Canadian Mathematical Society

CANADA JAY MATHEMATICAL COMPETITION
NOVEMBER 16TH, 2023

CJMC.MATH.CA

#CJMC2023

Official Exam Booklet

DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR OR TEACHER PERMITS

1) **Do not open the exam booklet** until instructed to do so by your proctor (supervising teacher).

2) **Only your Student Answer Sheet is marked** – all your answers and your identity must be recorded there. You can write in this booklet if you wish, but it has no value for marking.

3) **Check your name is on the Student Answer Sheet.** If it isn't, make sure your proctor/teacher knows and write your name in.

4) **Make sure your pencil marks are dark enough** (e.g. **2B**).

5) **Fill in the circles completely** on your answer sheet for the choice you make for each question. If you change your answer, erase your mark. *Be sure it is very clear* to the markers if you intend to leave a question unanswered (blank).

6) **Diagrams provided are *not* drawn to scale; they are intended as aids only.**

7) **Points:**
   Questions answered *correctly* get 4, 5 or 7 points for sections A, B, and C, respectively.
   Questions answered *incorrectly* get zero points.
   Questions you leave *blank* get one point. So don't guess if you are really unsure.

8) **Time**: You get a total of 1 hour and 30 minutes to complete the competition.

9) **Scrap paper**: May be used, but only your answer sheet will be marked.

10) **Don't talk about it online or publicly before Monday**. This is to make sure others across Canada or around the world don't get an advantage before they write the exam.

Mobile phones and calculators are NOT permitted.

THIS BOOKLET IS NOT MARKED – WRITE YOUR ANSWERS ON THE ANSWER SHEET INSTEAD

Part A: Each correct answer is worth 4 points. Unanswered questions are worth 1 point each.

Problem A1

Nicholas solved a math homework question and wrote the answer, 2023, on a piece of paper. His dog Curly bit the piece of paper, ate the last digit, and now the paper has 202 written on it. Later in the day, Curly ate the last digit again, and now the paper has 20 written on it. Before going to sleep, Curly bit the paper one more time, eating the last digit, and now the paper has 2 written on it. Find the sum of these four numbers:
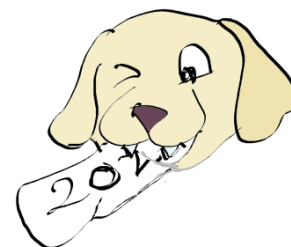
$$
\begin{array}{r}
2023 \\
202 \\
20 \\
+\quad 2 \\
\hline
\end{array}
$$

A. 2027   B. 2047   C. 2245   D. 2247
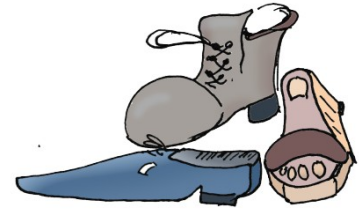
Problem A2

Hanif wants to read every page on Mathematics in the online encyclopedia. There are 952 pages, and on each day Hanif opens and reads exactly one page. If he starts today, Thursday, November 16, 2023, on what day of the week will he open the last page?

A. Monday   B. Tuesday   C. Wednesday   D. Thursday

Problem A3

Shoe sizing systems in North America use a standard measurement between sizes based on barleycorns.

One barleycorn is equal to $1/3$ inch. When shoe sizing systems were first created, the largest shoe size was set to 12 for a shoe length of 12 inches. For each decrease of one shoe size the shoe length decreases by one barleycorn. So a size 11 shoe has a shoe length of $11^2/3$ inches (12 inches $- 1/3$ inches $= 11^2/3$ inches).

What is the shoe length of a size 7 shoe?

A. $10^1/3$ inches.   B. 7 inches.   C. 5 inches.   D. $9^2/3$ inches.

Problem A4

The Kaktovik numerals are a system of numerical digits used by the Alaskan Iñupiat people. Each of the 20 Kaktovik digits consists of up to 3 side-to-side strokes and up to 4 up-and-down strokes. Each side-to-side stroke represents a 5 and each up-and-down stroke represents a 1.

For example, the number 3 consists of three up-and-down strokes: ꟍ, while the number 11 consists of two side-to-side strokes (5+5) and one up-and-down stroke (+1): ꚕ.

The Kaktovik numerals from 0 to 19 are listed below.

0	ꚙ	5	＼	10	＞	15	≶
1	＼	6	ꚕ	11	ꚕ	16	ꚕ
2	ꯟ	7	ꯟ	12	ꯟ	17	ꯟ
3	ꟍ	8	ꟍ	13	ꟍ	18	ꟍ
4	W	9	W	14	W	19	W

If we write all Kaktovik numerals from 1 to 15, inclusive, how many side-to-side strokes do we draw?

A. ꚕ   B. ≶   C. ꯟ   D. ꟍ

Problem A5

Each of the following figures, not drawn to scale, can be obtained by joining together a few squares.
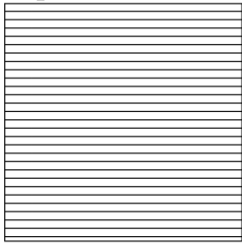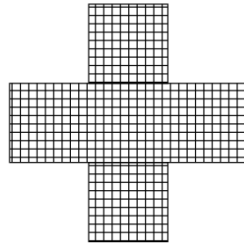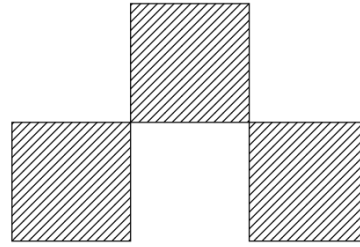

Figure X   Figure Y   Figure Z

If the three figures have the same perimeter, order them in increasing order of area.

A. Area of Y < Area of X < Area of Z B. Area of Z < Area of Y < Area of X
C. Area of Z < Area of X < Area of Y D. Area of Y < Area of Z < Area of X

Part B: Each correct answer is worth 5 points. Unanswered questions are worth 1 point each.

Problem B1

The last digit of

$$15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 + 10 \cdot 9 \cdot 8 \cdot 7 + 6 \cdot 5 \cdot 4 + 3 \cdot 2 + 1$$

is

A. 1   B. 3   C. 5   D. 7   E. 9

Problem B2

At a school dance there are 76 students wanting orange juice.

There is enough orange juice for 90 students to each have a 200 ml cup filled to exactly three-quarters (75%). By mistake, the teachers handed out to the students 300 ml cups filled to exactly three-quarters (75%). How many students received orange juice?

A. 60   B. 64   C. 70   D. 76   E. 90

Problem B3

Consecutive numbers are ones that follow each other with a difference of 1: for example, 3,4,5,6 are consecutive numbers, as are 12, 13, 14.

The median of some numbers is a certain number separating the top half from the bottom half. For example, the median of $1, 1, 2, \mathbf{3}, 4, 9, 21$ is 3, while the median of $2, 2, \mathbf{4}, \mathbf{6}, 11, 12$ is the average of $4, 6$: that is, the median is 5.

I write down 7 consecutive numbers whose sum is 2023. What is their median?

A. 286   B. 289   C. 292   D. 2023   E. No median exists

Problem B4

An "L" shaped pool with dimensions as shown in the diagram below requires a "L"-shape safety cover that must be 0.5 m larger than the pool edge on all sides, drawn with dotted lines. What is the area of the pool cover?
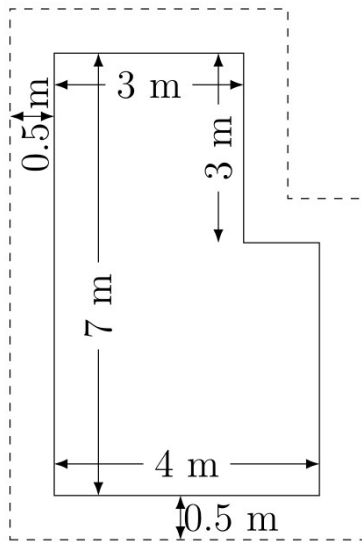
A. 37 m^2 B. 24 m^2 C. 31.5 m^2 D. 40 m^2 E. 33.75 m^2

Problem B5

A mail delivery company charges the following for delivering letters and parcels:

- $ 2.50 for weight up to 60 g.

- $ 0.50 each extra 10 g or part thereof.

A woman wants to use this company to send a 138 g manuscript, either as a single package or as two or more packages. What is the lowest cost of postage for this weight with this company?

A. $ 5.90   B. $ 6.00   C. $ 6.50   D. $ 7.00   E. $ 7.50

Part C: Each correct answer is worth 7 points. Unanswered questions are worth 1 point each.

Problem C1

Belinda sells watermelons at the local market. She sells a whole watermelon for $ 7 and half of a watermelon for $ 4. When a customer purchases a half watermelon, she takes a whole one and cuts it in half for them.

Belinda started one particular day with 20 whole watermelons. At the end of the day, she had made $ 101 in watermelon sales. What is the greatest number of watermelons that Belinda could have left at the end of the day?

  A. 7   B. 9   C. 4   D. 2   E. 6   F. 1

Problem C2

The police has 5 suspects for a robbery: Ana, Bob, Chu, Dana and Ezra. During the interrogation, the 5 suspects said:

- Ana: I only commit crimes with Chu

- Bob: Ezra and Dana did it together

- Chu: I did not do it

- Dana: Chu did it

- Ezra: Dana did not do it

Knowing that only one person is guilty, and that at most one suspect is telling the truth, find the guilty person.

A. Ana   B. Bob   C. Chu   D. Dana   E. Ezra   F. There is more than one possibility.

Problem C3

Two cars, named Lightning McQueen and Sally, are traveling from their garage towards a mountain. At any moment, each car can only carry enough fuel for at most 150 km. Both cars are consuming fuel at the same rate per kilometer, and the rate is constant no matter which direction they travel.

They start together from the garage, and at some stage, McQueen transfers some fuel to Sally and returns to the garage. If McQueen made it back to the garage, what is the furthest distance Sally can travel away from the garage?

     A. 150 km   B. 175 km   C. 200 km   D. 225 km   E. $233\frac{1}{3}$ km   F. 250 km

Problem C4

At each corner of a rhombus, I write down a whole number and call it a *corner-number*. I multiply the corner-numbers at the ends of each side and write each product on that side, calling it a *side-number*. Here are two examples:
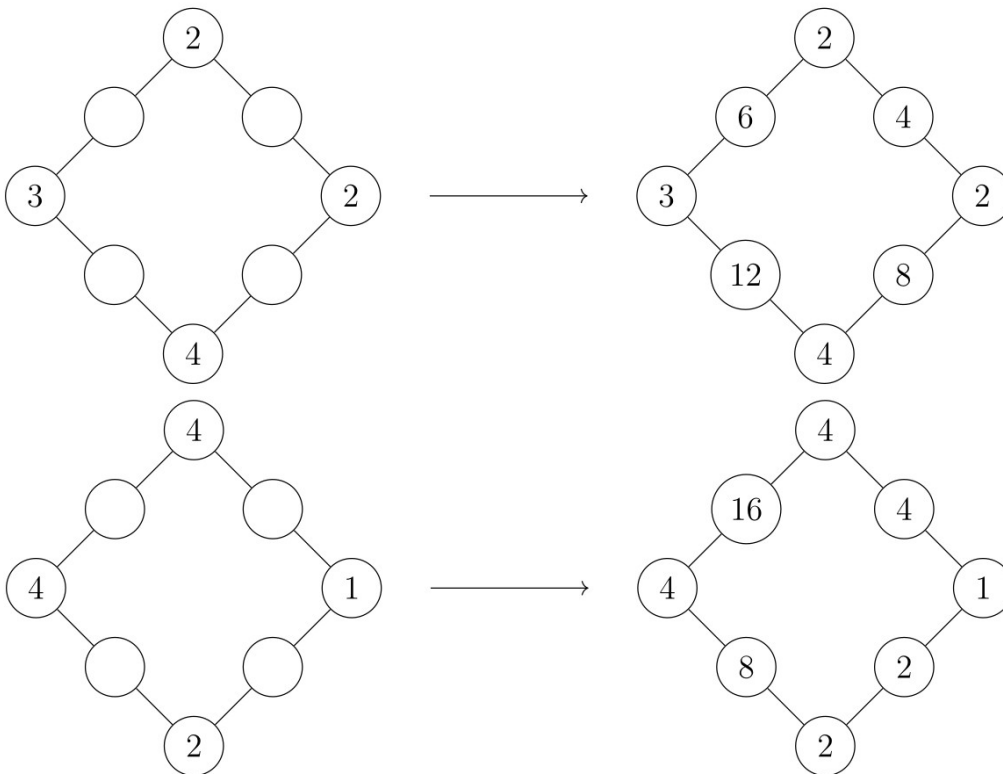
When I add all four side-numbers of my rhombus, that sum is an odd number. How many corner-numbers are even?

     A. 0   B. 1   C. 2   D. 3   E. 4   F. There is more than one possibility.

Problem C5

Meijuan plays *subtraction sudoku*: numbers around the grid show the difference of the nearest 3 numbers in that row or column: the result when subtracting two smaller numbers from the largest. For example,

$$\begin{array}{c|ccc} & 1 & -1 & -3 \\ \hline 1 & 8 & 2 & 5 \\ -3 & 6 & 4 & 7 \\ 5 & 1 & 3 & 9 \end{array}$$

has the rows

$$1 = 8 - 5 - 2$$
$$-3 = 7 - 6 - 4$$
$$5 = 9 - 3 - 1$$

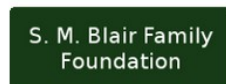and columns $-1 = 4 - 3 - 2$, etc.

Every row and column and each of the 3-by-3 grid with darker lines has the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, in some order.

Meijuan finished this puzzle, but when she left the room, her little sister stuck pretty stickers over the numbers, so that the same sticker always covers the same digit. What number is covered up by the 🐦 sticker?

Top clues: 0 0 −1 3 1 −1 −2 −1 2

Left clues: 0, −3, 0, −2, 3, 2, −3, −1, 3

Right clues: 2, −4, 1, 5, −3, 1, −1, 3, −3

Bottom clues: −2 −5 6 1 4 −2 3 0 −4

A. 1   B. 2   C. 4   D. 5   E. 8   F. 9

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