again
⚁	write down the letter "e" and toss the die again,
⚂	write down the letter "i" and toss the die again,
⚃	write down the letter "o" and toss the die again,
⚄	write down the letter "u" and toss the die again,
⚅	write down the next letter in the sequence: $\{\boxed{\text{b}}, \boxed{\text{r}}, \boxed{\text{d}}\}$ and toss the die again.
	Once the letter "d" is written, stop.

Yuchen refers to each outcome as an word. Even nonsensical words like "baerioioad" are words.
Which of the words below is the most likely to have been written?

(A) bird **(B)** bread **(C)** board **(D)** aboard **(E)** abroad

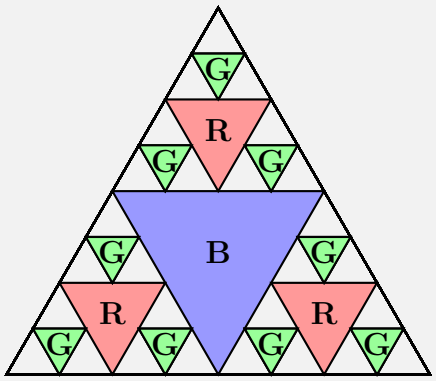
Solution: Since each of the letters is equally likely to have happened, the shortest word will be the most likely to occur. The longer Yuchen plays, the more possibilities open up, and the less the likelihood of each one particular k-letter word occurring.

Answer: $\boxed{\textbf{(A)}}$

2. A white equilateral triangle with area 2048 cm^2 is decorated in the following manner:

- The midpoints of the sides are joined to form a small triangle, which is shaded blue (B).
- Then the midpoints of the three new unshaded triangles are joined to form small triangles, which are shaded red (R).
- The process is repeated with the nine new unshaded triangles to make the final central triangles, shaded green (G).

The final figure is shown below.



What is the area of the white unshaded region?

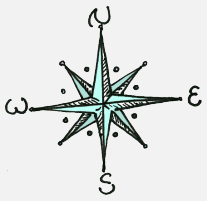
(A) 476 cm^2 **(B)** 512 cm^2
(C) 864 cm^2 **(D)** 896 cm^2
(E) 1152 cm^2

Solution: At each step, $\frac{1}{4}$ of the unshaded area is shaded, so $\frac{3}{4}$ of the area remains unshaded. Thus, the final unshaded area is

$$2048\text{cm}^2 \times \left(\frac{3}{4}\right)^3 = 864\text{cm}^2$$

Answer: (C)

3. A Canada Jay leaves its home and goes on a journey. Each day the jay does each of the following three actions once:



- in the morning, it flies 3 km North,
- in the afternoon, it flies 4 km South, and
- in the evening, it flies 5 km North.

Where is the jay located once it has traveled exactly 2025 km?

(A) 336 km North of its home **(B)** 672 km North of its home
(C) 673 km North of its home **(D)** 674 km North of its home
(E) 675 km North of its home

Solution: Each day, the jay flies exactly 12 km and moves 4 km North.

Therefore, after traveling

$$2016 = 12 \times 168\text{km},$$

the jay is exactly

$$4 \times 168 = 672\text{km}$$

North of his home.

From here, the jay travels 3 km North, 4 km South and 2 km north. The jay ends his journey 673 km North of his home.

Answer: **(C)**

4. The number 236 is an example of a three-digit number for which one of the digits is the product of the other two digits.
How many numbers between 100 and 999 inclusive have this property?

(A) 50 **(B)** 51 **(C)** 52 **(D)** 53 **(E)** 61

Solution: We split the problem into three cases.

Case 1: One of the digits is zero. Then, a second digit must be zero. The non-zero digit must be the first one.

There are 9 numbers in this case.

Case 2: Zero is not a digit and one of the digits is 1. Then, the other two digits must be equal. Therefore, the digits are $1, x, x$ for some $1 \leq x \leq 9$.

When $x = 1$, the number must be 111.

When $2 \leq x \leq 9$ then, there are 8 possibilities for choosing x and three possibilities of choosing the position of the digit 1 in the three digit number.

There are

$$1 + 8 \times 3 = 25$$

numbers in this case.

Case 3: No digit is 0 or 1. Therefore, all digits are between 2 and 9, inclusive, and the product of two of them is the third one. It follows that the digits must be one of the following groups

$$\{2, 2, 4\}; \{2, 3, 6\}; \{2, 4, 8\}, \{3, 3, 9\}\,.$$

There are 3 numbers which can be formed with the first group, 6 numbers which can be formed with the each of the second and third group, and 3 with the last group.

Therefore, in this case there are

$$3 + 6 + 6 + 3 = 18$$

numbers.

In total there are

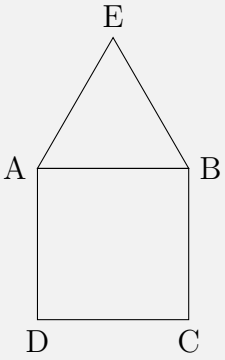
$$9 + 25 + 18 = 52$$

numbers in total.

We can list them all out:
100, 111, 122, 133, 144, 155, 166, 177, 188, 199,
200, 212, 221, 224, 236, 242, 248, 263, 284,
300, 313, 326, 331, 339, 362, 393,
400, 414, 422, 428, 441, 482,
500, 515, 551,
600, 616, 623, 632, 661,
700, 717, 771,
800, 818, 824, 842, 881,
900, 919, 933, 991.
So, there are 52.

Answer: $\boxed{\textbf{(C)}}$

5. Two runners, Anna and Bob, run around the following track. Each side (AB, BC, CD, DA, AE, BE) is exactly 100 m long.



They start running at the same time.
Anna starts at point A and runs around the triangle at a constant speed of 100 m/min from A to E to B to A to E to B ....
Bob starts at point B and runs around the square at a constant speed of 200 m/min from B to C to D to A to B to C ...
How many seconds after they start running do they meet for the second time?

(A) 140 **(B)** 200 **(C)** 240 **(D)** 300 **(E)** 340

Solution: Anna and Bob meet only between A and B. Let us check all times both of them are on this side.

It takes Anna 60 seconds to run one full side and it takes Bob 30 seconds to run a full side.

Anna arrives first time at point B after 120 seconds. Between 120 and 180 seconds she runs from B to A. As it takes her 180 seconds to complete the track, she is running from B to A in the following intervals:

$$120 - 180; 300 - 360; 480 - 540; 660 - 720; ...$$

Bob arrives at A after 90 seconds and at B after 120 seconds. At this point, the two meet for the first time. Since it takes Bob 120 seconds to complete the track, he is running from A to B in the following intervals:

$$90 - 120; 210 - 240; 330 - 360; 450 - 480; ...$$

The second meeting occurs when Anna run from B to A in the $300 - 360$ seconds interval and Bob run from A to B in the $330 - 360$ seconds interval, at some point X between A and B.

Let us denote the moment they meet by $360 - t$ seconds.

In the next t seconds, Anna run from X to A and Bob run from X to B. Therefore

$$100 = AB = AX + XB = 100 \cdot \frac{t}{60} + 200 \cdot \frac{t}{60} = 5t$$

meaning $t = 20$ seconds.

They meet for the second time after $360 - 20 = 340$ seconds.

Answer: $\boxed{\textbf{(E)}}$

