# Official 2024 Problem Set

**J1.** Centuries ago, the pirate Captain Blackboard buried a vast amount of treasure in a single cell of a $2 \times 4$ grid-structured island. You and your crew have reached the island and have brought special treasure detectors to find the cell with the treasure. For each detector, you can set it up to scan a specific subgrid $[a, b] \times [c, d]$ with $1 \leq a \leq b \leq 2$ and $1 \leq c \leq d \leq 4$. Running the detector will tell you whether the treasure is in the region or not, though it cannot say where in the region the treasure was detected. You plan on setting up $Q$ detectors, which may only be run simultaneously after all $Q$ detectors are ready. What is the minimum $Q$ required to guarantee your crew can determine the location of Blackboard's legendary treasure?

**J2.** Let $n$ be a positive integer. Let

$$I_n = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \min\left(\frac{1}{i}, \frac{1}{j}, \frac{1}{k}\right)$$

and $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$. Determine $I_n - H_n$ in terms of $n$.

**J3.** Let $ABC$ be a triangle with incenter $I$. Suppose the reflection of $AB$ across $CI$ and the reflection of $AC$ across $BI$ intersect at a point $X$. Prove that $XI$ is perpendicular to $BC$.

(The incenter is the point where the three angle bisectors meet.)

**J4.** Jane writes down 2024 natural numbers around the perimeter of a circle. She wants the 2024 products of adjacent pairs of numbers to be exactly the set $\{1!, 2!, \ldots, 2024!\}$. Can she accomplish this?

**J5.** Let $N$ be the number of positive integers with 10 digits $\overline{d_9 d_8 \cdots d_1 d_0}$ in base 10 (where $0 \leq d_i \leq 9$ for all $i$ and $d_9 > 0$) such that the polynomial

$$d_9 x^9 + d_8 x^8 + \cdots + d_1 x + d_0$$

is irreducible in $\mathbb{Q}$. Prove that $N$ is even.

(A polynomial is irreducible in $\mathbb{Q}$ if it cannot be factored into two non-constant polynomials with rational coefficients.)

 