

2025 CMO Qualifying Repêchage

Problems



1. **[10 points]** Solve the following equation, where A, B , and C are digits and A and C are non-zero: $\overline{ABCB} + 1434 = \overline{CABA}$.
2. **[10 points]** Let triangle ABC be a right triangle with $\angle BAC = 90^\circ$. Let I and O be the incentre and the circumcentre of triangle ABC , respectively. It is given that $\angle IOB = 45^\circ$. Determine all possible angles of $\angle CBA$.
3. **[10 points]** Initially, there are 2024 green balls and 1 red ball in a box. Every minute, Kate chooses a random ball from the box. If it is green, she paints it blue and puts it back into the box. If it is blue, she paints it green and puts it back into the box. Finally, if it is red, then she stops the process. What is the expected number of green balls at the end of her process?
4. **[10 points]** Assume that $\{a_n\}_{n \geq 1}$ is an infinite arithmetic sequence and $\{b_n\}_{n \geq 1}$ is an infinite geometric sequence. If it is given that $a_1 < a_2$, $b_i = a_i^2$ for $i = 1, 2, 3$, and

$$\lim_{n \rightarrow \infty} (b_1 + b_2 + \cdots + b_n) = \sqrt{2} + 1,$$

determine, with proof, all possible sequences $\{a_n\}$.

5. **[10 points]** Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $xy = f(x)f(y) - f(x + y)$ for all real numbers x and y .
6. **[15 points]** In scalene triangle ABC , the circumcentre and incentre are respectively O and I . Let AD be the altitude to line BC , with D lying on line BC . Given that the radius of the circumcircle and A -excircle are equal, prove that the points O, I , and D are collinear.
7. **[15 points]** Is it possible to arrange the numbers $1, 2, \dots, 54^2$ in a 54×54 grid such that any two vertically or horizontally adjacent cells are relatively prime?
8. **[20 points]** Let n be a positive integer, and let $2n$ points be equally spaced on a circle. Prove that for any integer $0 \leq k \leq \frac{n(n-1)}{2}$, there exists a way to draw n line segments, each connecting two distinct points, such that exactly k pairs of these line segments intersect.