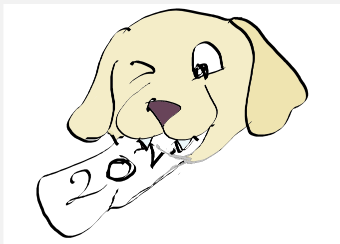
2023 Canada Jay Mathematical Competition

Official Solutions

A competition of the Canadian Mathematical Society.

Part A: 4 marks each

1. Nicholas solved a math homework question and wrote the answer, 2023, on a piece of paper. His dog Curly bit the piece of paper, ate the last digit, and now the paper has 202 written on it. Later in the day, Curly ate the last digit again, and now the paper has 20 written on it. Before going to sleep, Curly bit the paper one more time, eating the last digit, and now the paper has 2 written on it. Find the sum of these four numbers:



$$\begin{array}{r} 2023 \\ 202 \\ 20 \\ +\quad 2 \\ \hline \end{array}$$

(A) 2027 **(B)** 2047 **(C)** 2245 **(D)** 2247

Solution:

$$2023 + 202 + 20 + 2 = 2247$$

Answer: $\boxed{\text{(D)}}$

2. Hanif wants to read every page on Mathematics in the online encyclopedia. There are 952 pages, and on each day Hanif opens and reads exactly one page. If he starts today, Thursday, November 16, 2023, on what day of the week will he open the last page?

(A) Monday **(B)** Tuesday **(C)** Wednesday **(D)** Thursday

Solution: $952 = 7 \times 136\, R\, 0$. This means that it will take a whole number of weeks for Hanif to read all the pages. Since he starts on Thursday, the last day is going to be Wednesday.

Answer: $\boxed{\text{(C)}}$

3. Shoe sizing systems in North America use a standard measurement between sizes based on barleycorns.
One barleycorn is equal to $1/3$ inch. When shoe sizing systems were first created, the largest shoe size was set to 12 for a shoe length of 12 inches. For each decrease of one shoe size the shoe length decreases by one barleycorn. So a size 11 shoe has a shoe length of $11\,2/3$ inches (12 inches $- 1/3$ inches $= 11\,2/3$ inches).
What is the shoe length of a size 7 shoe?

(A) $10\,1/3$ inches. **(B)** 7 inches. **(C)** 5 inches. **(D)** $9\,2/3$ inches.

Solution:

$$12 \text{ inches } - (12-7)\cdot 1/3 \text{ inches } = 10\,1/3 \text{ inches }.$$

Answer: $\boxed{\text{(A)}}$

4. The Kaktovik numerals are a system of numerical digits used by the Alaskan Iñupiat people. Each of the 20 Kaktovik digits consists of up to 3 side-to-side strokes and up to 4 up-and-down strokes. Each side-to-side stroke represents a 5 and each up-and-down stroke represents a 1.
For example, the number 3 consists of three up-and-down strokes: 𝋃, while the number 11 consists of two side-to-side strokes (5+5) and one up-and-down stroke (+1): 𝋋.
The Kaktovik numerals from 0 to 19 are listed below.

0	𝋀	5	𝋅	10	𝋊	15	𝋏
1	𝋁	6	𝋆	11	𝋋	16	𝋐
2	𝋂	7	𝋇	12	𝋌	17	𝋑
3	𝋃	8	𝋈	13	𝋍	18	𝋒
4	𝋄	9	𝋉	14	𝋎	19	𝋓

If we write all Kaktovik numerals from 1 to 15, inclusive, how many side-to-side strokes do we draw?

(A) 𝋋 **(B)** 𝋏 **(C)** 𝋑 **(D)** 𝋒

Solution: Counting, there are 18 side-to-side strokes.

Answer: $\boxed{\textbf{(D)}}$

5. Each of the following figures, not drawn to scale, can be obtained by joining together a few squares.

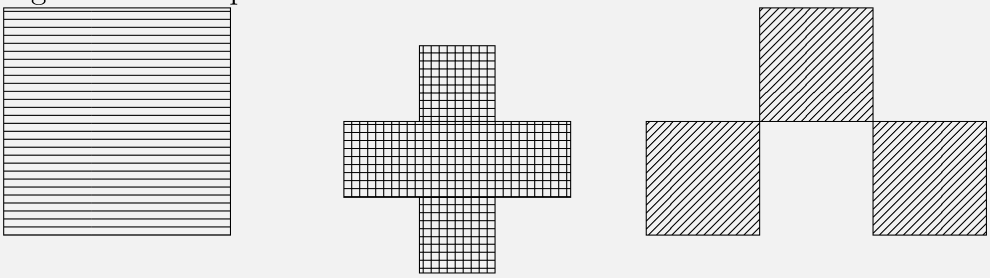


Figure X Figure Y Figure Z

If the three figures have the same perimeter, order them in increasing order of area.

(A) Area of $Y <$ Area of $X <$ Area of Z
(B) Area of $Z <$ Area of $Y <$ Area of X
(C) Area of $Z <$ Area of $X <$ Area of Y
(D) Area of $Y <$ Area of $Z <$ Area of X

Solution: Insert the following points on Figure X:

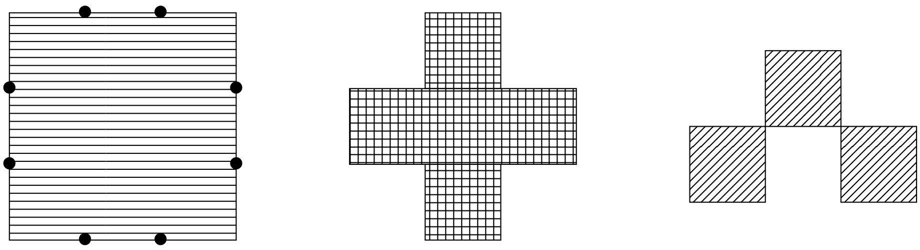


Figure X Figure Y Figure Z

Now, each figure has exactly 12 equal segments on its perimeter, so all the segments in the picture are equal. Let's call the common length of these segments a unit. Then,

$$\text{Area of } X = 9 \text{ units}^2$$
$$\text{Area of } Y = 5 \text{ units}^2$$
$$\text{Area of } Z = 3 \text{ units}^2$$

Answer: **(B)**

Part B: 5 marks each

1. The last digit of

$$15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 + 10 \cdot 9 \cdot 8 \cdot 7 + 6 \cdot 5 \cdot 4 + 3 \cdot 2 + 1$$

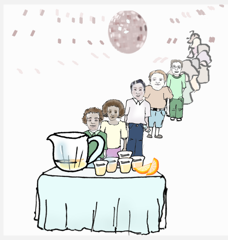
is

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

Solution: Note that in the first three terms there is an even number and a multiple of 5 in the product. Therefore, the first three terms end in 0.

Answer: $\boxed{\textbf{(D)}}$

2. At a school dance there are 76 students wanting orange juice.



There is enough orange juice for 90 students to each have a 200 ml cup filled to exactly three-quarters (75%). By mistake, the teachers handed out to the students 300 ml cups filled to exactly three-quarters (75%). How many students received orange juice?

(A) 60 **(B)** 64 **(C)** 70 **(D)** 76 **(E)** 90

Solution: Note that three 200 ml cups have the same capacity as two 300 ml cups. This means that ninety 200 ml cups have the same capacity as sixty 300 ml cups. Therefore, there was enough juice to fill 60 cups.

Answer: $\boxed{\textbf{(A)}}$

3. Consecutive numbers are ones that follow each other with a difference of 1: for example, 3,4,5,6 are consecutive numbers, as are 12, 13, 14.

The median of some numbers is a certain number separating the top half from the bottom half. For example, the median of $1, 1, 2, \mathbf{3}, 4, 9, 21$ is 3, while the median of $2, 2, \mathbf{4}, \mathbf{6}, 11, 12$ is the average of $4, 6$: that is, the median is 5.

I write down 7 consecutive numbers whose sum is 2023. What is their median?

(A) 286 **(B)** 289 **(C)** 292 **(D)** 2023

(E) No median exists

Solution 1: Since the numbers are consecutive, they are already in order from smallest to largest, so the median is the middle number. If we think of the numbers as $m-3, m-2, m-1, m, m+1, m+2, m+3$ (or we notice a pattern), we see that the sum is $7 \times m$, where m stands in for the middle number of those listed. Through long division, we find that $2023 \div 7 = 289$.

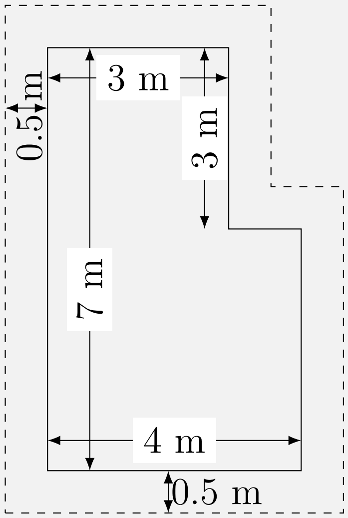
Solution 2: Since the numbers are consecutive, they are already in order from smallest to largest, so the median is the middle number.

Observe that the sets $\{m-3, m-2, m-1, m, m+1, m+2, m+3\}$, $\{m-3, m-2, m, m, m, m+2, m+3\}$, $\{m-3, m, m, m, m, m, m+3\}$ and $\{m, m, m, m, m, m, m\}$ all have the same sum and the same median.

The last set consists of seven numbers, each equal to the median, with sum 2023. This means that the median is $2023 \div 7 = 289$.

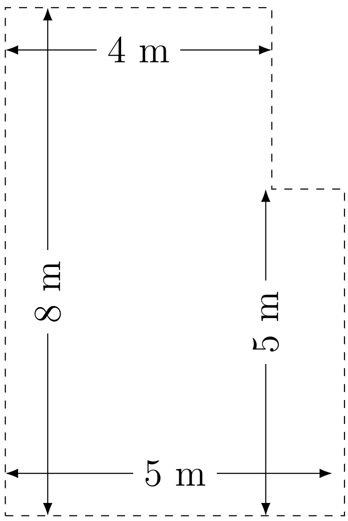
Answer: $\boxed{\textbf{(B)}}$

4. An "L" shaped pool with dimensions as shown in the diagram below requires a "L"-shape safety cover that must be 0.5 m larger than the pool edge on all sides, drawn with dotted lines. What is the area of the pool cover?

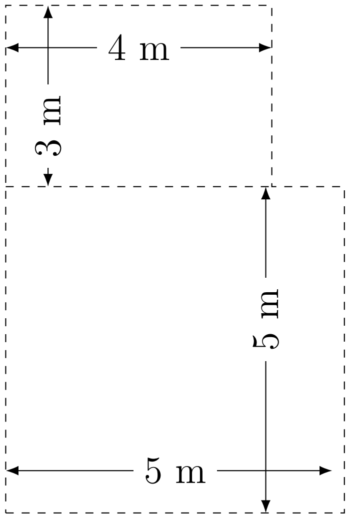


(A) 37 m^2 **(B)** 24 m^2 **(C)** 31.5 m^2 **(D)** 40 m^2 **(E)** 33.75 m^2

Solution 1: The cover has the following dimensions:



It is made up of the following two rectangles, one 5×5 m^2 and the second 4×3 m^2



Therefore, the area is

$$25 + 12 = 37 \text{ m}^2 .$$

Solution 2: The pool is made up of a 3×3 and a 4×4 square. If we cover each of these squares individually, the covers are 4×4 and 5×5 and they overlap on a 4×1 rectangle.

Therefore, the area is

$$16 + 25 - 4 = 37 \text{ m}^2 .$$

Solution 3: The cover is a 8×5 rectangle, with a 3×1 rectangle removed in the top right corner. Therefore, the cover has an area of

$$40 - 3 = 37 \text{ m}^2 .$$

Answer: $\boxed{\textbf{(A)}}$

5. A mail delivery company charges the following for delivering letters and parcels:

- \$ 2.50 for weight up to 60 g.
- \$ 0.50 each extra 10 g or part thereof.

A woman wants to use this company to send a 138 g manuscript, either as a single package or as two or more packages. What is the lowest cost of postage for this weight with this company?

(A) 5.90 **(B)** 6.00 **(C)** 6.50 **(D)** 7.00 **(E)** 7.50

Solution: If she sends one single package, the cost would be

$$2.50 + 8 \times 0.50 = \$\,6.50\,.$$

Sending two packages, she would pay \$ 5.00 for up to 120 g. She would need to pay an extra \$1 for the remaining 18 g. Therefore, with two packages, the lowest cost would be

$$2 \times 2.50 + 2 \times 0.50 = \$\,6.00\,.$$

Note that sending 3 or more packages would cost at least $3 \times 2.5 = \$\,7.50$, which would be more expensive than these two options.

Therefore, the cheapest option would be with 2 packages, and would cost \$ 6.00.

Answer: $\boxed{\textbf{(B)}}$

Part C: 7 marks each

1. Belinda sells watermelons at the local market. She sells a whole watermelon for \$ 7 and half of a watermelon for \$ 4. When a customer purchases a half watermelon, she takes a whole one and cuts it in half for them.
Belinda started one particular day with 20 whole watermelons. At the end of the day, she had made \$ 101 in watermelon sales. What is the greatest number of watermelons that Belinda could have left at the end of the day?

(A) 7 **(B)** 9 **(C)** 4 **(D)** 2 **(E)** 6 **(F)** 1

Solution 1: In order for the number of watermelons left to be the greatest, Belinda must have sold the smallest possible number of watermelons. As two halves bring \$ 8 and a whole watermelon only brings \$ 7, she must have sold as many halves as possible, or, equivalently, as few whole watermelons as possible.

Note also that the half watermelons brought in an even amount of money. Since she made \$ 101, which is odd, she must have sold an odd number of whole watermelons.

We are now looking for the smallest odd number of whole watermelons which works.

If she sold one whole watermelon for \$ 7, then she made \$ 94 from half watermelons, which is not possible.

If she sold three whole watermelons for \$ 21, then she made \$ 80 from half watermelons, meaning she sold 20 half watermelons. Since this works, she sold 3 whole watermelons and 20 halves, meaning 13 watermelons in total. She has 7 watermelons left.

Solution 2: Let x be the number of whole watermelons sold and y be the number of half watermelon sold. Then, we have

$$7x + 4y = 101\,.$$

At the end of the day, Belinda has

$$z = 20 - x - \frac{y}{2}$$

watermelons left.

We have

$$\begin{aligned} 8z &= 160 - 8x - 4y \\ 101 &= 7x + 4y \end{aligned}$$

and hence

$$8z + 101 = 160 - x \Rightarrow z = \frac{59 - x}{8}\,.$$

In order for this to be the greatest possible, x needs to be as small as possible.

Now,

$$x + (6x + 4y) = 101$$

is odd. Since $6x + 4y$ is even, we get that x must be odd. Therefore, x is an odd number. Since we are looking for the smallest possible x let us first check if $x = 1$ works.

If $x = 1$ then

$$7 + 4y = 101 \Rightarrow y = 23.5$$

which is not possible.

Next possibility is $x = 3$. In this case

$$7 \cdot 3 + 4y = 101 \Rightarrow y = 20,$$

which means that Belinda has

$$\frac{59 - 3}{8} = 7$$

watermelons left.

Therefore, Belinda sold 3 whole watermelons, 20 halves and had 7 watermelons left.

Answer: $\boxed{\textbf{(A)}}$

2. The police has 5 suspects for a robbery: Ana, Bob, Chu, Dana and Ezra. During the interrogation, the 5 suspects said:

- Ana: I only commit crimes with Chu
- Bob: Ezra and Dana did it together
- Chu: I did not do it
- Dana: Chu did it
- Ezra: Dana did not do it

Knowing that only one person is guilty, and that at most one suspect is telling the truth, find the guilty person.

(A) Ana **(B)** Bob **(C)** Chu **(D)** Dana **(E)** Ezra

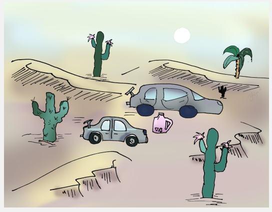
(F) There is more than one possibility.

Solution: Chu and Dana are giving opposite information, hence one of them must be telling the truth. Since only one suspect is telling the truth, Ana, Bob and Ezra must be lying.

As Ezra is lying, it means that Dana is guilty (and Chu is the only one telling the truth).

Answer: $\boxed{\textbf{(D)}}$

3. Two cars, named Lightning McQueen and Sally, are travelling from their garage towards a mountain. At any moment, each car can only carry enough fuel for at most 150 km.



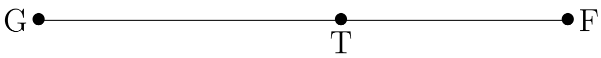
Both cars are consuming fuel at the same rate per kilometer, and the rate is constant no matter which direction they travel.

They start together from the garage, and at some stage, McQueen transfers some fuel to Sally and returns to the garage. If McQueen made it back to the garage, what is the furthest distance Sally can travel away from the garage?

(A) 150 km **(B)** 175 km **(C)** 200 km **(D)** 225 km

(E) $233\frac{1}{3}$ km **(F)** 250 km

Solution: Denote by G the garage, T the point where McQueen transferred the fuel and F the furthest point Sally can travel.



Note that in total, the cars have travel the distance between G and T 3 times and the distance from T to F once.

At point T, Sally can only have enough fuel for 150 km. This means that the distance TF is at most 150 km.

Since each km travelled together uses three times more fuel, to maximize the distance travelled we need to make TF as large as possible. Thus we must have $TF = 150$ km.

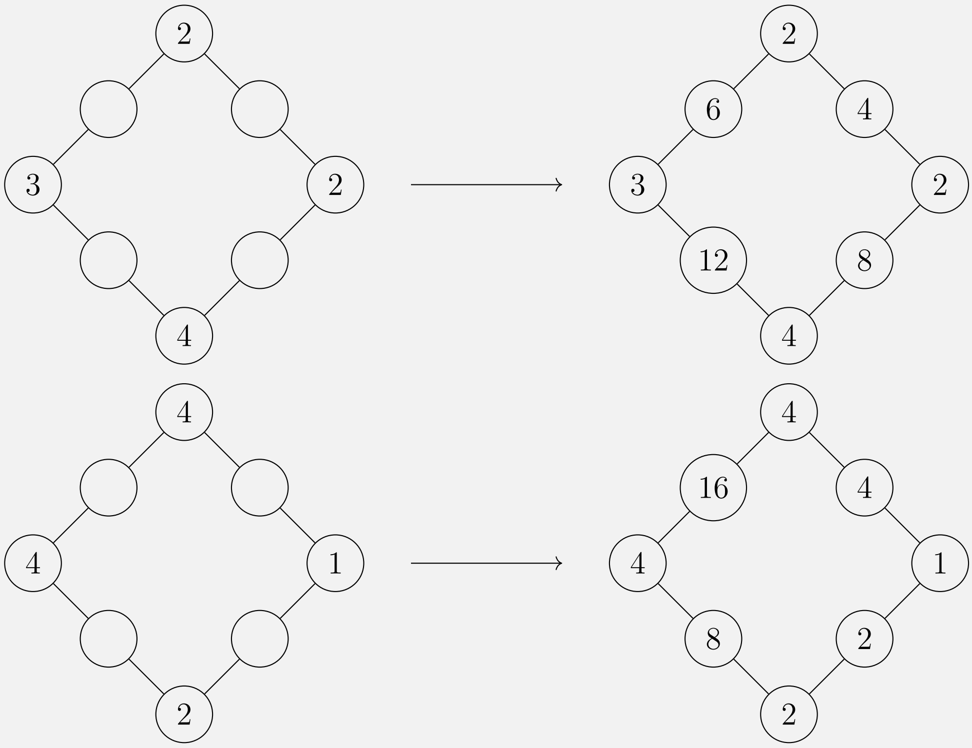
In total the two cars travel at most 300 km. Since Sally travelled alone for 150 km, the other 150 km are the common portion GT. Since this is travelled a total of 3 times, the distance from G to T is at most 50 km.

Therefore, Sally could travel at most $150 + 50 = 200$ km.

Let us note that 200 km is possible. To do this, they travel together 50 km, then McQueen transfers 50 km worth of fuel to Sally and returns to the garage. Sally spent 50 km worth of fuel to travel between points G and T and receives 50 km worth of fuel at point T. Therefore, Sally can travel another 150 km from that point.

Answer: $\boxed{\text{(C)}}$

4. At each corner of a rhombus, I write down a whole number and call it a *corner-number*. I multiply the corner-numbers at the ends of each side and write each product on that side, calling it a side-number. Here are two examples:



When I add all four side-numbers of my rhombus, that sum is an odd number. How many corner-numbers are even?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

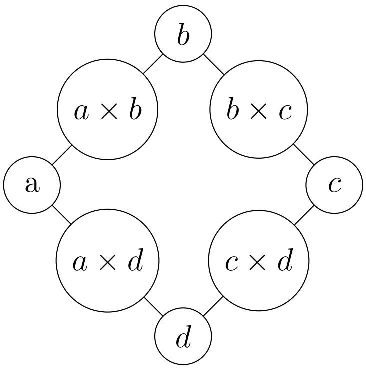
(F) There is more than one possibility.

Solution 1: Let us write o when we want to mention an odd number and e when we want to mention an even number. By the rules of parity (since $o+o$ and $e+e$ would both be even), we have

$$\begin{aligned} o =& o+e+e+e \qquad \textit{or} \\ o =& o+o+o+e. \end{aligned}$$

An even side-number must be next to an even corner-number, and each even corner-number is next to two side-numbers which must both be even, so that second equality is not possible. For the first we know that the two corner-numbers around the o side number must be odd (only $o \times o = o$), so we must have two odd numbers. To make the side-numbers around them even, we must have the other two corner-numbers be even.

Solution 2: Labelling the corner-numbers as a, b, c, d, we note that the sum is $a \times b + b \times c + c \times d + d \times a$ which is $(a + c) \times (b + d)$.



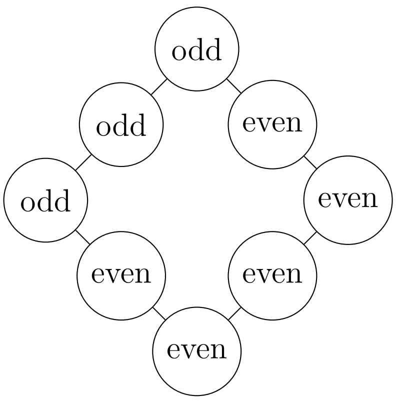
The sum is odd only if $a + c$ and $b + d$ are both odd, and each sum is odd only if one of the summands is odd and the other even. So, we must have two even and two odd corner-numbers.

Solution 3: At least one of the side numbers must be odd, otherwise their sum would be even. Then, the corners of that side must also be odd. Each of the remaining two corners could be odd/even. There are three possibilities.

Case 1: The two remaining corners are also odd. Then, all four side numbers are odd and their sum is even. This case is not possible.

Case 2: One of the remaining corners is odd and one is even. This means that exactly one corner is even. Then, the two side numbers on the sides meeting at that corner are even, and the other two are odd. Again, the sum is even and hence this case is not possible.

Case 3: The remaining two corners are even. If we rotate the rhombus so that the odd numbers are at the top and right corners (which does not change the sum of the four side-numbers), we get the following situation.



This leads to an odd sum. Therefore, this is the only possible scenario.

Answer: $\boxed{\textbf{(C)}}$

5. Meijuan plays *subtraction sudoku*: numbers around the grid show the difference of the nearest 3 numbers in that row or column: the result when subtracting two smaller numbers from the largest.

For example,

	1	−1	−3	
1	8	2	5	
−3	6	4	7	
5	1	3	9	

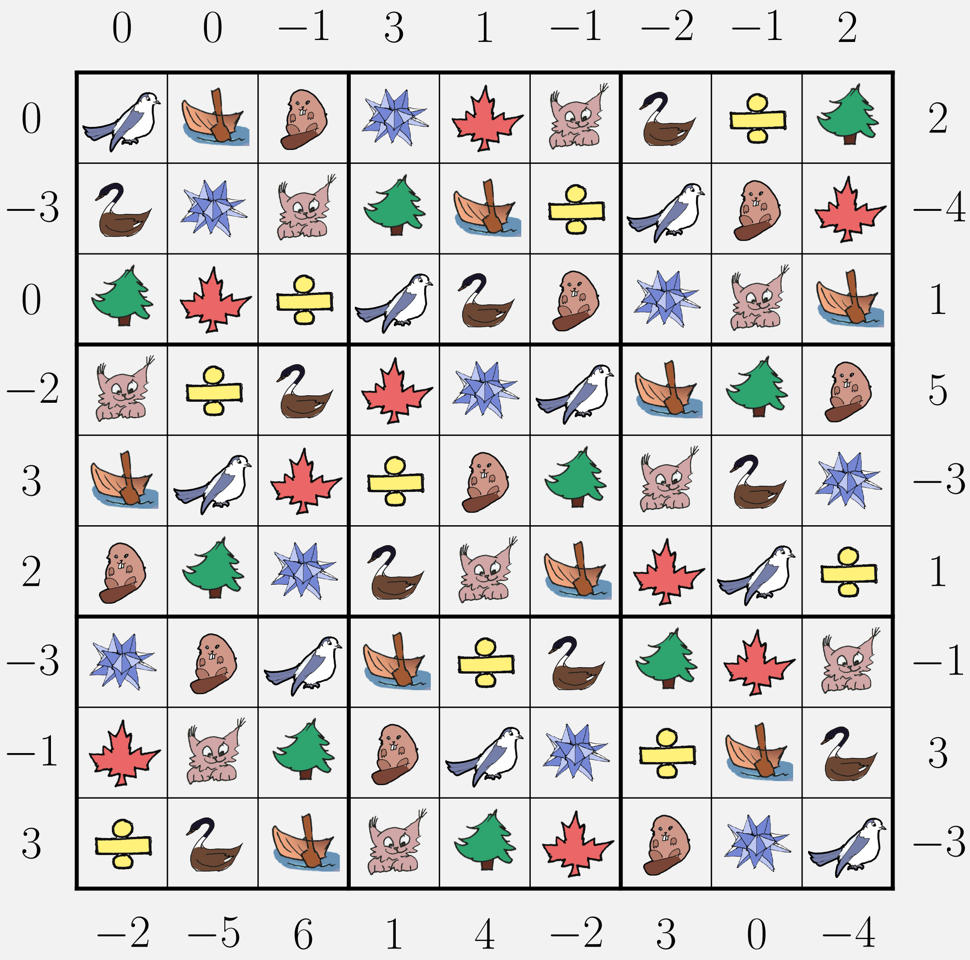
has the rows

$$1 = 8 - 5 - 2$$
$$-3 = 7 - 6 - 4$$
$$5 = 9 - 3 - 1$$

and columns $-1 = 4 - 3 - 2$, etc.

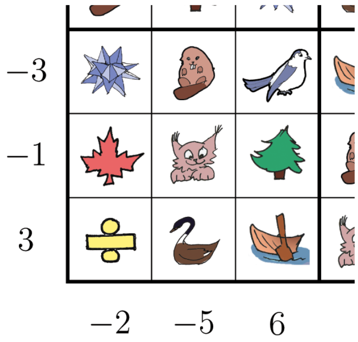
Every row and column and each of the 3-by-3 grid with darker lines has the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, in some order.

Meijuan finished this puzzle, but when she left the room, her little sister stuck pretty stickers over the numbers, so that the same sticker always covers the same digit. What number is covered up by the  sticker?



(A) 1 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 8 **(F)** 9

Solution 1: Working with the lower left 3×3 square, we notice that $6 = 9 - 1 - 2$ is the only solution for its rightmost column.



First, consider any 3×1 row/column containing a 1. Adding the 1 back to the difference operation performed yields the difference between the largest and the smallest number, and hence a positive result. This implies that the output of any 3x1 row/column containing a 1 must be at least 0. This implies that 1 must be in the bottom row. Since we know that 1 is in the rightmost column, we get that  covers 1.

Similarly, consider any 3×1 row/column containing a 2. Adding the 2 back to the difference operation performed yields the difference between the largest and the smallest number, and hence a positive result. This implies that the output of any 3x1 row/column containing a 2 must be at least -1. This implies that in the rightmost column  is 2.

Therefore,  covers a 9.

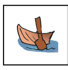
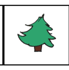
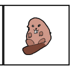
Solution 2: Exactly like in Solution 1, the output of any 3×1 row/column containing a 1 must be at least 0 and the output of any 3×1 row/column containing a 2 must be at least -1. Similarly, the output of a 3×1 row/column containing a 3 must be at least -2, and the output of a 3×1 row/column containing a 4 must be at least -3.

Noting the locations of all the stickers, this gives the following chart of where things cannot be:

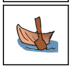
	1	2	3	4	5	6	7	8	9
	✓	✓							
	✗	✗	✗	✗					
	✓	✓							
	✗	✗	✗	✗					
	✗	✗	✗						
	✗	✗							
	✗	✗	✗	✗					
	✗	✗	✗	✗					
	✗	✗	✗	✗					

It follows that  and  cover the numbers 1 and 2 in some order.

Next, working with the lower left 3×3 square, we notice that $6 = 9 - 1 - 2$ is the only solution for its rightmost column. It follows that  covers a 9.

Solution 3: Notice the row  **5** and the column



.

6

Six has the unique solution $6 = 9 - 2 - 1$ in our subtraction sudoku. Five, on the other hand, has two solutions: $5 = \begin{cases} 9 - 3 - 1 \\ 8 - 2 - 1 \end{cases}$.

2 and 1 appear in both:  and  appear in both, so they must correspond to 1 and 2 in some order and the $6 = 9 - 2 - 1$ solution means that  is 9.

Solution 4: We could use some of the insights above to solve the whole sudoku: after all, once a number is known, all its appearances are indicated! Here is the finished solution:

9	1	8	4	5	6	7	3	2
7	4	6	2	1	3	9	8	5
2	5	3	9	7	8	4	6	1
6	3	7	5	4	9	1	2	8
1	9	5	3	8	2	6	7	4
8	2	4	7	6	1	5	9	3
4	8	**9**	1	3	7	2	5	6
5	6	2	8	9	4	3	1	7
3	7	1	6	2	5	8	4	9

Answer: $\boxed{\textbf{(F)}}$

