2023 Samuel Beatty Report

64th International Mathematical Olympiad in Chiba, Japan

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Probably the best-known math competition, the International Mathematical Olympiad is an annual math competition to which each country sends their 6 best high school-aged math contestants to solve 6 problems over 2 days to the best of their ability. The 64th IMO, held this year in Chiba, Japan, had 112 countries send a total of 618 students to compete against and get to know each other.

The team selection process this year was different from previous years, with Canada moving away from relying on the USAMO. First, based on a combination of results from last year's olympiads and this year's Canadian Open Mathematics Challenge, students qualified to write the Canadian Mathematical Olympiad and Asian Pacific Mathematics Olympiad. In previous years, these students would have also written the United States of America Mathematical Olympiad; instead, this year, the students whose combined scores on the CMO and APMO were high enough wrote the Canadian Team Selection Test, whose problems were selected from the 2022 IMO Shortlist. Then, based on the combined score from those 3 contests, 6 people qualified for the IMO team: Warren Bei (Vancouver, BC) and Kaixin Wang (Vancouver, BC), who had been on the team previously, and Ming Yang (Arizona, USA), Marvin Mao (New Jersey, USA), myself (Ottawa, ON), and Haozhe Yang (Saskatoon, SK). We also had 3 team leaders who came along with us to the camp and the IMO: Antonio Molina Lovett, as leader, Caleb Ji, as deputy leader, and Howard Halim, as observer.

Usually, Canada hosts a summer training camp for the IMO team. However, this year, along with over 20 other countries, Canada's team was invited to attend the International Mathematics Summer Camp in Beijing, China, at the Beijing Institute of Mathematical Sciences and Applications. Here, along with the Brazilian and Saudi Arabian teams, we had lectures most mornings from instructors and leaders from various countries on topics such as algebraic integers, conics in olympiad geometry, generating functions, and inequalities. Each of these lectures was approximately 1.5 hours long, and consisted of a leader or instructor lecturing, and often some time to solve problems from handouts alone or with each other.

In the afternoons, we would take mock 4.5-hour contests with problems only from one of the four math olympiad subjects -- algebra, combinatorics, geometry, and number theory -- and then

grade each others' tests the next day. Additionally, at the end of the camp, there was a two-day mock IMO, with problems contributed by leaders from various countries. In the evenings, we would often play card games with the leaders, and occasionally with members of other teams as well. We also had the chance to visit Tsinghua University and a section of the Great Wall.

We left for and arrived at Chiba on July 6th, a day before the opening ceremony. We met some members of other teams over meals that day, and we went to a local store to buy supplies for the IMO such as pens, rulers, and lots of chocolate to keep us going during the contest. The opening ceremony was held the next day. It included speeches from people such as Japan's Minister of Education, Culture, Sports, Science and Technology, the president of the IMO board, and the chairman of the organising committee. It also had a Japanese drumming performance and a performance by a brass band. However, perhaps the highlight of the ceremony was getting to see each team come up to the stage and present themselves in their own way, with team mascots, flags, and traditional costumes. After the opening ceremony, we went back to the hotel to eat, rest, and get a good night's sleep to prepare for the contest the next day.

The IMO consists of two 4.5-hour days of problem-solving. Each day, there are 3 problems in roughly increasing order of difficulty; typically, a fairly strong team such as Canada's is expected to get 6 correct solutions to Problems 1 and 4, have most but perhaps not all members solve Problems 2 and 5, and have at least a few solves on Problems 3 and 6 -- there is, however, a fair bit of variance in the problem difficulty. The problem selection process is a multi-step process: first, a problem selection committee will select around 7-9 problems for each subject from proposed problems sent by participant countries. Then, a few days before the contest, leaders from each country will vote on which problems should actually go on the contest, with a goal of making sure that the difficulties are correct and the problems are relatively evenly distributed across the four subject areas, as well as simply choosing high-quality problems for the contest. The remaining problems are then given to the leaders to use for next year's team selection process and training.

The first day, upon beginning the contest, we were immediately disappointed by the problem distribution: collectively, our best subject of the four was probably combinatorics, but there were no combinatorics problems to be seen. Additionally, our collectively worst subject was geometry, with me and Kaixin being essentially incapable of solving an olympiad geometry problem, and Problem 2 was a geometry problem. However, given the problem distribution, the first day of the contest went quite well.

The first problem, a number theory problem about divisors of a number dividing each other in sequence, was easy, even for an IMO Problem 1, and we all solved it relatively quickly, achieving 42/42 points. The second problem was, compared to past years, a bit on the easy side; however, due to our relative lack of geometry skill, we scored significantly worse than other

teams at our level on this problem, only managing 29/42 possible points, with 4 solves from Warren, Ming, Marvin, and Haozhe, and 1 partial mark from me. However, we were fairly happy with that afterwards; neither me nor Kaixin expected to get any points on geometry, so 29 was better than we had dared to hope. The third problem went well; it was an algebra problem about a recursive sequence based on a polynomial with integer coefficients. In absolute terms, it was fairly difficult, but it was quite easy for an IMO Problem 3, and we achieved 3 solves by Warren, Kaixin, and me, as well as 3 partial marks from Marvin and 1 from Ming for a total of 25/42 points; this was about average for teams at our level. These partial marks would turn out to be very important later on.

The second day went very well. Problem 4 was an algebra problem about an inequality on a sequence with a condition that the sequence had to only take on integer values; it was hard for a Problem 4, and it took some of us some time, but we all solved it, getting 42/42 points. Problem 5 was a combinatorics problem about “ninja paths” in “Japanese triangles” -- based on most countries’ solve rates, fairly difficult for a Problem 5 -- and it was where our team’s strengths really shone. We managed to get 5 solves by Warren, Ming, Marvin, Kaixin, and me, as well as 3 very important partial marks from Haozhe, for a total of 38/42 points, and were skeptical of its supposed difficulty until we saw other teams’ scores. This was better than many top teams such as the American, South Korean, and Romanian teams’ performances on that problem, tying for second-best score on that problem with Taiwan (behind, of course, China), and was what really gave us the edge in the end. Problem 6 was an unusual geometry problem with equilateral triangles -- an uncommon sight in olympiad geometry problem statements -- and was very hard, even for a Problem 6. However, we still did relatively well. Ming managed to get 2 partial marks for proving some important claims, and Warren managed 5 points out of 7 with an analytic solution involving moving points -- reportedly very messy and painful, but it got the job done for a total of 7/42 points for the team. In fact, only 6 people in the entire contest managed to get all 7 points on the problem, and only 5 more managed to get at least 5 points. All things considered, the day went extremely well for us; if one were to just consider the scores for that day, Canada would have placed 4th.

Over the next couple of days, we spent some time sightseeing, seeing parts of Tokyo and visiting Tokyo Disneyland. However, while the team members got to have fun, the leaders were arguing tooth and nail for every partial mark. While each problem has a specific grading rubric to assign a set amount of points for each successfully proved claim, leaders will engage in a process called coordination, where they make the case for why their team members deserve as many points as they can soundly argue for, and there is often leeway in exactly how many points a contestant can receive. This can entail explaining how a badly written but ultimately correct solution has all the correct claims written down even if they are hard to understand, or arguing for how a contestant must have obviously known what ended up going unsaid, or a million other things. For us, there were two major points of contention: Marvin’s solution to Problem 3, in which we

were arguing that he should receive 5 points while the graders thought he should only receive 3, and Warren's analytic solution to Problem 6, in which we were trying to make the case for as many as 6 points while the some graders were arguing for as low as 0 points. Ultimately, while the leaders were unable to budge the graders on Marvin's Problem 3 solution, they did manage to convince the graders to give Warren 5 points on Problem 6, which would end up making a huge difference.

Finally, on the second night of coordination and the night before the closing ceremony, some scores were posted online and on boards in the hotel. However, these scores were incomplete to try and build suspense. We spent hours fishing for rumours and quizzing other teams on the scores they knew until we finally determined, with some confidence, other teams' scores, and realised that we had come 5th with 183 points, tying Canada's best performance at the IMO! It was very close -- the 6th place team, Japan, was only 2 points behind us, so those partial marks the leaders had spent hours arguing for made all the difference in the end.

The next day was the closing ceremony, where we got to see medals presented. In absolute terms, medals went well for us: Warren got a gold medal; Ming, Marvin, myself, and Kaixin got a silver medal; and Haozhe got a bronze medal. However, the medal cutoffs were disappointing. While this IMO was not as easy as last year's in absolute terms, the cutoffs were still higher than usual, perhaps due to how many people solved Problems 2 and 3. Marvin and Ming both ended up being one point below the cutoff for a gold medal -- 32 points -- and Haozhe ended up being one point below the cutoff for a silver medal -- 25 points. However, Warren, Ming, and Marvin are still eligible for the IMO, and I hope they get a chance to earn many more gold medals for Canada!

Overall, the experience was amazing -- everyone who does a lot of math olympiads dreams of being able to attend the IMO, and it was incredible to realise this dream and participate in the 64th International Mathematical Olympiad. I'm very grateful for all of the work that the leaders, the instructors, the Canadian Mathematical Society, and the IMO organisers put in to make this happen, and for sponsors such as Jane Street and the Samuel Beatty Fund. Thank you very much, and I hope many more people get to have this opportunity in the future.