The Canadian Mathematical Society La Société mathématique du Canada

The Canadian Mathematical Society

in collaboration with

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

presents the

Sun Life Financial Canadian Open Mathematics Challenge

Sun Life Financial

Wednesday, November 19, 2008

Time: $2\frac{1}{2}$ hours

Calculators are NOT permitted.

Do not open this booklet until instructed to do so.

There are two parts to this paper.

PART A

This part of the paper consists of 8 questions, each worth 5 marks. You can earn full value for each question by entering the correct answer(s) in the space provided. If your answer is incorrect, any work that you do will be considered for part marks, **provided that it is done in the space allocated** to that question in your answer booklet.

PART B

This part of the paper consists of 4 questions, each worth 10 marks. Finished solutions must be written in the appropriate location in the answer booklet. Rough work should be done separately. If you require extra pages for your finished solutions, paper will be provided by your supervising teacher. Any extra papers should be placed inside your answer booklet. Be sure to write your name and school name on any inserted pages.

Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

NOTES:

At the completion of the contest, insert the information sheet inside the answer booklet.

The names of top scoring competitors will be published on the Web sites of the CMS and CEMC.

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NOTE:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write solutions in the answer booklet provided.
	3.	It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
	4.	Calculators are **not** allowed.
	5.	Diagrams are not drawn to scale. They are intended as aids only.

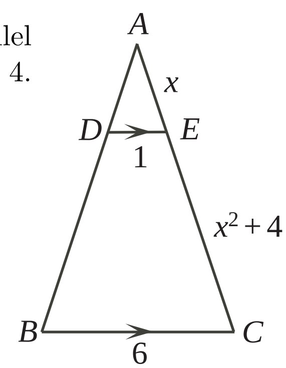
PART A

1. If $2x + y = 13$ and $x + 2y = 11$, what is the value of $x + y$?

2. Determine the units digit of the integer equal to $9 + 9^2 + 9^3 + 9^4$.
(The *units digit* of an integer is its rightmost digit. For example, the units digit of the integer 1234 is 4.)

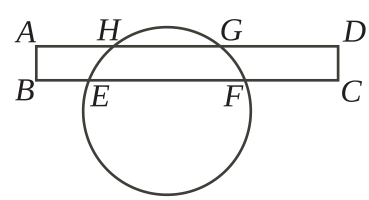
3. If the average of four *different* positive integers is 8, what is the largest possible value of any one of these integers?

4. In the diagram, D is on AB and E is on AC with DE parallel to BC. Also, $DE = 1$, $BC = 6$, $AE = x$, and $EC = x^2 + 4$. Determine all possible values of x.

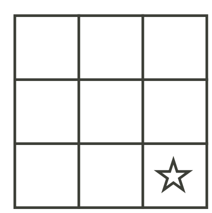


5. Four consecutive integers p, q, r, s with $p < q < r < s$ satisfy $\frac{1}{2}p + \frac{1}{3}q + \frac{1}{4}r = s$. What is the value of s?

6. Rectangle $ABCD$ intersects a circle at points E, F, G, and H, as shown. If $AH = 4$, $HG = 5$ and $BE = 3$, determine the length of EF.



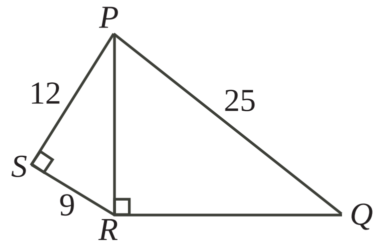
7. A star is placed in the bottom right corner square of a 3×3 grid, as shown. A fair coin is flipped repeatedly. Each time that the coin shows heads, the star is moved one space upwards; each time that the coin shows tails, the star is moved one space to the left. (The star may move off the grid.) Determine the probability that the star reaches the top left corner square of the grid.



8. Determine the sum of all integer values of the parameter r for which the equation $x^3 - rx + r + 11 = 0$ has at least one positive integer solution for x.

PART B

1. In the diagram, $\triangle PSR$ is right-angled at S and $\triangle PRQ$ is right-angled at R. Also, $PS = 12$, $SR = 9$, and $PQ = 25$.



   (a) Determine the length of RQ.

   (b) Determine the area of figure $PQRS$.

   (c) Show that $\angle QPR = \angle PRS$.

   (d) Determine the length of SQ.

2. (a) Determine all real numbers x such that $(x + 3)(x - 6) = -14$.

   (b) Determine all real numbers x such that $2^{2x} - 3(2^x) - 4 = 0$.

   (c) Determine all real numbers x such that $(x^2 - 3x)^2 = 4 - 3(3x - x^2)$.

3. (a) An infinite sequence $a_0, a_1, a_2, a_3, \ldots$ satisfies

$$a_{m-n} + a_{m+n} = \tfrac{1}{2}a_{2m} + \tfrac{1}{2}a_{2n}$$

   for all non-negative integers m and n with $m \geq n \geq 0$.

   (i) Show that $a_0 = 0$.

   (ii) If $a_1 = 1$, determine the value of a_2 and the value of a_3.

   (b) An infinite sequence $b_0, b_1, b_2, b_3, \ldots$ satisfies

$$b_{m-n} + b_{m+n} = b_{2m} + b_{2n}$$

   for all non-negative integers m and n with $m \geq n \geq 0$. Prove that all terms in the sequence have the same value.

4. A triangle is called *automedian* if its three medians can be used to form a triangle that is similar to the original triangle.

   (a) Show that the triangle with sides of length 7, 13 and 17 is automedian.

   (b) $\triangle ABC$ has side lengths $AB = c$, $AC = b$ and $BC = a$, with $a < b < c$. If $\triangle ABC$ is automedian, prove that $a^2 + c^2 = 2b^2$.

   (c) Determine, with proof, an infinite family of automedian triangles with integer side lengths, such that no two of the triangles in the family are similar.

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(English)