Official 2025 Problem Set

P1. The n players of a hockey team gather to select their team captain. Initially, they stand in a circle, and each person votes for the person on their left.

The players will update their votes via a series of rounds. In one round, each player a updates their vote, one at a time, according to the following procedure: At the time of the update, if a is voting for b, and b is voting for c, then a updates their vote to c. (Note that a, b, and c need not be distinct; if $b = c$, then a's vote does not change for this update.) Every player updates their vote exactly once in each round, in an order determined by the players (possibly different across different rounds).

They repeat this updating procedure for n rounds. Prove that at this time, all n players will unanimously vote for the same person.

P2. Determine all positive integers a, b, c, p where p and $p + 2$ are odd primes and

$$2^a p^b = (p + 2)^c - 1.$$

P3. A polynomial $c_d x^d + c_{d-1}x^{d-1} + \cdots + c_1 x + c_0$ with degree d is *reflexive* if there is an integer $n \geq d$ such that $c_i = c_{n-i}$ for every $0 \leq i \leq n$, where $c_i = 0$ for $i > d$. Let $\ell \geq 2$ be an integer and $p(x)$ be a polynomial with integer coefficients. Prove that there exist reflexive polynomials $q(x), r(x)$ with integer coefficients such that

$$(1 + x + x^2 + \cdots + x^{\ell-1})p(x) = q(x) + x^\ell r(x).$$

P4. Let ABC be a triangle with circumcircle Γ and $AB \neq AC$. Let D and E lie on the arc BC of Γ not containing A such that $\angle BAE = \angle DAC$. Let the incenters of BAE and CAD be X and Y respectively, and let the external tangents of the incircles of BAE and CAD intersect at Z. Prove that Z lies on the common chord of Γ and the circumcircle of AXY.

P5. A rectangle R is divided into a set S of finitely many smaller rectangles with sides parallel to the sides of R such that no three rectangles in S share a common corner. An ant is initially located at the bottom-left corner of R. In one operation, we can choose a rectangle $r \in S$ such that the ant is currently located at one of the corners of r, say c, and move the ant to one of the two corners of r adjacent to c.

Suppose that after a finite number of operations, the ant ends up at the top-right corner of R. Prove that some rectangle $r \in S$ was chosen in at least two operations.