

2023 Canadian Open Mathematics Challenge



Official Competition Booklet

Please completely fill the appropriate circles:

Were you born *on or before* June 30, 2004?

☐ Y

☐ N

Why we ask this: The COMC has an age limit for official participation.

Are you a Canadian Citizen or Permanent Resident of Canada?

☐ Y

☐ N

Why we ask this: We need to know which of our two primary divisions you qualify in: the Canadian Division or the Foreign/International Division.

Your email address: _____

*Why we ask this: If your score on the COMC is high enough, we'll want to contact you for higher level invitation-only competitions such as the Canadian Mathematical Olympiad. **Please print clearly!***

DO NOT PHOTOCOPY BLANK EXAMS

Each page has a unique pre-registered bar code corresponding to a particular student.

Question A1 (4 points)

Ty took a positive number, squared it, then divided it by 3, then cubed it, and finally divided it by 9. In the end he received the same number as he started with.

What was the number?

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question A2 (4 points)

A point with coordinates $(a, 2a)$ lies in the 3rd quadrant and on the curve given by the equation $3x^2 + y^2 = 28$.

Find a .

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question A3 (4 points)

Tanya and Katya made an experiment and obtained two positive real numbers, each of which was between 4 and 100, inclusive. Tanya wrote the two numbers x and y and found their average while Katya wrote the second number twice and took the average of the three numbers x , y , and y .

What is the maximum number by which their results may differ?

Your solution:

Your final answer:

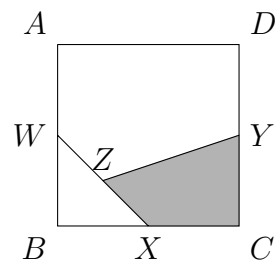
[A correct answer here earns full marks]

Question A4 (4 points)

Square ABCD has side length 10 cm. Points W , X , Y , and Z are the midpoints of segments AB , BC , CD and WX , respectively.

Determine the area of quadrilateral $XCYZ$ (in cm^2).

Your solution:



Your final answer:

[A correct answer here earns full marks]

Question B1 (6 points)

A bug moves in the coordinate plane, starting at $(0,0)$. On the first turn, the bug moves one unit up, down, left, or right, each with equal probability. On subsequent turns the bug moves one unit up, down, left, or right, choosing with equal probability among the three directions other than that of its previous move. For example, if the first move was one unit up then the second move has to be either one unit down or one unit left or one unit right.

After four moves, what is the probability that the bug is at $(2,2)$?

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B2 (6 points)

This month, I spent 26 days exercising for 20 minutes or more, 24 days exercising 40 minutes or more, and 4 days of exercising 2 hours exactly. I never exercise for less than 20 minutes or for more than 2 hours.

What is the minimum number of hours I could have exercised this month?

Your solution:

Your final answer:

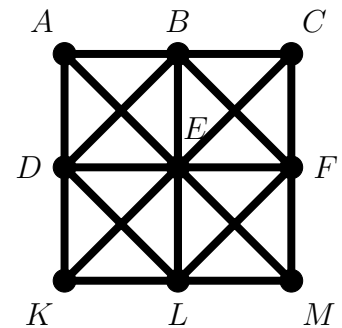
[A correct answer here earns full marks]

Question B3 (6 points)

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A 3×3 grid of 9 dots labelled by A, B, C, D, E, F, K, L , and M is shown in the figure. There is one path connecting every pair of adjacent dots, either orthogonal (i.e. horizontal or vertical) or diagonal. A turtle walks on this grid, alternating between orthogonal and diagonal moves. One could describe any sequence of paths in terms of the letters A, \dots, M . For example, $A - B - F$ describes a sequence of two paths AB and BF .

What is the maximum number of paths the turtle could traverse, given that it does not traverse any path more than once?



Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B4 (6 points)

Consider triangle ABC with angles $\angle BAC = 24^\circ$ and $\angle ACB = 28^\circ$. Point D is constructed such that AB is parallel to CD , $AD = BC$, and AD and BC are not parallel. Similarly, point E is constructed such that AE is parallel to BC , $AB = CE$, and AB and CE are not parallel. Lines DE and AC intersect at point P .

Determine angle $\angle CPE$ (in degrees).

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question C1 (10 points)

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Let F be a function which maps integers to integers by the following rules:

$$\begin{aligned} F(n) &= n - 3 \text{ if } n \geq 1000; \\ F(n) &= F(F(n + 5)) \text{ if } n < 1000. \end{aligned}$$

- (a) Find $F(999)$.
- (b) Show that $F(984) = F(F(F(1004)))$.
- (c) Find $F(84)$.

Your solution:

You **must** show all your work.

Question C1 (continued)

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Question C1 (continued)

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Question C2 (10 points)

- (a) Find the distance from the point $(1, 0)$ to the line connecting the origin and the point $(0, 1)$.
- (b) Find the distance from the point $(1, 0)$ to the line connecting the origin and the point $(1, 1)$.
- (c) Find the distance from the point $(1, 0, 0)$ to the line connecting the origin and the point $(1, 1, 1)$.

Your solution:

You **must** show all your work.

Question C2 (continued)

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Question C2 (continued)

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Question C3 (10 points)

Alice and Bob are playing a game. There are initially $n \geq 1$ stones in a pile. Alice and Bob take turns, with Alice going first. On their turn, Alice or Bob roll a die with numbered faces 1, 1, 2, 2, 3, 3, and take at least one and at most as many stones from the pile as the rolled number on the die. The person who takes the last stone wins.

- (a) If $n = 2$, what is the probability that Alice wins?
- (b) What is the smallest value of n for which Bob is more likely to win than Alice?
- (c) Find all values n for which Bob is more likely to win than Alice.

Your solution:

You **must** show all your work.

Question C3 (continued)

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Question C3 (continued)

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Question C4 (10 points)

For a positive integer n , let $\tau(n)$ be the sum of its positive divisors (including 1 and itself), and let $\phi(n)$ be the number of integers x , $1 \leq x \leq n$, such that x and n are relatively prime. For example, if $n = 18$, then $\tau(18) = 1 + 2 + 3 + 6 + 9 + 18 = 39$ and $\phi(18) = 6$ since the numbers 1, 5, 7, 11, 13, and 17 are relatively prime to 18.

- (a) Prove that $\phi(n)\tau(n) < n^2$ for every positive integer n .
- (b) Determine all positive integers n such that $\phi(n)\tau(n) + 1 = n^2$.
- (c) Prove that there are no positive integers n such that $\phi(n)\tau(n) + 2023 = n^2$.

Your solution:

You **must** show all your work.

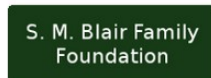
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Question C4 (continued)

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