

Canadian Junior Mathematical Olympiad 2023



A competition of the Canadian Mathematical Society.

Official Problem Set

- P1. Let a and b be non-negative integers. Consider a sequence s_1, s_2, s_3, \dots such that $s_1 = a$, $s_2 = b$, and $s_{i+1} = |s_i - s_{i-1}|$ for $i \geq 2$. Prove that there is some i for which $s_i = 0$.
- P2. An acute triangle is a triangle that has all angles less than 90° (90° is a Right Angle). Let ABC be a right-angled triangle with $\angle ACB = 90^\circ$. Let CD be the altitude from C to AB , and let E be the intersection of the angle bisector of $\angle ACD$ with AD . Let EF be the altitude from E to BC . Prove that the circumcircle of BEF passes through the midpoint of CE .
- P3. William is thinking of an integer between 1 and 50, inclusive. Victor can choose a positive integer m and ask William: “does m divide your number?”, to which William must answer truthfully. Victor continues asking these questions until he determines William’s number. What is the minimum number of questions that Victor needs to guarantee this?
- P4. There are 20 students in a high school class, and each student has exactly three close friends in the class. Five of the students have bought tickets to an upcoming concert. If any student sees that two or more of their three close friends have bought tickets, then they will buy a ticket too.
- Is it possible that the entire class buys tickets to the concert?
- (Assume that friendship is mutual; if student A is close friends with student B , then B is close friends with A .)
- P5. Let ABC be an acute triangle with altitudes AD , BE , and CF meeting at H . The circle passing through points D , E , and F meets AD , BE , and CF again at X , Y , and Z respectively. Prove the following inequality:

$$\frac{AH}{DX} + \frac{BH}{EY} + \frac{CH}{FZ} \geq 3.$$

Important!

Please do not discuss this problem set online for at least 24 hours!
