1) **Do not open the exam booklet** until instructed to do so by your proctor (supervising teacher).

2) **Only your Student Answer Sheet is marked** – all your answers and your identity must be recorded there. You can write in this booklet if you wish, but it has no value for marking.

3) **Make sure your pencil marks are dark enough** and your writing is neat.

4) Please **fill in the circles completely** on your answer sheet for the choice you make for each question. If you change your answer, erase your mark. **Be sure it is very clear** to the markers if you intend to leave a question unanswered (blank).

5) **Don’t talk about it online or publicly**: The questions and solutions of the exam must not be publicly discussed or shared online **before Monday**. This is to make sure others across Canada or around the world don’t get an advantage before they write the exam.

6) **Length**: You get a total of 1 hour and 30 minutes to complete this fifteen-question exam.

7) **Scores**:
   - Questions you get **correct** get 4, 5 or 7 points for sections A, B, and C, respectively.
   - Questions you answer **wrong** get zero points.
   - Questions you leave **blank** get one point. So don’t guess if you are really unsure.

8) **Diagrams** provided are **not** drawn to scale; they are intended as aids only.

9) **Scrap paper**: May be used, but only your answer sheet will be marked.

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**THIS BOOKLET IS NOT MARKED – WRITE YOUR ANSWERS ON THE ANSWER SHEET INSTEAD**
Part A: Each correct answer is worth 4 points. Unanswered questions are worth 1 point each.

Question #1: If \( N + 2N + 3N + 4N = 10 + 20 + 30 + 40 \), what is the value of \( N \)?
(a) 1     (b) 10     (c) 100     (d) 1000

Question #2: Of the four numbers \( 2^5, 3^4, 4^3, 5^2 \), which is the largest number?
(a) \( 2^5 \)     (b) \( 3^4 \)     (c) \( 4^3 \)     (d) \( 5^2 \)

Question #3: What is the value of the expression \( \frac{6}{1000} + \frac{7}{100} + \frac{8}{10} + \frac{9}{1} \)?
(a) 0.6789     (b) 0.9876     (c) 6.789     (d) 9.876

Question #4: Let \( x, y, \) and \( z \) be positive integers for which
\[
\begin{align*}
x + 2y &= 60 \\
y + 2z &= 70 \\
z + 2x &= 110
\end{align*}
\]
What is the value of \( x + y + z \)?
(a) 80     (b) 100     (c) 120     (d) 140

Question #5: There are 6 permutations (i.e., rearrangements) of the word EAT, namely
\[ \{ \text{AET, ATE, EAT, ETA, TAE, TEA} \} \]
There are 24 permutations of the word LYNX. Suppose we write these 24 permutations in alphabetical order, starting with the 1st word (LNXY) and ending with the 24th word (YXNL).

What is the 11th word we will write down?
(a) NXYL     (b) NYXL     (c) NYLX     (d) XLNY
**Part B:** Each correct answer is worth 5 points. Unanswered questions are worth 1 point each.

**Question #6:** Let $T$ be the area of an equilateral triangle with side length 12 units, and let $H$ be the area of a regular hexagon with side length 4 units.

What is the value of the ratio $\frac{T}{H}$?

(a) $\frac{2}{3}$  (b) 1  (c) $\frac{4}{3}$  (d) $\frac{3}{2}$  (e) 2

**Question #7:** There are 20 students in a Physical Education class. Each of these 20 students is wearing a shirt that is either red or white, and is wearing shoes that are either red or white.

You are given the following information.

(i) Exactly 10 students are wearing a red shirt.
(ii) Exactly 12 students are wearing red shoes.
(iii) Exactly 14 students are wearing a shirt and shoes of the same colour.

How many students are wearing a red shirt and white shoes?

(a) 2  (b) 4  (c) 6  (d) 8  (e) Impossible to determine from the given information

**Question #8:** Let $A$ be the vertex of the parabola $y = x^2 - 2$ and let $C$ be the vertex of the parabola $y = -\frac{1}{3}x^2 + 10$. Let $B$ and $D$ be the two points where these two parabolas intersect.

What is the area of the quadrilateral $ABCD$?

(a) 18  (b) 24  (c) 36  (d) 48  (e) 72
**Question #9:** Let $ABCD$ be a quadrilateral with coordinates $(2, 2)$, $(14, 2)$, $(16, 0)$, and $(0, 0)$. This quadrilateral is inscribed in a circle.

![Diagram of a quadrilateral inscribed in a circle]

What is the area of this circle?

(a) $61\pi$  (b) $64\pi$  (c) $79\pi$  (d) $81\pi$  (e) $100\pi$

**Question #10:** Consider the sequence $t_1, t_2, t_3, \ldots, t_{15}, t_{16}, t_{17}$. This sequence has the following three properties.

(i) Each of the 17 integers from 1 to 17 appears exactly once in the sequence.

(ii) The sum of each pair of consecutive terms is a perfect square. (For example, $t_1 + t_2$ is a perfect square, $t_2 + t_3$ is a perfect square, and so on.)

(iii) $t_1 = 17$.

What is the value of $t_5$?

(a) 3  (b) 6  (c) 10  (d) 13  (e) 15
Part C: Each correct answer is worth 7 points. Unanswered questions are worth 1 point each.

Question #11: Alphonse drives to Beryl’s house, intending to arrive at a certain time. There is no traffic on the road between the two houses, and so Alphonse can drive at a constant speed.

If Alphonse drives at \( x \) kilometres per hour, then he will arrive 7 minutes early.
If Alphonse drives at \( y \) kilometres per hour, then he will arrive 7 minutes late.
If Alphonse drives at 72 kilometres per hour, then he will arrive exactly on time.

If \( x \) and \( y \) are positive integers, what is the smallest possible value of \( x + y \)?

(a) 144  (b) 147  (c) 150  (d) 156  (e) 162  (f) 180

Question #12: Suppose we remove two diagonally-opposite corner squares from an 8 by 8 board, as shown in the diagram below.

![Diagram](image)

How many squares and rectangles, of all sizes, appear on this modified board?

(a) 1127  (b) 1158  (c) 1159  (d) 1169  (e) 1200  (f) 1296
**Question #13:** You are given a biased coin, where Heads comes up with probability \( \frac{2}{3} \) and Tails comes up with probability \( \frac{1}{3} \).

You play a game where you start with 0 points. Each time you flip Heads, you add 2 points to your score. Each time you flip Tails, you add 1 point to your score.

If you reach a total of *exactly* \( n \) points, then you win. However, if you *go over* \( n \) points, then you lose.

Let \( P_n \) be the probability that you win the game with a target score of \( n \) points. For example, \( P_2 = \frac{7}{9} \) because you win the game by flipping H (probability \( \frac{2}{3} \)) or TT (probability \( \frac{1}{9} \)) but lose by flipping TH (probability \( \frac{2}{9} \)).

Determine the value of \( P_8 \), rounded to three decimal places.

(a) 0.576  
(b) 0.584  
(c) 0.592  
(d) 0.600  
(e) 0.608  
(f) 0.616

**Question #14:** Let \( A, B, C, D, E, F \) be six points equally spread out around a circle. Draw all 15 edges connecting two of these six points. Aponi picks 8 of these 15 edges and colours them red; the remaining 7 edges are then coloured blue.

Consider all triangles that can be formed from three of these six points. For each triangle that has only red edges, Aponi scores 1 point. For each triangle that has only blue edges, Aponi scores 2 points. For all other triangles (e.g. a triangle with two red edges and one blue edge), Aponi scores 0 points.

Let \( X \) be the maximum score that Aponi can obtain, and let \( Y \) be the minimum score that Aponi can obtain.

What is the value of \( X - Y \)?

(a) 5  
(b) 6  
(c) 7  
(d) 8  
(e) 9  
(f) 10
**Question #15:** Let $\triangle ABC$ be an isosceles triangle with $AB = AC$ and $\cos \angle A = \frac{1}{N}$, where $N \geq 2$ is a positive integer.

Let $O$ be the circumcentre of the triangle and let $I$ be the incentre of the triangle.

There is one positive integer $N$ with $289 < N < 2023$ for which $\frac{IA}{IO}$ is a rational number.

What is the last digit of $N$?

(a) 0  (b) 1  (c) 2  (d) 4  (e) 5  (f) 6
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