Canadian Mathematical Olympiad
Official 2024 Problem Set

P1. Let ABC be a triangle with incenter I. Suppose the reflection of AB across CI and the reflection of AC across BI intersect at a point X. Prove that XI is perpendicular to BC.

(The incenter is the point where the three angle bisectors meet.)

P2. Jane writes down 2024 natural numbers around the perimeter of a circle. She wants the 2024 products of adjacent pairs of numbers to be exactly the set $\{1!, 2!, \ldots, 2024!\}$. Can she accomplish this?

P3. Let N be the number of positive integers with 10 digits $\overline{d_9 d_8 \cdots d_1 d_0}$ in base 10 (where $0 \leq d_i \leq 9$ for all i and $d_9 > 0$) such that the polynomial

$$d_9 x^9 + d_8 x^8 + \cdots + d_1 x + d_0$$

is irreducible in $\mathbb{Q}$. Prove that N is even.

(A polynomial is irreducible in $\mathbb{Q}$ if it cannot be factored into two non-constant polynomials with rational coefficients.)

P4. Centuries ago, the pirate Captain Blackboard buried a vast amount of treasure in a single cell of an $M \times N$ $(2 \leq M, N)$ grid-structured island. You and your crew have reached the island and have brought special treasure detectors to find the cell with the treasure. For each detector, you can set it up to scan a specific subgrid $[a, b] \times [c, d]$ with $1 \leq a \leq b \leq M$ and $1 \leq c \leq d \leq N$. Running the detector will tell you whether the treasure is in the region or not, though it cannot say where in the region the treasure was detected. You plan on setting up Q detectors, which may only be run simultaneously after all Q detectors are ready. In terms of M and N, what is the minimum Q required to guarantee your crew can determine the location of Blackboard's legendary treasure?

P5. Initially, three non-collinear points, A, B, and C, are marked on the plane. You have a pencil and a double-edged ruler of width 1. Using them, you may perform the following operations:

- Mark an arbitrary point in the plane.
- Mark an arbitrary point on an already drawn line.
- If two points P_1 and P_2 are marked, draw the line connecting P_1 and P_2.
- If two non-parallel lines ℓ_1 and ℓ_2 are drawn, mark the intersection of ℓ_1 and ℓ_2.
- If a line ℓ is drawn, draw a line parallel to ℓ that is at distance 1 away from ℓ (note that two such lines may be drawn).

Prove that it is possible to mark the orthocenter of ABC using these operations.