

# The 2020 Canadian Mathematical Olympiad

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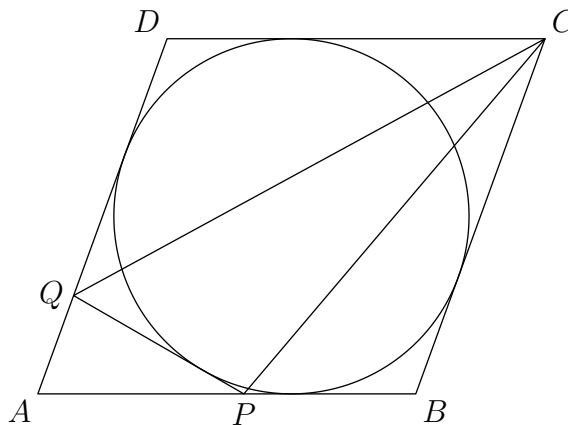
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## Official Problem Set

1. Let  $S$  be a set of  $n \geq 3$  positive real numbers. Show that the largest possible number of distinct integer powers of three that can be written as the sum of three distinct elements of  $S$  is  $n - 2$ .
2. A circle is inscribed in a rhombus  $ABCD$ . Points  $P$  and  $Q$  vary on line segments  $\overline{AB}$  and  $\overline{AD}$ , respectively, so that  $\overline{PQ}$  is tangent to the circle. Show that for all such line segments  $\overline{PQ}$ , the area of triangle  $CPQ$  is constant.



3. A purse contains a finite number of coins, each with distinct positive integer values. Is it possible that there are exactly 2020 ways to use coins from the purse to make the value 2020?

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4. Let  $S = \{1, 4, 8, 9, 16, \dots\}$  be the set of perfect powers of integers, i.e. numbers of the form  $n^k$  where  $n, k$  are positive integers and  $k \geq 2$ . Write  $S = \{a_1, a_2, a_3, \dots\}$  with terms in increasing order, so that  $a_1 < a_2 < a_3 < \dots$ . Prove that there exist infinitely many integers  $m$  such that 9999 divides the difference  $a_{m+1} - a_m$ .
5. There are 19,998 people on a social media platform, where any pair of them may or may not be *friends*. For any group of 9,999 people, there are at least 9,999 pairs of them that are friends. What is the least number of friendships, that is, the least number of pairs of people that are friends, that must be among the 19,998 people?

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**Important!**

*Please do not discuss this problem set online for at least 24 hours.*

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