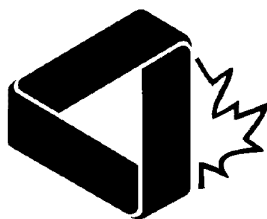


A TASTE OF MATHEMATICS



AIME-T-ON LES MATHÉMATIQUES

Volume / Tome XI

PROBLEMS FOR
JUNIOR MATHEMATICS LEAGUES

Bruce L.R. Shawyer

Memorial University of Newfoundland

Bruce B. Watson

Memorial University of Newfoundland

The ATOM series

The booklets in the series, **A Taste of Mathematics**, are published by the Canadian Mathematical Society (CMS). They are designed as enrichment materials for high school students with an interest in and aptitude for mathematics. Some booklets in the series will also cover the materials useful for mathematical competitions at national and international levels.

La collection ATOM

Publiés par la Société mathématique du Canada (SMC), les livrets de la collection Aime-t-on les mathématiques (ATOM) sont destinés au perfectionnement des étudiants du cycle secondaire qui manifestent un intérêt et des aptitudes pour les mathématiques. Certains livrets de la collection ATOM servent également de matériel de préparation aux concours de mathématiques sur l'échiquier national et international.

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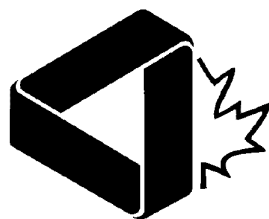
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Foreword

This volume is similar to previous publications on *Problems for Mathematics Leagues*. See inside back cover for details.

The problems were originally designed for mathematics competitions aimed at students in the Junior High Schools (grades 7, 8 and 9) in Newfoundland and Labrador.

While we have tried to make the text as correct as possible, some mathematical and typographical errors, for which we accept full responsibility, might remain. We would be grateful to any reader drawing our attention to errors as well as to alternative solutions.

It is the hope of the Canadian Mathematical Society that this collection may find its way to school students including those who may have the talent, ambition and mathematical expertise to represent Canada internationally.

It is the hope of the authors that this book may serve a wider audience. Their accessibility may allow the problems herein to function as a source of ‘out of classroom’ mathematical enrichment that teachers and parents/guardians of appropriate students may assign to their charges. To aid in this, answers and complete solutions are provided to all the problems (except the relays where there are answers only) and problems and solutions are presented in separate chapters.

There is repetition of various mathematical themes, but not the actual problems, among the questions. This is deliberate. Fundamental ideas like divisibility, simple counting, comparison of areas and proportionality are important enough to be revisited.

We have also deliberately avoided the temptation to discuss the various mathematical concepts or to intrude in any way with what is done in the school system. For example, the term ‘tangent to a circle’ may not be well known amongst the students at whom this booklet is aimed. We would hope that the concept would be explained in advance, perhaps with a diagram, by the teacher or parent/guardian.

The authors would like to thank Henry Andrews for some suggestions of improvements to problems.

Bruce L.R. Shawyer

Department of Mathematics and Statistics
Memorial University of Newfoundland
St. John's, NL
Canada A1C 5S7

Bruce B. Watson

Department of Mathematics and Statistics
Memorial University of Newfoundland
St. John's, NL
Canada A1C 5S7

The Authors

- **Bruce Shawyer**

Bruce Shawyer is Professor Emeritus in the Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland and Labrador, Canada.

His professional interests in mathematics research include approximation of the sums of slowly convergent series, the summability of series and integrals, and Euclidean geometry. In the area of mathematics education he is interested in mathematics enrichment, why certain mathematical ideas are not well-learned and the propagation of mathematics.

He is the founder of the Newfoundland and Labrador Teachers Association Senior Mathematics League and of the mathematical challenge for junior high school students (now replaced by a Mathematics League) and former head coach of the Canadian team for the International Mathematical Olympiad.

- **Bruce Watson**

Bruce Watson is Professor of Mathematics (retired) in the Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland and Labrador, Canada.

His professional interests include classical summability theory and fixed point theory in metric spaces. On the pedagogical side, he is interested in how technology can be incorporated into mathematics teaching.

History of the NLTA Junior Mathematics League

The Newfoundland and Labrador Junior Mathematics League began in 2004 as a competition amongst the Junior High Schools in the St. John's area. It has grown since then into a province-wide competition with many schools competing in local leagues in several districts all over the province. The same game takes place simultaneously in each place, with the schools competing at the district level. There are two games per school year. Teachers from the participating schools attend and act as proctors for different schools.

Jurisdictions interested in starting a league may ask to use our game materials. Master copies of materials will be provided for a modest fee.

The NLTA Junior Math League stresses cooperative problem solving. Each participating school sends up to two teams of four students, who will work cooperatively on each problem. Students may, if they wish, submit individual answers. However, to reward cooperative work, a bonus mark is given for a correct team answer. Non-symbolic calculators are allowed unless prohibited for a specific question. In this case the person directing the game will make an appropriate announcement. Games usually run to about two hours in length.

A typical contest consists of eight questions and a relay. For example, the first eight questions on page 1 plus Relay # 1 on page 41 could constitute a game. Unlike most mathematics competitions where the contestant receives all of the questions at once, the team receives only the first problem at the start.

Each team is seated around a table, and is given two copies of the first problem. Thus, everyone can read the problem easily. There is a time limit announced for each problem (usually between three and ten minutes). When time is called, the team hands one sheet to a proctor for marking.

While the contestants' answers are being marked, a solution to the problem is presented, preferably by a student, usually using an overhead projector. A correct team answer gets five marks whereas an incorrect team answer gets zero marks. If the team members do not agree on the answer, each may submit an answer for one possible mark each. This results in much discussion and debate, especially if two team members arrive at different answers. Such debate is indeed encouraged. The next problem is now given to the teams, and the above sequence of events is repeated.

After eight problems, which usually increase in difficulty and in time allotted, students do the relay question.

The relay question has four parts: the answer to part # 1 is an input to part # 2; the answer to part # 2 is an input to part # 3; and the answer to part # 3 is an input to part #4.

The relay question has fifteen minutes allotted and a total of fifteen points are available. Over and above the five normal points for a complete solution, ten bonus points are available for speed of solution. For incomplete solutions, one point is awarded if only part # 1 is correct; two points are awarded if only both parts # 1 and # 2 are correct; and three points are awarded if only all of parts # 1, # 2 and # 3 are correct. So, for complete solutions, five points are awarded if all four parts are correct. If time has not

been called when the team hands the relay answer sheet to the proctor, the proctor will say either “CORRECT” or “WRONG”. If the proctor answers “WRONG”, the relay answer sheet is returned to the team. No indication of the place of any error is given.

Bonus points, up to ten, are awarded as follows: there are ten bonus points for a CORRECT answer sheet completed within the first six minutes; then the number of bonus points for a CORRECT answer sheet is reduced by one at the end of each subsequent minute until there are no more available.

The theoretical maximum score consists of five points for each question plus five for the relay and ten bonus time points, giving a grand total of 55. Scores above 50 are rarely achieved.

After the scores have been tallied, the day’s winners are announced unless there is a tie for first place. In this case, a tiebreaker question is used to determine the contest’s winner. No points are awarded for winning a tiebreaker.

The rules for the tiebreaker are a little different. Essentially, the first team to solve the question wins. But to prevent repetitive guessing games, the following rule applies: a team that has offered an incorrect answer to the tiebreaker may not offer any other answer until a “one minute penalty” period has expired.

There is usually a “nutrition break” at a convenient time. Students and proctors are provided with a soft drink, juice, tea or coffee and often a slice or two of pizza. This allows for social interactions amongst the competitors and amongst the teachers and the organisers.

Attending an NLTA Junior Math League contest is a very rewarding experience. They usually take place on Friday after school. It is very gratifying to be in a room full of young men and women, usually in approximately equal numbers, doing mathematics, enjoying mathematics, and having fun. Parents are welcome to observe the fun!

Questions

1. A book has 64 pages which are numbered in the usual way — when you open the cover, the page facing you is page 1.

If you open the book at page 37, what is the sum of the digits of the two page numbers that you can see?

2. A person went to a corner store and bought \$25 worth of candy.

The weight (in kilograms) of the candy was the cost divided by twice the square root of the cost (in dollars) of the candy.

What is the cost per kilogram of the candy?

3. A book has its pages numbered in the usual way — when you open the front cover, the page facing you is page 1.

You open the book at random and notice that the product of the two page numbers showing is 272. What is the sum of the digits of the two page numbers that you can see?

4. Determine how many odd numbers, exactly divisible by 7, lie between 1 and 100.

5. What is the units digit of 2^{2004} ?

6. Two workers can produce five widgets in three hours. How many minutes are required for six workers to make eight widgets?

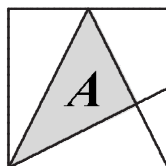
7. How many squares can be drawn by using, as vertices, four of the dots in this diagram?



8. A family has three children. Their ages in 2004 were in the ratio 3 : 4 : 5. The sum of the years in which they were born is 5976.

Determine the sum of the children's ages in the year 2000.

9. The side lengths of a square are 1. Two of the sides are bisected and some lines are drawn (see diagram). Determine the area of the triangle marked *A*.



10. A book has 504 pages, which are numbered in the usual way — when you open the cover, the page facing you is page 1.

If you open the book at page 376, what is the sum of the digits of the two page numbers that you can see?

11. A person went to a shop and bought \$144 worth of filet mignon.

The weight (in kilograms) of the filet mignon was three times the square root of the cost (in dollars) of the filet mignon.

What is the cost (per kilogram) of the filet mignon?

12. A book has its pages numbered in the usual way — when you open the front cover, the page facing you is page 1.

You open the book at random and notice that the product of the two page numbers showing is 4422. What is the sum of the digits of the two page numbers that you can see?

13. Determine how many odd numbers, exactly divisible by 13, lie between 1 and 200.

14. What is the units digit of 3^{3003} ?

15. Two workers can produce three widgets in five hours.

How many minutes are required for five workers to make five widgets?

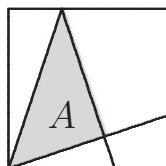
16. How many rectangles (including squares) can be drawn by using, as vertices, four of the dots in this diagram?



17. A family has four children. Their ages in 2004 were in the ratio 2 : 3 : 4 : 5. The sum of the years in which they were born is 7960.

Determine the sum of the children's ages in the year 2000.

18. A square has side length of 1. Three of the sides are trisected and some lines are drawn (see diagram). Determine the area of the right triangle marked A .



19. A book has 504 pages, which are numbered in the usual way — when you open the cover, the page facing you is page 1.

If you open the book at page 369, what is the sum of the digits of the two page numbers that you can see?

20. A person went to a shop and bought \$512 worth of steak for a barbecue.

It so happened that the cost per kilogram of steak was equal to the square root of the number of kilograms bought.

Calculate the number of kilograms bought for the barbecue.

21. A book has its pages numbered in the usual way — when you open the front cover, the page facing you is page 1.

You open the book at random and notice that the product of the two page numbers showing is 60762. What is the sum of the digits of the two page numbers that you can see?

22. Determine how many odd numbers, exactly divisible by 17, lie between 201 and 400.

23. What is the units digit of 7^{2004} ?

24. Three workers can produce four widgets in five hours.

How many widgets are produced by seven workers in thirty hours?

25. How many rectangles (including squares) can be drawn by using, as vertices, four of the dots in this diagram?



26. A family has four children. Their ages in 2004 were in the ratio $3 : 4 : 5 : 6$. The sum of the years in which they were born is 7962.

Determine the sum of the children's ages in the year 2010.

27. A **prime** number is a positive whole number that has no factors other than 1 and itself. However, 1 is not considered to be a prime number. The smallest (and only even) prime number is 2.

The page numbers in a book are numbered in the usual way; that is, the odd numbers are on the right hand pages.

A book is opened, and it is observed that the odd page number is a prime number and the even page number is the product of four distinct prime numbers.

Determine the least possible odd page number showing.

28. How many squares can be formed by using four dots in this diagram as vertices?



29. A farmer wishes to divide a circular field into as many separate portions as possible. The farmer can only afford to construct three fences, all of which must be straight lines (but may intersect).

Determine the maximum number of separate portions.

30. Pat observes that the distance (in kilometres) from St. Clare's to Johnsville is the square of the time (in hours) to cycle between the places.

One day, the journey takes 4 hours longer since the speed was reduced by 3 km/h.

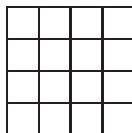
Determine the distance (in kilometres) from St. Clare's to Johnsville.

31. The figure shown is to be divided into non-overlapping isosceles right angled triangles. The longer sides are twice the length of the shorter sides.



Determine the smallest number of pieces.

32. On a 4×4 checkerboard, determine the number of squares that are shown.



33. A student sees a great iPod advertised at \$235.00. The student decides to start saving. In the first month, the student puts \$2.00 into the piggy bank. The next month, \$4.00, the next month, \$6.00, and so on.

Determine the least number of months that the student must save in order to buy the iPod (don't forget HST was 15% when the problem was originally set!).

34. Danny Billions has \$8 000 000 000 to invest.

After one year, the value has increased by 10%. The next year, that value decreased by 8%. The following year, that value increased by 6%. The following year, that value decreased by 4%. The following year, that value increased by 2%.

What was the profit or loss on Danny Billions investment?

35. [NO CALCULATORS PERMITTED]

Determine the exact value of

$$\left(\sqrt{13 - \sqrt{21 - \sqrt{25}}} \right)^{\left(\sqrt{11 + \sqrt{19 + \sqrt{36}}} \right)}$$

36. The sum of two whole numbers is 116. Their product is 3003. Determine the two numbers.

37. Determine the exact value of

$$1 + 3 + 5 + 7 + \cdots + 199.$$

38. The factorial sign ! after a number means that one must multiply all the consecutive numbers starting at 1 and ending with the given number.

For example, $4! = 1 \times 2 \times 3 \times 4 = 24$ and $7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$.

The number 210 is said to have one **trailing zero** and the number 5310000 is said to have four trailing zeros.

Determine how many trailing zeros the number $30!$ has.

39. Jennifer wrote four perfect tests. She wants an average of at least 90% after the fifth test. Determine the lowest percentage mark that will ensure her wish. (All tests are of equal value in calculating the average.)

40. Determine the value of

$$1\,000\,000 - 999\,999 + 999\,998 - 999\,997 + \cdots + 4 - 3 + 2 - 1.$$

41. Five towns, A , B , C , D and E are plotted on a chart.

The distance from A to B is 3 km. The distance from A to D is 5 km. The distance from A to E is 13 km.

The road from A to B goes straight on for an additional 9 km to C . The roads for B to D and from C to E are parallel to each other, and are perpendicular to the road from A to C . D and E are on the same side of AC .

Determine the distance from D to E correct to the nearest metre.

42. A diagonal line in a polygon is a line joining two non-adjacent vertices.

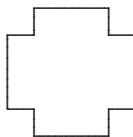
How many diagonal lines does a hexagon have?

43. If you add up all the whole numbers from 1 to n and divide by 23, determine the smallest value of n that ensures that your answer is a whole number.

44. A **palindrome** is one which reads exactly the same from either direction. For example, 123454321 is a palindrome, whereas 123456789 is not.
Determine how many palindromes there are between 1000 and 10000.
45. The sum of two whole numbers is 201. Their product is 10010. Determine the two numbers.
46. Determine the exact value of

$$1 + 4 + 7 + \cdots + 301.$$

47. A number is said to be **perfect** if it is equal to the sum of its proper divisors (the divisors must be less than the given number). For example, 6 has proper divisors of 1, 2 and 3, and $6 = 1 + 2 + 3$.
Find the next smallest perfect number.
48. Given that one-fifth of a positive whole number plus one-seventh of another positive whole number gives $\frac{29}{35}$, determine these whole numbers.
49. A square of side 24 has four squares of side 7 removed from the corners, leaving a cross shaped figure:



This shape is placed inside a circle of as small a radius as possible. Determine that radius.

50. Determine the number of distinct triangles that can be formed by using three points in the following diagram as the vertices. Remember that three points in a straight line do not form a triangle.



51. Danny Mullions has an amount of \$1 000 000 to invest for three years at 5% compound interest. (This means that the interest added at the end of a year is added to the amount invested, and the new amount is then invested for the next year.)

Determine the profit that Danny Mullions makes after three years.


52. Determine the value of the denominator when $1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}}$ is written out


as a rational number (fraction) in lowest terms; that is, with no common factors between the numerator and the denominator.




53. A man spent two-thirds of his money and misplaced two-thirds of the remainder, leaving him with \$18.00.

With how much money did he start?

54. Suppose that a and b represent numbers.

The symbol , drawn to any size, means $a - 3$.

The symbol , drawn to any size, means b^2 .

Find the value of  $+$  $-$ .

55. Your math teacher has recently bought a new tape deck.

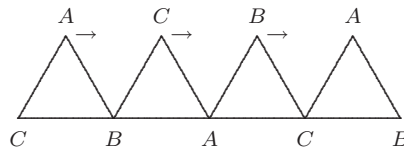
Being well organised, your math teacher had labelled each tape with the number on the counter when the song began. Now, a whole tape ran to 634 on the old counter.

On the new deck, the same tape runs to 2219 on the new counter.

A favourite song was at counter 164 on the old tape deck. What is it at on the new tape deck?

Note that the total listening time of the tape was the same on each deck.

56. An equilateral triangle of side length 5 cm is “rolled” over along a line, pivoting at the bottom right hand vertex, until the triangle is again in its original configuration.



Determine the **exact** distance travelled by the vertex A .

57. Nan and Pop have twelve children. Nan spends \$3.51 for five apples and seven oranges. Pop spends \$4.89 for seven apples and five oranges at the same store at the same time. Each child is given one apple and one orange.

How much did Nan and Pop spend on each child?

58. One day, one member of the class was sick, and missed the test. The class average was 75%.

The sick person was allowed to write an equivalent test the next week, and the result was that the class average rose to 75.5%.

If there were a total of 28 people in that class, what mark did the sick person get?

59. The residents of Mathland have a secret code indecipherable to residents of Nonmathland.

They write down the prime numbers (starting, of course, with 2) in ascending order, and assign to each the letters of the alphabet in alphabetical order. They then multiply the numbers together to give the code for the word.

Decipher 110618.

60. A circle is drawn inside a 3, 4, 5 right triangle. The circle is tangent to the three sides.

Determine the exact value of the radius of this circle.

HINT: when two tangents are drawn to a circle from a point outside it, the tangents are of the same length.

61. Determine the sum of all prime numbers which are less than 50.
62. A man spent four-fifths of his money and misplaced four-fifths of the remainder, leaving him with \$17.00.

With how much money did he start?

63. A girl walks three-quarters of the way home in 18 minutes.
- If she continues to walk at the same speed, how long will it take her to walk the rest of the way home?

64. Pat had collected some pennies in a jar. They would make 14 rows of equal length, or 15 rows of equal length, or 20 rows of equal length.

Determine the minimum number of pennies that Pat had.

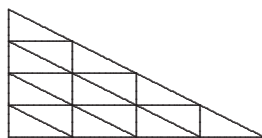
65. Determine the largest prime factor of 52710.
66. A cruise ship left St. John's for New York travelling at 16 km/h. One hour later, another cruise ship left St. John's for New York travelling along the same route at 18 km/h. How long did it take for the second cruise ship to overtake the first cruise ship?

67. A circle is drawn inside a 5, 12, 13 right triangle. The circle is tangent to the three sides.

Determine the exact value of the radius of this circle.

HINT: when two tangents are drawn to a circle from a point outside it, the tangents are of the same length.

68. Determine the total number of right triangles in the following figure.



69. A committee is made up of three boys and five girls.
- They decide to have a subcommittee of five persons, and they agree that it must have at least one boy and at least two girls.
- How many different subcommittees are possible?
70. Pamela has two daughters. One is five years older than the other. The sum of their ages is 27. How old is the younger daughter?

71. A man spent six-sevenths of his money and misplaced six-sevenths of the remainder, leaving him with \$11.00.
With how much money did he start?
72. Find the average of all the even whole numbers between 27 and 1003 inclusive.
73. Determine the exact value of

$$\sqrt{20 + \sqrt{21 + \sqrt{12 + \sqrt{13 + \sqrt{6 + \sqrt{7 + \sqrt{3 + \sqrt{1}}}}}}}}$$

NOTE: $\sqrt{\dots}$ means the POSITIVE value of the square root.

74. Three people went to mow a meadow.
When they worked together, it took 6 days.
Alice worked three times as quickly as Charlie and Bruce worked twice as quickly as Charlie.
Later, there were three more meadows, identical to the original meadow, to mow, and each worked separately.
Determine how long each took to mow their meadow.
The answer we are seeking is the average number of days that it took to mow the three more meadows.
75. A cruise ship left St. John's for New York travelling at 16 km/h. One hour later, another cruise ship left St. John's for New York travelling along the same route at 20 km/h. How long did it take for the second cruise ship to overtake the first cruise ship?
76. Josh saved \$250 from a summer job. He put it into a very special bank account that gave 5% interest per month. The bank calculated the interest each month, and rounded it down to the nearest cent.
After leaving it in the bank for four months, Josh withdrew the money. How much did he then have?
77. Start with a first square with side length 1.
Construct a second square by adding isosceles right triangles on the outside of the sides of the first square.
Construct a third square by adding isosceles right triangles on the outside of the sides of the second square.
Continue this process until you get to the 99th square.
Determine the length of the side of the 99th square.
78. A committee is made up of three boys and five girls.
They decide to have a subcommittee of four persons, and they agree that it must have an equal number of boys and girls.
How many different subcommittees are possible?
79. A gardener added 1 kg of sand to 15 kg of earth. If the resulting mixture was uniform, how many grams of sand are there in 1 kg of the mixture. Exact answer required.

80. [NO CALCULATORS] My hard drive has a capacity of 120 billion bytes. The average size of a jpeg image taken by my digital camera is 1.5 million bytes. What is the maximum number of average sized jpeg images that I can store on my hard drive?
81. The product of two positive numbers is 2160. The ratio of the two numbers is 60. Determine the two numbers.
82. Determine the measure of the angle between the hands of a clock when it shows a time of 3:30.
83. When a painting is framed, it is placed behind a rectangular hole in a piece of card, known as a picture frame mat. The outside dimensions of a picture frame mat measure 32 cm by 24 cm. The mat is cut with a border of 4 cm on each side. Determine the percentage of the mat that remains after the hole is cut out.
84. In a group of 75 people, 49 have brown hair, 31 have blue eyes, and 24 have both brown hair and blue eyes. How many people in the group have neither brown hair nor blue eyes?
85. Determine the exact value of

$$\sqrt{3 \frac{\sqrt{6 + \sqrt{7 + \sqrt{3 + \sqrt{1}}}}}{\sqrt{7 + \sqrt{76 + \sqrt{22 + \sqrt{9}}}}}}$$

NOTE: $\sqrt{\dots}$ means the POSITIVE value of the square root.

86. A square of side length 4 is divided into two regions using arcs of circles of radius 2.



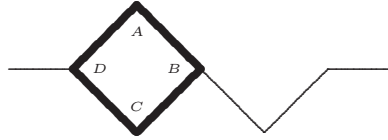
Determine the area of the smaller region.

87. Determine the smallest number which can be divided exactly by all of 3, 9, 12 and 15.
88. The product of the digits in a two digit number is 24. The difference of the digits is 5. Determine the sum of the digits.
89. A class of 18 students wrote a math test, and the class average was 70%. The average of the top 12 students was 85%. Determine the average of the bottom 6 students.
90. A number of students are standing equally spaced around a circle. The 4th student is standing diametrically opposite the 16th student.



Determine how many students are standing in the circle.

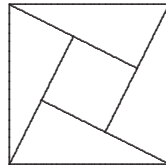
91. In the plane, a square $ABCD$ of side length 1 sits in a 90° hole (which has an adjacent 90° hole).



In the plane, the square rotates about B until it sits in the adjacent hole.

Determine the exact value of the length of the path described by the point A .

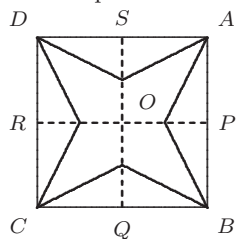
92. Triangle ABC is equilateral. Point E is on side BC such that $\angle EAC = 40^\circ$. Determine the value of $\angle BEA$.
93. In his grade 7 math course, Nonnash got a final grade of 70%. Nonnash wrote five tests, but the worst mark of 20% was discarded in calculating the final grade. Determine Nonnash's average on all five tests.
94. Four congruent right triangles with legs of 1 and 3 are drawn inside a square, forming a smaller square.



Determine the area of the larger square.

95. Determine the sum of the digits of $2^{2009} \times 5^{2007}$.
96. A day is called **multiplicative** if the product of the day number and the month number is equal to the number given by the tens and units digits of the year. For example, 03/08/24 was multiplicative in 1924. (Assumed to mean **day/month/year**.) Determine how many multiplicative dates there are in the year 2009.
97. Four people can dig four holes in four days. Determine the number of days required for two people to dig two holes.
98. A three digit number consists of one each of the digits 7, 8 and 9. Determine which of these possible numbers has the smallest sum of its distinct prime factors. The required answer is that sum.
99. Determine the number of odd positive whole number factors of 840.
100. The sum of 17 consecutive whole numbers is 2244. Determine the smallest number of the 17.
101. A sequence of three letters represents a three digit number. Determine how many three digit numbers, abc , satisfy the equation $abc = cba$.

102. Rent-a-Lemon Vehicle Hire Company has 36 vehicles. All vehicles have 2, 3, or 4 road wheels each (excluding any spares). Exactly six of the vehicles have 4 wheels each. All together the vehicles have 100 wheels.
- Determine how many vehicles have 2 wheels each.
103. A square $ABCD$ has side length 2. The mid-points P , Q , R and S of opposite sides are joined, meeting at the point O . The mid-points of OP , OQ , OR and OS are joined to their nearer vertices of square $ABCD$ to form a star-like figure.



Determine the area of the star-like figure.

Solutions

1. A book has 64 pages which are numbered in the usual way — when you open the cover, the page facing you is page 1.

If you open the book at page 37, what is the sum of the digits of the two page numbers that you can see?

Solution. The open pages of the book have an even number on the left and the following odd number on the right.

If the odd number is 37, then the even number is 36. The sum of the digits is $3 + 6 + 3 + 7 = \boxed{19}$.

2. A person went to a corner store and bought \$25 worth of candy.

The weight (in kilograms) of the candy was the cost divided by twice the square root of the cost (in dollars) of the candy.

What is the cost per kilogram of the candy?

Solution. Now, $\sqrt{25} = 5$. Thus, the weight of candy is $\frac{25}{10} = 2.5$ kilograms.

The cost per kilogram of candy is then $\frac{25}{2.5} = \boxed{10}$ dollars.

3. A book has its pages numbered in the usual way — when you open the front cover, the page facing you is page 1.

You open the book at random and notice that the product of the two page numbers showing is 272. What is the sum of the digits of the two page numbers that you can see?

Solution. The product 272 is the product of two consecutive numbers.

Note that $16^2 = 256$ and $17^2 = 289$. This suggests that the numbers are 16 and 17 — check that this is correct.

The sum of the digits is $1 + 6 + 1 + 7 = \boxed{15}$.

4. How many odd numbers, exactly divisible by 7, lie between 1 and 100.

Solution. The multiples of 7, less than 100 are

$$1 \times 7, \quad 2 \times 7, \quad 3 \times 7, \quad 4 \times 7, \quad \dots, \quad 12 \times 7, \quad \text{and} \quad 13 \times 7.$$

Only those that are an odd multiple of 7 are odd.

Thus, there are $\boxed{7}$ such numbers.

5. What is the units digit of 2^{2004} ?

Solution. The units digits of the powers of 2 are shown in the following table:

Power	1	2	3	4	5	6	7	8	...
Units digit	2	4	8	6	2	4	8	6	...

We see a repeating pattern: every fourth digit is the same as the one that is four positions before it.

Now, 4 is a factor of 2004, so that the answer is $\boxed{6}$.

6. Two workers can produce five widgets in three hours. How many minutes are required for six workers to make eight widgets?

Solution. We work by proportion:

# workers	# widgets	hours
2	5	3
6	15	3
6	$8 = (\frac{8}{15} \times 15)$	$\frac{8}{15} \times 3$

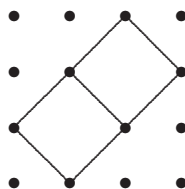
The answer is $3 \times \frac{8}{15} = \frac{8}{5}$ hours, which is 96 minutes.

7. How many squares can be drawn by using, as vertices, four of the dots in this diagram?

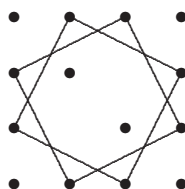


Solution. There are:

- (a) squares of side length 1 — number is 2;
- (b) squares of side length 2 — number is 2;
- (c) squares of side length 3 — number is 1;
- (d) diagonal squares at 45°
squares of side length $\sqrt{2}$ — number is 2;



- (e) diagonal squares at about 60°
squares of side length $\sqrt{5}$ — number is 2.



The total number is 9.

8. A family has three children. Their ages in 2004 were in the ratio 3 : 4 : 5. The sum of the years in which they were born is 5976.

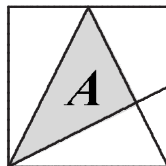
Determine the sum of the children's ages in the year 2000.

Solution. Let the ages be $3n$, $4n$ and $5n$.

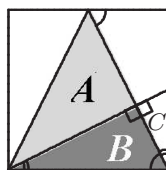
They were born in $2004 - 3n$, $2004 - 4n$ and $2004 - 5n$. The sum of these years is $6012 - 12n = 5976$, showing that $12n = 36$, so that $n = 3$.

From this, we see that the years of their birth are 1995, 1992, and 1989. In the year 2000, they were 5, 8 and 11 years old, so that the sum of their ages in 2000 is 24.

9. The side lengths of a square are 1. Two of the sides are bisected and some lines are drawn (see diagram). Determine the area of the triangle marked A .



Solution. We are dealing with right triangles whose legs are in the ratio of $1 : 2$. Thus, including the hypotenuse, the sides are in the ratio of $1 : 2 : \sqrt{5}$. Let B be the triangle below A , and C the triangle to the right of B .



The triangle B is similar to the triangle C with B having double the dimensions of C (look at the hypotenuse). Therefore, $\text{Area } B = 4 \text{ Area } C$.

Put this together with $\text{Area } B + \text{Area } C = \frac{1}{4}$ and $\text{Area } A + \text{Area } B = \frac{1}{2}$ to give that the area of B is $\frac{1}{5}$ and the area of A is $\boxed{\frac{3}{10}}$.

10. A book has 504 pages, which are numbered in the usual way — when you open the cover, the page facing you is page 1.

If you open the book at page 376, what is the sum of the digits of the two page numbers that you can see?

Solution. The open pages on the book have an even number on the left and the following odd number on the right.

If the even number is 376, then the odd number is 377, and the sum of the digits is $3 + 7 + 6 + 3 + 7 + 7 = \boxed{33}$.

11. A person went to a shop and bought \$144 worth of filet mignon.

The weight (in kilograms) of the filet mignon was three times the square root of the cost (in dollars) of the filet mignon.

What is the cost (per kilogram) of the filet mignon?

Solution. $\sqrt{144} = 12$. Thus, the weight of filet mignon is $3 \times 12 = 36$ kilograms.

The cost (per kilogram) of filet mignon is then $\frac{144}{36} = \boxed{4}$ dollars.

12. A book has its pages numbered in the usual way — when you open the front cover, the page facing you is page 1.

You open the book at random and notice that the product of the two page numbers showing is 4422. What is the sum of the digits of the two page numbers that you can see?

Solution. The product 4422 is the product of two consecutive numbers.

If we take the square root of 4422, we get $66.4981\dots$

This suggests that the numbers are 66 and 67 — check that this is correct.

The sum of the digits is $6 + 6 + 6 + 7 = \boxed{25}$.

13. Determine how many odd numbers, exactly divisible by 13, lie between 1 and 200.

Solution. The odd multiples of 13, less than 200 are

$$1 \times 13, \quad 3 \times 13, \quad 5 \times 13, \quad 91, \dots, \quad 13 \times 13 \text{ and } 15 \times 13.$$

Thus, there are $\boxed{8}$ such numbers.

14. What is the units digit of 3^{3003} ?

Solution. The units digits of the powers of 3 are shown in the following table:

Power	1	2	3	4	5	6	7	8	...
Units digit	3	9	7	1	3	9	7	1	...

We see a repeating pattern: every fourth digit is the same as the one in the power four positions before it.

Divide 4 into 3003: we get $\frac{3003}{4} = 750 + \frac{3}{4}$.

Now, the $4 \times 750 = 3000^{\text{th}}$ power of 3 ends in a 1.

Thus, the answer to our question is $\boxed{7}$.

15. Two workers can produce three widgets in five hours.

How many minutes are required for five workers to make five widgets?

Solution. We work by proportion:

# workers	# widgets	hours
2	3	5
1	$\frac{3}{2}$	5
5	$\frac{3}{2}$	1
5	$5 = \left(\frac{10}{3} \times \frac{3}{2}\right)$	$\frac{10}{3}$

The answer is $\frac{10}{3}$ hours, which is $\boxed{200}$ minutes.

16. How many rectangles (including squares) can be drawn by using, as vertices, four of the dots in this diagram?



Solution. There are:

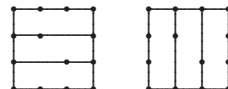
- (a) squares of side length 1 — number is 2;
- (b) squares of side length 2 — number is 2;
- (c) squares of side length 3 — number is 1;
- (d) rectangles of side lengths 1 and 2 — number is 4;



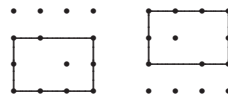
- (e) diagonal squares at 45° — number is 2;



- (f) diagonal squares at about 60° — number is 2;



- (g) rectangles 3×1 — the number is 6;



- (h) rectangles 3×2 — the number is 4;



- (i) diagonal rectangles at 45° — the number is 2.

The total number is then 25.

17. A family has four children. Their ages in 2004 were in the ratio $2 : 3 : 4 : 5$. The sum of the years in which they were born is 7960.

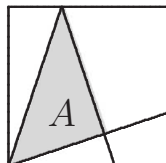
Determine the sum of the children's ages in the year 2000.

Solution. Let the ages be $2n$, $3n$, $4n$ and $5n$.

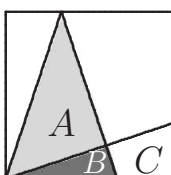
They were born in $2004 - 2n$, $2004 - 3n$, $2004 - 4n$ and $2004 - 5n$. The sum of these years is $8016 - 14n = 7960$, showing that $14n = 56$, so that $n = 4$.

From this, we see that the years of their birth are 1996, 1992, 1988 and 1984. In the year 2000, they were 4, 8, 12 and 16 years old, so that the sum of their ages in 2000 is 40.

18. A square has side length of 1. Three of the sides are trisected and some lines are drawn (see diagram). Determine the area of the right triangle marked A .



Solution. Label regions B and C as shown.



Now B and $B \cup C$ (which denotes regions B and C combined) are similar right triangles whose sides are in the ratio $1:3:\sqrt{10}$.

First, the area of region $A \cup B$ is $\frac{1}{2} \cdot \frac{2}{3} \cdot 1 = \frac{1}{3}$. We need to find the area of right triangle B .

If x represents the length of the shorter leg of B then

$$\frac{x}{\frac{2}{3}} = \frac{1}{\sqrt{10}},$$

or

$$x = \frac{2}{3\sqrt{10}}.$$

Hence,

$$\text{area of } B = \frac{1}{2} \cdot x \cdot 3x = \frac{3}{2}x^2 = \frac{3}{2} \cdot \frac{4}{90} = \frac{1}{15}.$$

Finally, the area of A is $\frac{1}{3} - \frac{1}{15} = \boxed{\frac{4}{15}}$.

19. A book has 504 pages, which are numbered in the usual way — when you open the cover, the page facing you is page 1.

If you open the book at page 369, what is the sum of the digits of the two page numbers that you can see?

Solution. Since the odd page is on the right side, the page on the left will have a number of one less; that is 368.

The answer is $3 + 6 + 8 + 3 + 6 + 9 = \boxed{35}$.

20. A person went to a shop and bought \$512 worth of steak for a barbecue.

It so happened that the cost per kilogram of steak was equal to the the square root of the number of kilograms bought.

Calculate the number of kilograms bought for the barbecue.

Solution. If the cost per kilogram of the steak is $\$n$, then n^2 kilograms were bought, and the total cost is $\$n^3 = \512 , giving $n = 8$.

Hence, $\boxed{64}$ kilograms of steak were bought.

21. A book has its pages numbered in the usual way — when you open the cover, the page facing you is page 1.

You open the book at random and notice that the product of the two page numbers showing is 60762. What is the sum of the digits of the two page numbers that you can see?

Solution. The book has an even number page number on the left and the next odd number as the page number on the right. If the left page number is n , then the right page number is $n + 1$, and the product is $n(n + 1) > n^2$.

Now, $\sqrt{60762} = 246.499 \dots$, giving that $n = 246$ and $n + 1 = 247$. (Check that $246 \times 247 = 60762$.) Hence, the required sum is 25.

22. Determine how many odd numbers, exactly divisible by 17, lie between 201 and 400.

Solution. First, note that $\frac{201}{17} = 11.823 \dots$ and $\frac{400}{17} = 23.529 \dots$.

This means that the smallest possible number is 13×17 and the largest is 23×17 .

The other eligible numbers are 15×17 , 17×17 , 19×17 and 21×17 , giving a total of 6 odd numbers, exactly divisible by 17.

23. What is the units digit of 7^{2004} ?

Solution. Note that $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, $7^5 = 16807$, \dots , showing that the powers of 7 end in 7, 9, 3, 1, and this pattern repeats.

Thus, any power of 7 that is a multiple of 4 will end in the digit 1.

But 2004 is a multiple of 4, so that the answer is 1.

24. Three workers can produce four widgets in five hours.

How many widgets are produced by seven workers in thirty hours?

Solution.

Workers	Time	Widgets
3	5	4
1	5	$\frac{4}{3}$
1	1	$\frac{4}{15}$
7	1	$\frac{28}{15}$
7	30	$\frac{30 \times 28}{15} = \span style="border: 1px solid black; padding: 0 2px;">56$

25. How many rectangles (including squares) can be drawn by using, as vertices, four of the dots in this diagram?

Solution.

With lines parallel to the top and sides, we have two squares, 1×1 , two squares, 2×2 and two rectangles 3×1 .

With lines at 45° to the sides, we have two squares and two rectangles.

And there are two squares at other angles as shown to the right:

The total number of rectangles is 12.



26. A family has four children. Their ages in 2004 were in the ratio $3 : 4 : 5 : 6$.

The sum of the years in which they were born is 7962.

Determine the sum of the children's ages in the year 2010.

Solution. If the ages in 2004 are $3n$, $4n$, $5n$ and $6n$, they were born in $2004 - 3n$, $2004 - 4n$, $2004 - 5n$, and $2004 - 6n$.

The sum of these years is $8016 - 18n = 7962$, giving $n = 3$.

Their ages in 2004 are then 9, 12, 15 and 18.

In 2010, they will be 15, 18, 21 and 24, so that the sum of their ages is 78.

27. A **prime** number is a positive whole number that has no factors other than 1 and itself.

However, 1 is not considered to be a prime number.

The smallest (and only even) prime number is 2.

The page numbers in a book are numbered in the usual way; that is, the odd numbers are on the right hand pages.

A book is opened, and it is observed that the odd page number is a prime number and the even page number is the product of four distinct prime numbers.

Determine the least possible odd page number showing.

Solution.

Take $2 \times 3 \times 5 \times 7$, the smallest possible product of four distinct primes, and add 1, to get 211, which is a prime number.

28. How many squares can be formed by using four dots in this diagram as vertices?



Solution.

Parallel to the sides:

Size	1×1	2×2
#	5	1

And on the slant:

Size	$\sqrt{2} \times \sqrt{2}$	$\sqrt{5} \times \sqrt{5}$
#	3	1

The total is 10.

29. A farmer wishes to divide a circular field into as many separate portions as possible. The farmer can only afford to construct three fences, all of which must be straight lines (but may intersect).

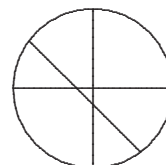
Determine the maximum number of separate portions.

Solution.

After two fences are built, it is not possible for another straight fence to cut all four portions.

But, it can cut three of the portions.

Hence, the answer is 7.



30. Pat observes that the distance (in kilometres) from St. Clare's to Johnsville is the square of the time (in hours) to cycle between the places.

One day, the journey takes 4 hours longer since the speed was reduced by 3 km/h. Determine the distance (in kilometres) from St. Clare's to Johnsville.

Solution. Use the magic formula “Distance Is Speed \times Time”.

On a normal day, from St. Clare's to Johnsville: $D = S \times T$, with $D = T^2$, so that $S = T$.

On the abnormal day, the time is $T + 4$, the speed is the normal speed less 3; that is, $T - 3$, so that the distance is $(T + 4)(T - 3) = T^2 - 3T + 4T - 12 = T^2 + T - 12$. This must be equal to T^2 , so that $T^2 = T^2 + T - 12$, which is the same as $T = 12$. Hence, the distance from St. Clare's to Johnsville is $12^2 = \boxed{144}$ kilometres.

31. The figure shown is to be divided into non-overlapping isosceles right angled triangles. The longer sides are twice the length of the shorter sides.

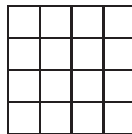


Determine the smallest number of pieces.

Solution. The answer is $\boxed{3}$, when divided as shown:



32. On a 4×4 checkerboard, determine the number of squares that are shown.



Solution. There are 16 squares 1×1 . There are 9 squares 2×2 . There are 4 squares 3×3 . There is one square 4×4 .

The total is $16 + 9 + 4 + 1 = \boxed{30}$.

33. A student sees a great iPod advertised at \$235.00. The student decides to start saving. In the first month, the student puts \$2.00 into the piggy bank. The next month, \$4.00, the next month, \$6.00, and so on.

Determine the least number of months that the student must save in order to buy the iPod (don't forget HST was 15% when the problem was originally set!).

Solution. The actual price is $\$235.00 \times 1.15 = \270.25 . The student needs at least this amount.

After two months, the student has $\$2 + 4 = \6 ; after three months, $\$6 + 6 = \12 ; and so on.

The answer is $\boxed{16}$ months, when the student will have \$272.

34. Danny Billions has \$8 000 000 000 to invest.

After one year, the value has increased by 10%. The next year, that value decreased by 8%. The following year, that value increased by 6%. The following year, that value decreased by 4%. The following year, that value increased by 2%.

What was the profit or loss on Danny Billions investment?

Solution. After one year, Danny Billions had $\$8\,000\,000\,000 \times 1.10$.

After two years, Danny Billions had $\$8\,000\,000\,000 \times 1.10 \times 0.92$.

After three years, Danny Billions had $\$8\,000\,000\,000 \times 1.10 \times 0.92 \times 1.06$.

After four years, Danny Billions had $\$8\,000\,000\,000 \times 1.10 \times 0.92 \times 1.06 \times 0.96$.

And after five years, Danny Billions had $\$8\,000\,000\,000 \times 1.10 \times 0.92 \times 1.06 \times 0.96 \times 1.02$.

Thus, Danny Billions had $\$8\,403\,259\,392$. He has a profit of \$403 259 392.

35. [NO CALCULATORS PERMITTED]

Determine the exact value of

$$\left(\sqrt{13 - \sqrt{21 - \sqrt{25}}} \right)^{\left(\sqrt{11 + \sqrt{19 + \sqrt{36}}} \right)}$$

Solution.

$$\begin{aligned} \left(\sqrt{13 - \sqrt{21 - \sqrt{25}}} \right)^{\left(\sqrt{11 + \sqrt{19 + \sqrt{36}}} \right)} &= \left(\sqrt{13 - \sqrt{21 - 5}} \right)^{\left(\sqrt{11 + \sqrt{19 + 6}} \right)} \\ &= \left(\sqrt{13 - \sqrt{16}} \right)^{\left(\sqrt{11 + \sqrt{25}} \right)} \\ &= (\sqrt{13 - 4})^{(\sqrt{11 + 5})} \\ &= (\sqrt{9})^{(\sqrt{16})} \\ &= (3)^4 = \boxed{81}. \end{aligned}$$

36. The sum of two whole numbers is 116. Their product is 3003. Determine the two numbers.

Solution. First note that $3003 = 3 \times 1001 = 3 \times 7 \times 143 = 3 \times 7 \times 11 \times 13$.

Thus, 1, 3, 7, 11, 13 and their various products are the numbers to investigate.

A little trial and error leads to the numbers being 39 and 77.

37. Determine the exact value of

$$1 + 3 + 5 + 7 + \cdots + 199.$$

Solution. We write the sum down twice:

$$\begin{array}{cccccccc} 1 & + & 3 & + & 5 & + & 7 & + & \cdots & + & 199 \\ 199 & + & 197 & + & 195 & + & 193 & + & \cdots & + & 1 \end{array}$$

Add the lines together to get

$$200 + 200 + 200 + 200 + \cdots + 200$$

How many times does 200 occur? That is the same as the number of terms in the sum. That is, 100 terms.

Adding up the last line gives a total of $200 \times 100 = 20\,000$. But this is twice the required sum. Thus, the answer is 10 000.

ALTERNATIVELY:

if you know or observe that

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2,$$

and notice that $199 = 2(100) - 1$, we get that the sum is $100^2 = \boxed{10\,000}$.

38. The factorial sign $!$ after a number means that one must multiply all the consecutive numbers starting at 1 and ending with the given number.

For example, $4! = 1 \times 2 \times 3 \times 4 = 24$ and $7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$.

The number 210 is said to have one **trailing zero** and the number 5310000 is said to have four trailing zeros.

Determine how many trailing zeros the number $30!$ has.

Solution. Note that in order to get a trailing zero, 10 must be a factor of the number. The number of trailing zeros is the number of times that 10 is a factor of the number.

Note that $30! = 1 \times 2 \times 3 \times 4 \times 5 \times \cdots \times 25 \times 26 \times 27 \times 28 \times 29 \times 30$.

There is an abundance of factors 2 since every other number in a factorial is even.

Thus, we need the number of times that 5 is a factor.

The multiples of 5 in $30!$ are 5, 10, 15, 20, 25 and 30; that is, 6 numbers have 5 as a factor. **BUT**, 25, being 5^2 , has 5 as a factor twice.

Hence, the number of times that 5 is a factor of $30!$ (that is, the number of trailing zeros) is $\boxed{7}$.

39. Jennifer wrote four perfect tests. She wants an average of at least 90% after the fifth test. Determine the lowest percentage mark that will ensure her wish. (All tests are of equal value in calculating the average.)

Solution. Jennifer's total number of marks after four tests is 400.

To have an average of 90%, she needs a total of $5 \times 90 = 450$ marks.

Thus, the lowest percentage mark that will ensure her wish is $\boxed{50}$.

40. Determine the value of

$$1\,000\,000 - 999\,999 + 999\,998 - 999\,997 + \cdots + 4 - 3 + 2 - 1.$$

Solution. Think of the numbers in pairs! We have

$$(1\,000\,000 - 999\,999) + (999\,998 - 999\,997) + \cdots + (4 - 3) + (2 - 1).$$

Each pair has value 1, and there are 500 000 such pairs.

The answer is then $\boxed{500\,000}$.

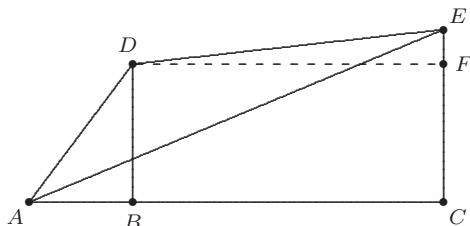
41. Five towns, A , B , C , D and E are plotted on a chart.

The distance from A to B is 3 km. The distance from A to D is 5 km. The distance from A to E is 13 km.

The road from A to B goes straight on for an additional 9 km. to C . The roads for B to D and from C to E are parallel to each other, and are perpendicular to the road from A to C . D and E are on the same side of AC .

Determine the distance from D to E correct to the nearest metre.

Solution. Let F be a point on the road from C to E , such that the distance from C to F is the same as the distance from B to D . Draw a diagram:



It is easy to see that the distance from B to D is 4 km., and that the distance from C to E is 5 km.

The distance from B to C is 9 km., and thus, the distance from D to F is also 9 km. The distance from F to E is 1 km.

Applying Pythagoras' Theorem, the distance from D to E is $\sqrt{9^2 + 1^2} = \sqrt{82} \approx 9.05538$. Hence, the answer is 9 km. and 55 m., or 9055 metres.

42. A diagonal line in a polygon is a line joining two non-adjacent vertices.

How many diagonal lines does a hexagon have?

Solution. There are six vertices on a hexagon. Each point can be joined to each of the five other points. But two of these points are adjacent vertices, and do not lead to diagonals.

Thus, there are $6 \times 3 = 18$ diagonal lines.

But each diagonal line has been counted twice. Hence, the answer is 9 diagonal lines.

43. If you add up all the whole numbers from 1 to n and divide by 23, determine the smallest value of n that ensures that your answer is a whole number.

Solution. Look at the sum of the numbers from 1 to n :

n	sum
1	1
2	3
3	6
4	10
5	15
\vdots	\vdots

It is not too hard to see that there is a formula for the sum: $\frac{n(n+1)}{2}$.

Since 23 is a prime number, it will not occur in the numerator of this number until $n = \text{22}$.

44. A **palindrome** is one which reads exactly the same from either direction. For example, 123454321 is a palindrome, whereas 123456789 is not.

Determine how many palindromes there are between 1000 and 10000.

Solution. A palindrome must begin and end with the same number. If it starts with 1, it will be of the form $1XX1$, where X stands for any digit. There are 10 digits — 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Thus, there are 10 palindromes between 1000 and 2000.

A similar argument holds between 2000 and 3000, etc.

Thus, there are $9 \times 10 = \boxed{90}$ palindromes between 1000 and 10000.

45. The sum of two whole numbers is 201. Their product is 10010. Determine the two numbers.

Solution. First note that $10010 = 2 \times 5 \times 1001 = 2 \times 5 \times 7 \times 143 = 2 \times 5 \times 7 \times 11 \times 13$.

Thus, 1, 2, 5, 7, 11, 13 and their various products are the numbers to investigate.

A little trial and error leads to the numbers being $2 \times 5 \times 11$ and 7×13 ; that is,

$\boxed{110}$ and $\boxed{91}$.

46. Determine the exact value of

$$1 + 4 + 7 + \cdots + 301.$$

Solution. We write the sum down twice:

$$\begin{array}{cccccccc} 1 & + & 4 & + & 7 & + & \cdots & + & 301 \\ 301 & + & 298 & + & 295 & + & \cdots & + & 1 \end{array}$$

Add the lines together to get

$$302 + 302 + 302 + \cdots + 302$$

How many times does 302 occur? That is the same as the number of terms in the sum. That is, 101 terms.

Adding up the last line gives a total of $302 \times 101 = 30\,502$. But this is twice the required sum. Thus, the answer is $\boxed{15\,251}$.

47. A number is said to be **perfect** if it is equal to the sum of its proper divisors (the divisors must be less than the given number).

For example, 6 has proper divisors of 1, 2 and 3, and $6 = 1 + 2 + 3$.

Find the next smallest perfect number.

Solution. This just requires checking! The answer is $\boxed{28}$.

48. Given that one-fifth of a positive whole number plus one-seventh of another positive whole number gives $\frac{29}{35}$, determine these whole numbers.

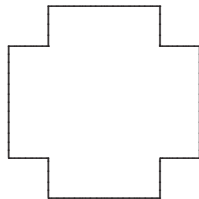
Solution. By algebra, we have $\frac{x}{5} + \frac{y}{7} = \frac{29}{35}$.

Multiply by 35 to get $7x + 5y = 29$.

Thus, x cannot exceed 4 and y cannot exceed 5.

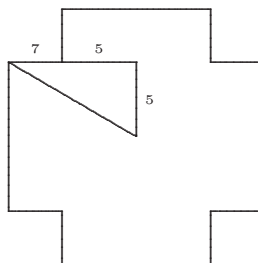
Guess and check leads to $x = \boxed{2}$ and $y = \boxed{3}$.

49. A square of side 24 has four squares of side 7 removed from the corners, leaving a cross shaped figure:



This shape is placed inside a circle of as small a radius as possible. Determine that radius.

Solution. By drawing a couple of line segments, we construct a right triangle with sides 5 and $5 + 7 = 12$.



Thus, the required radius is $\boxed{13}$.

50. Determine the number of distinct triangles that can be formed by using three points in the following diagram as the vertices. Remember that three points in a straight line do not form a triangle.



Solution. Start with triangles with two points on the base line.

There are 3 ways of choosing 2 points. For each pair, we can choose 2 of the remaining 3 points, giving a total of 9 triangles.

There are 3 choices for 1 point on the base line. With the 3 points not on the base line, we get 3 triangles for the middle point, but only 2 triangles for the end points (since we may not choose three points on a line), giving a total of 7 triangles.

If we do not take a point on the base line, there is only 1 possible triangle.

Thus, the total number is $9 + 7 + 1 = \boxed{17}$.

ALTERNATIVELY

How many ways can you choose three points from a collection of six?

The answer is 20. But exactly three of these give three points on a straight line.

Thus, the answer is $\boxed{17}$.

51. Danny Mullions has an amount of \$1 000 000 to invest for three years at 5% compound interest. (This means that the interest added at the end of a year is added to the amount invested, and the new amount is then invested for the next year.)

Determine the profit that Danny Mullions makes after three years.

Solution.

After 1 year, Danny Mullions has $\$1\,000\,000 \times 1.05 = \$1\,050\,000$.

After 2 years, Danny Mullions has $\$1\,050\,000 \times 1.05 = \$1\,102\,500$.

After 3 years, Danny Mullions has $\$1\,102\,500 \times 1.05 = \$1\,157\,625$.

Thus, the answer is $\boxed{\$157\,625}$.

52. Determine the value of the denominator when $1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}}$ is written

out as a rational number (fraction) in lowest terms; that is, with no common factors between the numerator and the denominator.

Solution.

$$\begin{aligned}
 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}} &= 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{\frac{21}{5}}}} \\
 &= 1 + \frac{1}{2 + \frac{1}{3 + \frac{5}{21}}} \\
 &= 1 + \frac{1}{2 + \frac{1}{\frac{68}{21}}} = 1 + \frac{1}{2 + \frac{21}{68}} \\
 &= 1 + \frac{1}{\frac{157}{21}} = 1 + \frac{21}{157} = \frac{178}{157}.
 \end{aligned}$$

Now, 157 is a prime number, and thus, the answer is $\boxed{157}$.

53. A man spent two-thirds of his money and misplaced two-thirds of the remainder, leaving him with \$18.00.


With how much money did he start?


Solution. If he spent two-thirds of his money, he then had one-third left.




He misplaced two-thirds of that; that is, he had one-third of the one-third left; that is one-ninth of the original amount.

Thus, one-ninth of the original amount is \$18.00, so that the original amount is $\$18.00 \times 9 = \boxed{\$162.00}$.

54. Suppose that a and b represent numbers.

The symbol , drawn to any size, means $a - 3$.

The symbol , drawn to any size, means b^2 .

Find the value of  $+$  $-$ .

Solution. The given expression is equal to

$$\boxed{6} + \triangle 2 - \triangle \boxed{4} = 36 + (-1) - 13 = \boxed{22}.$$

55. Your math teacher has recently bought a new tape deck.

Being well organised, your math teacher had labelled each tape with the number on the counter when each song began. Now, a whole tape ran to 634 on the old counter.

On the new deck, the same tape runs to 2219 on the new counter.

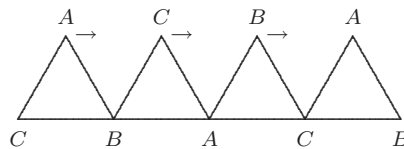
A favourite song was at counter 164 on the old tape deck. What is it at on the new tape deck?

Note that the total listening time of the tape was the same on each deck.

Solution.

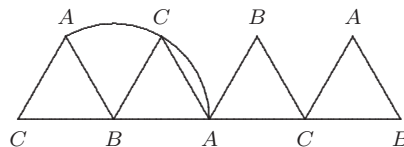
The number on the counter must be $\frac{2219}{634} \times 164 = \boxed{574}$.

56. An equilateral triangle of side length 5 cm is “rolled” over along a line, pivoting at the bottom right hand vertex, until the triangle is again in its original configuration.



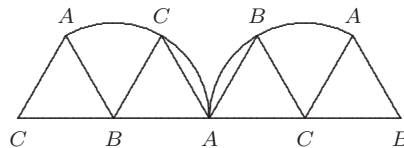
Determine the **exact** distance travelled by the vertex A .

Solution. In the first “rolling”, the vertex A traverses an arc of a circle with centre at B and radius $BA = 5$ cm. This arc is $\frac{1}{3}$ of a whole circle.



In the second “rolling”, the vertex A does not move.

In the third “rolling”, the vertex A again traverses an arc that is $\frac{1}{3}$ of a whole circle.



Thus, the vertex A has traverses $\frac{2}{3}$ of a circle of radius 5 cm.

The circumference of a circle is given by $2\pi r$; in this case 10π . We need $\frac{2}{3}$ of this;

that is, $\boxed{\frac{20\pi}{3}}$.

57. Nan and Pop have twelve children. Nan spends \$3.51 for five apples and seven oranges. Pop spends \$4.89 for seven apples and five oranges at the same store at the same time. Each child is given one apple and one orange.

How much did Nan and Pop spend on each child?

Solution.

Together, they have spent $\$3.51 + \$4.89 = \$8.40$ for twelve apples and twelve oranges.

Thus, one apple and one orange costs $\frac{\$8.40}{12} = \boxed{\$0.70}$.

58. One day, one member of the class was sick, and missed the test. The class average was 75%.

The sick person was allowed to write an equivalent test the next week, and the result was that the class average rose to 75.5%.

If there were a total of 28 people in that class, what mark did the sick person get?

Solution. The total number of marks on the test (without the sick person) was $75 \times 27 = 2025$.

The total number of marks after the sick person wrote was $75.5 \times 28 = 2114$.

The difference gives the sick person's mark: $2114 - 2025 = \boxed{89}$.

59. The residents of Mathland have a secret code indecipherable to residents of Nonmathland.

They write down the prime numbers (starting, of course, with 2) in ascending order, and assign to each the letters of the alphabet in alphabetical order. They then multiply the numbers together to give the code for the word.

Decipher 110618.

Solution. What are the prime factors of 110618? Clearly, 2 is a factor and, $110618 = 2 \times 55309$. Clearly we have all the 2's.

We must now factor 55309. This needs some trials. The next prime to work is 19, leaving $2911 = 41 \times 71$.

We have 2, 19, 41, 71, which correspond to A, H, M, T.

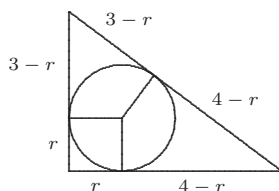
It is easy to see that the word is $\boxed{\text{MATH}}$.

60. A circle is drawn inside a 3, 4, 5 right triangle. The circle is tangent to the three sides.

Determine that exact value of the radius of this circle.

HINT: when two tangents are drawn to a circle from a point outside it, the tangents are of the same length.

Solution. Let the radius of the circle be r . The lengths of the tangents from the ends of the hypotenuse are then $3 - r$ and $4 - r$.



Looking at the hypotenuse, we see that its length is $(3 - r) + (4 - r) = 7 - 2r$. But we know that its length is 5.

Thus, $7 - 2r = 5$, giving $r = \boxed{1}$.

61. Determine the sum of all prime numbers which are less than 50.

Solution. The primes less than 50 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

Their sum is $\boxed{328}$.

62. A man spent four-fifths of his money and misplaced four-fifths of the remainder, leaving him with \$17.00.

With how much money did he start?

Solution. If he spent four-fifths of his money, he then had one-fifth left.

He misplaced four-fifths of that; that is, he had one-fifth of the one-fifth left; that is one-twenty fifth of the original amount.

Thus, one-twenty fifth of the original amount is \$17.00, so that the original amount is $\$17.00 \times 25 = \boxed{\$425.00}$.

63. A girl walks three-quarters of the way home in 18 minutes.

If she continues to walk at the same speed, how long will it take her to walk the rest of the way home?

Solution. She has still one-quarter of the way home to walk; that is, one-third of what she has already walked.

Thus, she will take $\frac{18}{3} = \boxed{6 \text{ minutes}}$ to complete the walk home.

64. Pat had collected some pennies in a jar. They would make 14 rows of equal length, or 15 rows of equal length, or 20 rows of equal length.

Determine the minimum number of pennies that Pat had.

Solution. The number of pennies is the least common multiple of 14, 15 and 20.

Note, $14 = 2 \times 7$, $15 = 3 \times 5$, and $20 = 2 \times 2 \times 5$.

Hence, the number of pennies is $2 \times 2 \times 3 \times 5 \times 7 = \boxed{420}$.

65. Determine the largest prime factor of 52710.

Solution. We clearly only need to consider 5271. Since $5 + 2 + 7 + 1 = 15 = 3 \times 5$, we have that 3 is a factor.

We then only have to consider $\frac{5271}{3} = 1757$.

Now, none of 2, 3 or 5 is a factor of this. So we try 7, and we discover that $1757 = 7 \times 251$.

Since $\sqrt{251} \approx 15.84 \dots$, we need only test for 7, 11 and 13 (none of which are factors), so that the required answer is $\boxed{251}$.

66. A cruise ship left St. John's for New York travelling at 16 km/h. One hour later, another cruise ship left St. John's for New York travelling along the same route at 18 km/h. How long did it take for the second cruise ship to overtake the first cruise ship?

Solution. In one hour, the first cruise ship will have travelled 16 km.

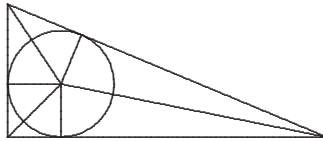
In each subsequent hour, the first ship will have travelled 16 km and the second ship will have travelled 18 km. Thus, the first ship will have travelled 2 km less than the second. Since the first ship had a lead of 16 km, it will take 8 hours for the second ship to overtake the first ship.

67. A circle is drawn inside a 5, 12, 13 right triangle. The circle is tangent to the three sides.

Determine that exact value of the radius of this circle.

HINT: when two tangents are drawn to a circle from a point outside it, the tangents are of the same length.

Solution. Let the radius of the circle be r . Join the centre of the circle to each vertex.



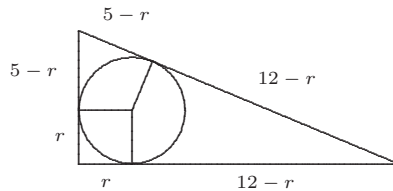
The areas of the three triangles (two vertices and the centre) sum to the area of the whole triangle:

$$\frac{12r}{2} + \frac{13r}{2} + \frac{5r}{2} = \frac{30r}{2} = 15r = \frac{12 \times 5}{2} = 30.$$

Hence, $r = \span style="border: 1px solid black; padding: 0 2px;">2.$

ALTERNATIVELY:

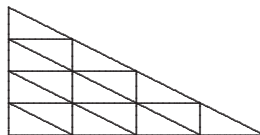
The lengths of the tangents from the ends of the hypotenuse are then $5 - r$ and $12 - r$.









Looking at the hypotenuse, we see that its length is $(5 - r) + (12 - r) = 17 - 2r$. But we know that its length is 13.

Thus, $17 - 2r = 13$, giving $r = \span style="border: 1px solid black; padding: 0 2px;">2.$

68. Determine the total number of right triangles in the following figure.



Solution. We do a systematic count.

1		10		1	6
2		6		2	1
3		3			
4		1			

Thus, the total is 27

69. A committee is made up of three boys and five girls.

They decide to have a subcommittee of five persons, and they agree that it must have at least one boy and at least two girls.

How many different subcommittees are possible?

Solution. From the three boys, we will have to choose one, two or all three.

We can choose one in 3 different ways. We can choose two in 3 different ways (this is the same as excluding one). We can choose all three in only 1 way.

From the five girls, we will have to choose four, three or two.

We can choose four in 5 different ways (this is the same as excluding one). We can choose three in 10 different ways (this is the same as excluding two). We can choose two in 10 different ways (as the previous statement shows).

So we have one boy with four girls: 3×5 ways; two boys with three girls: 3×10 ; and three boys with two girls: 1×10 .

Thus, the total is $15 + 30 + 10 =$ 55.

70. Pamela has two daughters. One is five years older than the other. The sum of their ages is 27. How old is the younger daughter?

Solution. The sum of their ages is twice the age of the younger plus 5. This is 27, so that twice the age of the younger is $27 - 5 = 22$. Thus, the younger is 11 years old.

71. A man spent six-sevenths of his money and misplaced six-sevenths of the remainder, leaving him with \$11.00.

With how much money did he start?

Solution. If he spent six-sevenths of his money, he then had one-seventh left.

He misplaced six-sevenths of that; that is, he had one-seventh of the one-seventh left; that is one-forty ninth of the original amount.

Thus, one-forty ninth of the original amount is \$11.00, so that the original amount is $\$11.00 \times 49 =$ \$539.00.

72. Find the average of all the even whole numbers between 27 and 1003 inclusive.

Solution. We need the average of the numbers 28, 30, ..., 1002.

These are twice the numbers 14, 15, ..., 501.

The average of a set of **consecutive** whole numbers is the average of the first and the last; in this case, $\frac{14 + 501}{2} = \frac{515}{2}$.

But, we need twice that number. Hence, the answer is 515.

73. Determine the exact value of

$$\sqrt{20 + \sqrt{21 + \sqrt{12 + \sqrt{13 + \sqrt{6 + \sqrt{7 + \sqrt{3 + \sqrt{1}}}}}}}}$$

NOTE: $\sqrt{\dots}$ means the POSITIVE value of the square root.

Solution.

$$\begin{aligned} & \sqrt{20 + \sqrt{21 + \sqrt{12 + \sqrt{13 + \sqrt{6 + \sqrt{7 + \sqrt{3 + \sqrt{1}}}}}}}} \\ &= \sqrt{20 + \sqrt{21 + \sqrt{12 + \sqrt{13 + \sqrt{6 + \sqrt{7 + \sqrt{4}}}}}}}} \\ &= \sqrt{20 + \sqrt{21 + \sqrt{12 + \sqrt{13 + \sqrt{6 + \sqrt{9}}}}}}}} \\ &= \sqrt{20 + \sqrt{21 + \sqrt{12 + \sqrt{16}}}}}} \\ &= \sqrt{20 + \sqrt{21 + \sqrt{16}}}} \\ &= \sqrt{20 + \sqrt{25}} = \sqrt{25} = \boxed{5}. \end{aligned}$$

74. Three people went to mow a meadow.

When they worked together, it took 6 days.

Alice worked three times as quickly as Charlie and Bruce worked twice as quickly as Charlie.

Later, there were three more meadows, identical to the original meadow, to mow, and each worked separately.

Determine how long each took to mow their meadow.

The answer we are seeking is the average number of days that it took to mow the three more meadows.

Solution. When they worked together, in six days, Alice mowed $\frac{1}{2}$ of the meadow,

Bruce mowed $\frac{1}{3}$ of the meadow and Charlie mowed $\frac{1}{6}$ of the meadow.

Hence, Alice can mow a meadow in 12 days, Bruce in 18 days and Charlie in 36 days.

The average of these three numbers is $\frac{12 + 18 + 36}{3} = \frac{66}{3} = \boxed{22}$ days.

75. A cruise ship left St. John's for New York travelling at 16 km/h. One hour later, another cruise ship left St. John's for New York travelling along the same route at 20 km/h. How long did it take for the second cruise ship to overtake the first cruise ship?

Solution. In one hour, the first cruise ship will have travelled 16 km.

In each subsequent hour, the first ship will have travelled 16 km and the second ship will have travelled 20 km. Thus, the first ship will have travelled 4 km less than the second. Since the first ship had a lead of 16 km, it will take 4 hours for the second ship to overtake the first ship.

76. Josh saved \$250 from a summer job. He put it into a very special bank account that gave 5% interest per month. The bank calculated the interest each month, and rounded it down to the nearest cent.

After leaving it in the bank for four months, Josh withdrew the money. How much did he then have?

Solution. After one month, Josh had $\$250 \times 1.05 = \262.50 in the bank.

After the second month, Josh would have had $\$262.50 \times 1.05 = \275.625 . The bank credited him with \$275.62.

After the third month, Josh would have had $\$275.62 \times 1.05 = \289.401 . The bank credited him with \$289.40.

After the fourth month, Josh would have had $\$289.40 \times 1.05 = \303.87 . Josh withdrew \$303.87.

77. Start with a first square with side length 1.

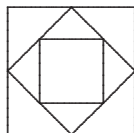
Construct a second square by adding isosceles right triangles on the outside of the sides of the first square.

Construct a third square by adding isosceles right triangles on the outside of the sides of the second square.

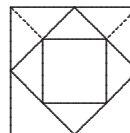
Continue this process until you get to the 99th square.

Determine the length of the side of the 99th square.

Solution. We draw a diagram only as far as the third square.



By adding two lines,



we can see that the length of the side of the third square is twice the length of the side of the first square (that is, 2).

By a similar argument, the length of the side of the fifth square is twice the length of the side of the third square (that is, $2 \times 2 = 2^2$).

Continuing in this way, the length of the side of the seventh square is 2^3 ; the length of the side of the ninth square, 2^4 , etc..

Note: to get the power of the 2, take number of the square, subtract 1, and divide that answer by 2.

Hence, the length of the 99th square is 2^{49} .

78. A committee is made up of three boys and five girls.

They decide to have a subcommittee of four persons, and they agree that it must have an equal number of boys and girls.

How many different subcommittees are possible?

Solution. From the three boys, we will have to choose two.

We can choose two in 3 different ways (this is the same as excluding one).

From the five girls, we will have to choose two.

We can choose two in 10 different ways.

Thus, the total is $3 \times 10 = \boxed{30}$.

79. A gardener added 1 kg of sand to 15 kg of earth. If the resulting mixture was uniform, how many grams of sand are there in 1 kg of the mixture. Exact answer required.

Solution. The proportion of sand in the mixture is $\frac{1}{1+15} = \frac{1}{16}$.

In 1 kg; that is 1000 grams, there will be $1000 \times \frac{1}{16} = \frac{1000}{16} = \boxed{62.5 \text{ grams of sand}}$.

80. [NO CALCULATORS] My hard drive has a capacity of 120 billion bytes. The average size of a jpeg image taken by my digital camera is 1.5 million bytes.

What is the maximum number of average sized jpeg images that I can store on my hard drive?

Solution. We need to calculate

$$\frac{120 \times 1\,000\,000\,000}{1.5 \times 1\,000\,000} = \boxed{80\,000}.$$

81. The product of two positive numbers is 2160. The ratio of the two numbers is 60. Determine the two numbers.

Solution. If we multiply the product (1st number times 2nd number) with the ratio (1st number divided by 2nd number), we get the square of the 1st number.

That is $2160 \times 60 = 129600$. The square root of 129600 is 360.

Hence, the 2nd number is $\frac{2160}{360} = 6$.

Thus, the numbers are $\boxed{360 \text{ and } 6}$.

82. Determine the measure of the angle between the hands of a clock when it shows a time of 3:30.

Solution. A complete circle has 360° . Since there are 12 hours, the hour hand travels $\frac{360}{12} = 30^\circ$ for each hour.

At 3 : 30, the minute hand is pointing down; that is, at 180° from the top.

At 3 : 30, the hour hand is half way between 3 and 4; that is $90 + \frac{30}{2} = 105^\circ$ from the top in a clockwise direction.

Hence, the answer is $180 - 105 = \boxed{75^\circ}$. [Accept an answer of $\boxed{75}$.]

83. When a painting is framed, it is placed behind a rectangular hole in a piece of card, known as a picture frame mat.

The outside dimensions of a picture frame mat measure 32 cm by 24 cm. The mat is cut with a border of 4 cm on each side.

Determine the percentage of the mat that remains after the hole is cut out.

Solution. The opening is 24 cm by 16 cm.

The area of the opening is $24 \times 16 = 384$ sq. cm.

The area of the mat is $32 \times 24 = 768$ sq. cm.

The required percentage is $\frac{768 - 384}{768} \times 100 = \boxed{50\%}$.

[Accept an answer of $\boxed{50}$.]

84. In a group of 75 people, 49 have brown hair, 31 have blue eyes, and 24 have both brown hair and blue eyes.

How many people in the group have neither brown hair nor blue eyes?

Solution. Look at this diagram:

	Brown hair	
Blue eyes		
	Brown hair	
Blue eyes	24	7
	25	

There are 31 in the top row, 49 in the left column and 24 in the top left box. This means that there are 7 in the top right box and 25 is the bottom left box.

And there are a total of 75 in all the boxes. This means that there are $75 - 24 - 7 - 25 = \boxed{19}$ with neither brown hair nor blue eyes.

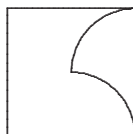
85. Determine the exact value of

$$\sqrt{3 \frac{\sqrt{6 + \sqrt{7 + \sqrt{3 + \sqrt{1}}}}}{\sqrt{7 + \sqrt{76 + \sqrt{22 + \sqrt{9}}}}}}.$$

NOTE: $\sqrt{\dots}$ means the POSITIVE value of the square root.

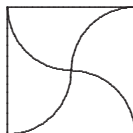
Solution. $\boxed{\frac{3}{2}}$.

86. A square of side length 4 is divided into two regions using arcs of circles of radius 2.



Determine the area of the smaller region.

Solution. We draw two more arcs.



By symmetry, the square is divided into four equal regions (equal to the desired region). Hence, the area of each is one quarter of the area of the square; that is $\boxed{4}$.

87. Determine the smallest number which can be divided exactly by all of 3, 9, 12 and 15.

Solution. Since $9 = 3 \times 3$, we can ignore 3 alone.

Note that $12 = 3 \times 4$ and $15 = 3 \times 5$.

Thus, we need to multiply all of 9, 4 and 5, giving the required number as $\boxed{180}$.

88. The product of the digits in a two digit number is 24.

The difference of the digits is 5. Determine the sum of the digits.

Solution. The possible pairs of digits, differing by 5, are

0, 5 1, 6 2, 7 3, 8 4, 9,

with products

0 6 14 24 36.

This shows that the two digits are 3 and 8, with sum $\boxed{11}$.

89. A class of 18 students wrote a math test, and the class average was 70%. The average of the top 12 students was 85%. Determine the average of the bottom 6 students.

Solution.

The total number of marks earned by the 18 students is $18 \times 70 = 1260$.

The total number of marks earned by the 12 students is $12 \times 85 = 1020$.

The total number of marks earned by the 6 students is $1260 - 1020 = 240$.

Hence, the average of the 6 students is $\frac{240}{6} = \boxed{40}$.

90. A number of students are standing equally spaced around a circle. The 4th student is standing diametrically opposite to the 16th student.

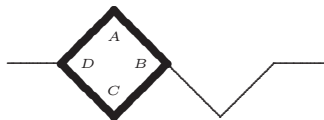


Determine how many students are standing in the circle.

Solution. Between the 4th student and the 16th student, there are 11 students.

Hence, there must be 22 students as well as the 4th student and the 16th student, making a total of $\boxed{24}$ students.

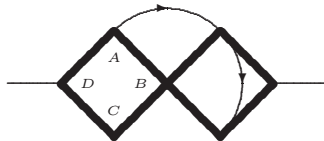
91. In the plane, a square $ABCD$ of side length 1 sits in a 90° hole (which has an adjacent 90° hole).



In the plane, the square rotates about B until it sits in the adjacent hole.

Determine the exact value of the length of the path described by the point A .

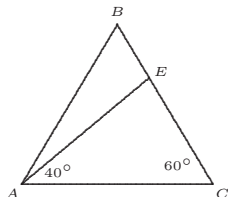
Solution. The point A describes an arc of a circle, radius $BA = 1$.



We see that A describes a half circle of radius 1. It follows that the distance is exactly $\boxed{\pi}$.

92. Triangle ABC is equilateral. Point E is on side BC such that $\angle EAC = 40^\circ$. Determine the value of $\angle BEA$.

Solution.



$\angle BCA = 60^\circ$ and $\angle EAC = 40^\circ$, so that $\angle AEC = 180^\circ - 60^\circ - 40^\circ = 80^\circ$.

Hence, $\angle BEA = 180^\circ - 80^\circ = \boxed{100^\circ}$.

93. In his grade 7 math course, Nonnash got a final grade of 70%. Nonnash wrote five tests, but the worst mark of 20% was discarded in calculating the final grade. Determine Nonnash's average on all five tests.

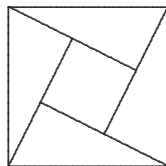
Solution.

On the four tests, Nonnash's total number of marks is $70 \times 4 = 280$.

Adding in the worst mark of 20%, we have a total of $280 + 20 = 300$ marks.

Hence, the average on all five tests is $\frac{300}{5} = \boxed{60\%}$.

94. Four congruent right triangles with legs of 1 and 3 are drawn inside a square, forming a smaller square.



Determine the area of the larger square.

Solution. The side of the smaller square is clearly 2. Thus, the area of the smaller square is 4.

The area of each triangle is $\frac{1}{2} \times 1 \times 3 = \frac{3}{2}$. There are four of them, so that the area of the triangles is $4 \times \frac{3}{2} = 6$.

Hence, the area of the outer square is $\boxed{10}$.

ALTERNATIVELY:

The hypotenuse of each triangle is $\sqrt{10}$, so that the area of the outer square is $\boxed{10}$.

95. Determine the sum of the digits of $2^{2009} \times 5^{2007}$.

Solution. We have $2^{2009} \times 5^{2007} = 2^2 \times 2^{2007} \times 5^{2007} = 4 \times 10^{2007} = 4000 \dots 0000$.

Hence, the answer is $\boxed{4}$.

96. A day is called **multiplicative** if the product of the day number and the month number is equal to the number given by the tens and units digits of the year.
For example, 03/08/24 was multiplicative in 1924. (Assumed to mean **day/month/year**.)

Determine how many multiplicative dates there are in the year 2009.

Solution. We need all pairs of positive integers whose product is 9; that is, 1 and 9 and 3 and 3.

These lead to the dates 01/09/2009, 09/01/2009 and 03/03/2009, showing that the required answer is $\boxed{3}$.

97. Four people can dig four holes in four days.

Determine the number of days required for two people to dig two holes.

Solution. Four people can dig four holes in four days.

Therefore, one person can dig one hole in four days.

Hence, two people can dig two holes in four days.

The required answer is $\boxed{4}$.

98. A three digit number consists of one each of the digits 7, 8 and 9.

Determine which of these possible numbers has the smallest sum of its prime factors. The required answer is that sum.

Solution. We have

$$987, 879, 789, 978, 897, 798.$$

Factoring gives

$$3 \times 7 \times 47, 3 \times 293, 3 \times 263, 2 \times 3 \times 163, 3 \times 13 \times 23, 2 \times 3 \times 7 \times 19.$$

The smallest prime factor sum is from 798, giving $2 + 3 + 7 + 19 = \boxed{31}$.

99. Determine the number of odd positive whole number factors of 840.

Solution. The prime factorisation of 840 is $2^3 \times 3 \times 5 \times 7$.

The odd prime factors are then 3, 5 and 7.

Thus, we have the possible odd factors of 3, 5, 7, 3×5 , 3×7 , 5×7 and $3 \times 5 \times 7$ (7 possibilities).

But, 1 is also an odd factor of 840.

Hence, the total number is $\boxed{8}$.

100. The sum of 17 consecutive whole numbers is 2244.

Determine the smallest number of the 17.

Solution. The average of the numbers is $\frac{2244}{17} = 132$.

This must be the 9th number of the set.

Hence, the least is $132 - 8 = \boxed{124}$.

101. A sequence of three letters represents a three digit number.

Determine how many three digit numbers, abc , satisfy the equation $abc = cba$.

Solution. If $abc = cba$, then we must have $a = c$ (with nine possibilities, since neither can be a zero), and ten possibilities for b .

Hence, the answer is $9 \times 10 = \boxed{90}$.

102. Rent-a-Lemon Vehicle Hire Company has 36 vehicles. All vehicles have 2, 3, or 4 road wheels each (excluding any spares).

Exactly six of the vehicles have 4 wheels each. All together the vehicles have 100 wheels.

Determine how many vehicles have 2 wheels each.

Solution. Since 36 vehicles have 100 wheels, if 6 vehicles have 4 wheels, then there are 30 vehicles and 76 wheels left.

It is reasonable to use guess and check to solve this.

Try 15 vehicles with each of 2 and 3 wheels.

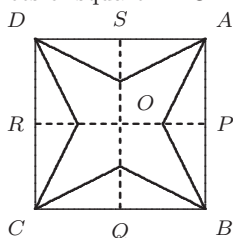
That would make $15 \times 2 + 15 \times 3 = 15 \times 5 = 75$ wheels. Not the solution. But note that $75 < 76$ — we need one more wheel!

Therefore, try 16 with 3 wheels and 14 with 2 wheels.

$16 \times 3 + 14 \times 2 = 48 + 28 = 76$. Solved!

Hence, the required number is $\boxed{14}$.

103. A square $ABCD$ has side length 2. The mid-points P , Q , R and S of opposite sides are joined, meeting at the point O . The mid-points of OP , OQ , OR and OS are joined to their nearer vertices of square $ABCD$ to form a star-like figure.



Determine the area of the star-like figure.

Solution. We start with a square of area 4, from which we must subtract four congruent triangles, each of which has base length 2 and height length $\frac{1}{2}$, and thus, has area $\frac{1}{2}$.

Hence, the required area is $4 - \frac{1}{2} \times 4 = 4 - 2 = \boxed{2}$.

Relays.**Relay #1 — Sample Relay**

- R1. Determine how many whole numbers strictly between 1 and 111 are exact multiples of 7.

Call your answer A .

Write A in Box #1 of the Relay Answer Sheet.

- R2. Pat's age is A . Pat's father is three times Pat's age.

Pat's mother is three years younger than her husband.

Pat's sibling is one third of the father's age.

Determine the sum of the four ages.

Then determine the distinct prime factors of this sum.

Call the sum of these prime factors B .

Write B in Box #2 of the Relay Answer Sheet.

- R3. Chris makes four widgets in B hours. Chris's friends makes five widgets in $B - 1$ hours.

How long will it take them working together to make seven widgets? Call your answer C .

Write C in Box #3 of the Relay Answer Sheet.

- R4. The perimeter of a rectangular area is, in centimetres, C .

The sides of the area are whole numbers, also in centimetres.

There are several possible dimensions for this area.

Determine them all, and write the sum of these areas in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

$\boxed{15}, \boxed{16}, \boxed{12}, \boxed{22}$.

It is normal practice not to give solutions to the Relay, since the game is usually over at the end of the Relay.

However, we will give one set of solutions here.

Relay #1 Solutions

R1.Soln. The multiples of 7 in the range are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98 and 105.

Thus, there are $\boxed{15}$ such multiples.

OR: note that $\frac{111-1}{7} = 15 + \frac{5}{7}$, showing that there are $\boxed{15}$ such multiples.

R2.Soln. Pat is 15 years old. Pat's father is then 45 years old, Pat's mother is 42 years old and Pat's sibling is also 15 years old (is Pat a twin?).

The sum of the ages is $15 + 45 + 42 + 15 = 117 = 3^2 \times 13$.

The prime factors are 3 and 13, giving the sum as $\boxed{16}$.

R3.Soln. Chris makes 4 widgets in 16 hours; that is, Chris makes $\frac{1}{4}$ of a widget in 1 hour.

Chris's friend makes 5 widgets in 15 hours; that is, Chris's friend makes $\frac{1}{3}$ of a widget in 1 hour.

Together, they make $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$ of a widget in 1 hour.

Thus, they make 7 widgets in $\boxed{12}$ hours.

R4.Soln. The rectangular area has a perimeter of 12 centimetres.

Thus, the sum of the adjacent sides is 6 centimetres.

The lengths of the possible sides are then 1 and 5, 2 and 4 and 3 and 3, giving possible areas of 5, 8 and 9.

The sum of these numbers is $\boxed{22}$.

Relay #2

- R1. Determine how many whole numbers strictly between 10 and 100 are exact multiples of 7.

Write your answer in Box #1 of the Relay Answer Sheet.

- R2. Pat's age is your answer to Relay Question 1.

Pat's father is three times Pat's age.

Pat's mother is three years younger than her husband.

Pat's sibling is one third of the mother's age.

Determine the sum of the four ages.

Then determine the distinct prime factors of this sum.

Write the product of these prime factors in Box #2 of the Relay Answer Sheet.

- R3. Chris makes three widgets in the number of hours that is the answer to Relay Question 2.

Chris's friend makes four widgets in one hour less than the number of hours that is the answer to Relay Question 1.

How long will it take them working together to make sixty-seven widgets?

Write your answer in Box #3 of the Relay Answer Sheet.

- R4. The perimeter of a rectangular area is, in centimetres, the answer to Relay Question 3.

The sides of the area are, in centimetres, whole numbers.

There are several possible values for this area.

Determine the largest such area, and write the value, in square centimetres, of this area in Box #4 of the Relay Answer Sheet. Hand the Answer Sheet to your proctor.

Answers.

13	,	10	,	90	,	506
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Relay #3

- R1. Determine how many whole numbers strictly between 20 and 200 are exact multiples of 7.

Write your answer in Box #1 of the Relay Answer Sheet.

- R2. Pat's age is your answer to Relay Question 1.

Pat's father is two times Pat's age.

Pat's mother is four years younger than her husband.

Pat's youngest sibling is one quarter of the mother's age.

Determine the sum of the four ages.

Then determine the distinct prime factors of this sum.

Write the sum of these prime factors in Box #2 of the Relay Answer Sheet.

- R3. Chris makes four widgets in the number of hours that is the answer to Relay Question 2.

Chris's friend makes three widgets in one hour less than the number of hours that is the answer to Relay Question 2.

How long will it take them working together to make sixteen widgets?

Write your answer in Box #3 of the Relay Answer Sheet.

- R4. The area of a rectangular area is, in square centimetres, the answer to Relay Question 3.

The sides of the area are, in centimetres, whole numbers.

There are several possible values for the perimeters of this area.

Determine all such perimeters, and write the value of the sum of all such perimeters, in centimetres in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

26	,	28	,	63	,	208	.
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Relay #4

R1. Determine all prime factors of 120.

Write the sum of these prime factors in Box #1 of the Relay Answer Sheet.

R2. When a number is divided by the answer to question R1, the result is 5 less than when the number is divided by one less than the answer to question R1.

Find this number and write it in Box #2 of the Relay Answer Sheet.

R3. The area of a rectangle is one half of the answer to question R2.

The sides are whole numbers.

There are several possible values for the sides in which one side is a strictly smaller number than the other side.

Find the sum of these smaller numbers, and write it in Box #3 of the Relay Answer Sheet.

R4. If you buy as many widgets as the answer to question R1 and as many wodguts as the answer to question R3, the cost, in dollars is the answer to question R2.

If you buy as many widgets as the answer to question R3 and as many wodguts as the answer to question R1, the cost, in dollars, is 614.

Determine the cost of a widget and the cost of a wodgut.

Write the cost, in dollars, of one widget plus one wodgut in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

10	,	450	,	18	,	38
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Relay #5

- R1. Determine the number of multiples of 13 that are greater than 20 and less than 260.

Call your answer A .

Write A in Box #1 of the Relay Answer Sheet.

- R2. If Pat is A years old and Chris is $(2A - 20)$ years old today, determine the sum of their ages when the younger of Chris and Pat was 5 years old.

Call your answer B .

Write B in Box #2 of the Relay Answer Sheet.

- R3. A regular polygon has B sides.

Determine the measure (in degrees) of each internal angle of the polygon.

Call your answer C .

Write C in Box #3 of the Relay Answer Sheet.

- R4. Find the largest prime factor of the number $2A + 3B + 5C$.

Write your answer in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

18	,	12	,	150	,	137
----	---	----	---	-----	---	-----

Relay #6

- R1. Determine the number of multiples of 17 that lie strictly between 40 and 340.

Call your answer A .

Write A in Box #1 of the Relay Answer Sheet.

- R2. If Pat is A years old and Chris is $(3A - 40)$ years old today, determine the sum of their ages when the younger of Chris and Pat was 6 years old.

Call your answer B .

Write B in Box #2 of the Relay Answer Sheet.

- R3. Each interior angle of a regular polygon has measure $9B$ degrees.

Determine the number of sides of this regular polygon.

Call your answer C .

Write C in Box #3 of the Relay Answer Sheet.

- R4. Determine the largest prime factor of $15A + 12B + 9C$.

Write your answer in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

17	,	18	,	20	,	31
----	---	----	---	----	---	----

Relay #7

- R1. The hour hand of a clock is 3 cm. long and the minute hand is 4 cm. long.

Determine the length A cm. of the distance between the tips of the hands at exactly 9 o'clock.

Write the number A in Box #1 of the Relay Answer Sheet.

- R2. Two numbers, A (from the previous question) and B are placed side by side in both ways (AB and BA) to make a two-digit number.

When you subtract the smaller (AB) of these numbers from the larger (BA), the answer is 36.

Determine the number B .

Write the number B in Box #2 of the Relay Answer Sheet.

- R3. Multiply the numbers A and B together.

Then add to that result A , B and 1 to get a new number.

Find all the prime factors of that new number.

Add all the distinct prime factors together and call that number C .

Write the number C in Box #3 of the Relay Answer Sheet.

- R4. Given a rectangular grid of unit squares with a length A on one side and a length C on the other, determine the total number of squares of all sizes with sides consisting of line segments already on the grid.

Call your answer D .

Write the number D in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

5

,

9

,

10

,

130

.

Relay #8

- R1. You are given two consecutive three digit numbers.

The sum of all the digits is 29.

Determine A , the larger of the two numbers which are the largest possible solution.

Write the number A in Box #1 of the Relay Answer Sheet.

- R2. Determine the number B , which is the sum of the distinct prime factors of A .

Write the number B in Box #2 of the Relay Answer Sheet.

- R3. Find the sum of the digits of A — call it X .

Find the sum of the digits of B — call it Y .

Find two numbers by writing these new numbers in order.

(For example, if it were true that $X = 35$ and $Y = 79$, then your new numbers would be 3579 and 7935.)

Determine the number C which is the difference of these two numbers (a positive value).

Write the number C in Box #3 of the Relay Answer Sheet.

- R4. You now have three numbers A , B and C .

From the largest, subtract the sum of the other two.

Call your answer D .

Write the number D in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

951

,

320

,

360

,

271

.

Relay #9

- R1. Determine A , the number of prime numbers greater than 10 and less than 50.

Write the number A in Box #1 of the Relay Answer Sheet.

- R2. Alicia has $\$(A + 7)$.

Betty has 50% more than Alicia.

Alicia has 50% more than Celia.

Determine B , the average amount that the three people have.

Write the number B in Box #2 of the Relay Answer Sheet.

- R3. A right angled triangle has a side of length $(B - A)$ and hypotenuse of length $\frac{A+B}{3}$.

Determine C , the length of the third side.

Write the number C in Box #3 of the Relay Answer Sheet.

- R4. Given that $xA + yB + zC = 12$, where x , y and z are integers, determine D , the smallest positive value of $x + y + z$.

Write the number D in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

$\boxed{11}, \boxed{19}, \boxed{6}, \boxed{1}$.

Relay #10

- R1. Let A be the sum of the digits of the sum of all the integers from 1 to 100 inclusive.

[HINT: do not waste time using a calculator to try to determine this sum.]

Write the number A in Box #1 of the Relay Answer Sheet.

- R2. Let X and Y represent the two prime numbers closest to A , X being greater than A and Y being smaller than A .

Let $Z = ((A \times X) \times (A \times Y))$.

Determine the number B which is the integer nearest to the square root of Z .

Write the number B in Box #2 of the Relay Answer Sheet.

- R3. A circle of radius $\frac{B}{C}$ has the property that the numerical value of the length of its circumference is equal to the numerical value of its area.

Write the number C in Box #3 of the Relay Answer Sheet.

- R4. Determine the largest prime factor of $A \times B + B \times C + C \times A$. Call your answer D .

Write the number D in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

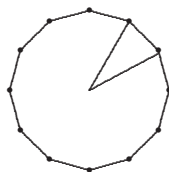
10	,	88	,	44	,	59
----	---	----	---	----	---	----

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Relay #11

- R1. A **quoderat** team has 4 players on the field at any instant.
 The coach has a bench strength of 4 more players.
 The positions are **right flot**, **left flot**, **right grot** and **left grot**.
 Determine A , the number of possible different arrangements of players that the coach can assign to the starting team.
 Write the number A in Box #1 of the Relay Answer Sheet.
- R2. You are given the number A just calculated.
 Determine the sum of its digits and then square that sum.
 Determine the sum of the digits of the new number and then square that sum.
 Continue doing this until the sum and the square repeats.
 Call that sum B .
 Write the number B in Box #2 of the Relay Answer Sheet.
- R3. A regular polygon has B sides.
 Draw lines joining two adjacent vertices to the centre.

For example:



- Determine the measure in degrees of the angle created at the centre.
 Call that number C .
 Write the number C in Box #3 of the Relay Answer Sheet.
- R4. Take the fraction defined by $\frac{A}{B \times C}$ and reduce it to its lowest terms $\frac{X}{Y}$. Add X and Y , and call your answer D .
 Write the number D in Box #4 of the Relay Answer Sheet, and hand it to your proctor.

Answers.

1680, 9, 40, 17.

Relay #12

- R1. Andrew has a number of hockey cards. He gives all but one (a Gretzky) to Janet. Janet gives half of the cards she got from Andrew to Anna. Anna gives half of the cards she received from Janet to Susanne, and Susanne gives half of the cards she got from Anna to Bruce. Bruce received 6 cards. How many did Andrew have originally? That number is A^2 .

Write the number A in Box #1 of the Relay Answer Sheet.

- R2. Determine first the product of the first A prime numbers.

Then, determine the sum of the digits of this number. Call your answer B .

Write the number B in Box #2 of the Relay Answer Sheet.

- R3. B workers can make 3 objects in 4 hours.

8 workers can make 4 objects in C hours.

Determine the number C .

Write the number C in Box #3 of the Relay Answer Sheet.

- R4. Add together the three numbers A , B and C .

Determine the total number of whole number factors of this sum.

Call your answer D .

Write the number D in Box #4 of the Relay Answer Sheet.

Hand the Answer Sheet to your proctor.

Answers.

7	,	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td style="padding: 2px 5px;">12</td></tr></table>	12	,	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td style="padding: 2px 5px;">8</td></tr></table>	8	,	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td style="padding: 2px 5px;">4</td></tr></table>	4
12									
8									
4									

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Tie Breakers

1. Add up all the whole numbers from 1 to n and divide by 23. Determine the smallest value of n that ensures that your answer is a whole number.

Solution. Look at the sum of the numbers from 1 to n :

n	sum
1	1
2	3
3	6
4	10
5	15
\vdots	\vdots

It is not too hard to see that there is a formula for the sum: $\frac{n(n+1)}{2}$.

Since 23 is a prime number, it will not occur in the numerator of this number until $n = \boxed{22}$.

2. Determine the value of the denominator when $1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}}$ is written

out as a rational number (fraction) in lowest terms; that is, with no common factors between the numerator and the denominator.

Solution.

$$\begin{aligned}
 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}} &= 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{\frac{21}{5}}}} \\
 &= 1 + \frac{1}{2 + \frac{1}{3 + \frac{5}{21}}} \\
 &= 1 + \frac{1}{2 + \frac{1}{\frac{68}{21}}} = 1 + \frac{1}{2 + \frac{21}{68}} \\
 &= 1 + \frac{1}{\frac{157}{21}} = 1 + \frac{21}{157} \\
 &= \frac{178}{157}.
 \end{aligned}$$

Now, 157 is a prime number, and thus, the answer is $\boxed{157}$.

3. Determine the sum of all prime numbers which are less than 50.

Solution. The primes less than 50 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

Their sum is $\boxed{328}$.

4. Pamela has two daughters. One is five years older than the other. The sum of their ages is 27. How old is the younger daughter?

Solution. The sum of their ages is twice the age of the younger plus 5. This is 27, so that twice the age of the younger is $27 - 5 = 22$. Thus, the younger is $\boxed{11}$ years old.

5. A committee is made up of three boys and five girls.

They decide to have a subcommittee of five persons and they agree that it must have at least one boy and at least two girls.

How many different subcommittees are possible?

Solution. Amongst the three boys, we will have to choose one, two or all three.

We can choose one in 3 different ways. We can choose two in 3 different ways (this is the same as excluding one). We can choose all three in only 1 way.

From the five girls, we will have to choose four, three or two.

We can choose four in 5 different ways (this is the same as excluding one).

We can choose three in 10 different ways (this is the same as excluding two).

We can choose two in 10 different ways (as the previous statement shows).

Thus, we have one boy with four girls: 3×5 ways; two boys with three girls: 3×10 ; and three boys with two girls: 1×10 .

Thus, the total is $15 + 30 + 10 = \boxed{55}$.

6. Determine the smallest number which can be divided exactly by each of 3, 9, 12 and 15.

Solution. Since $9 = 3 \times 3$, we can ignore 3 alone.

Note that $12 = 3 \times 4$ and $15 = 3 \times 5$.

Thus, we need to multiply all of 9, 4 and 5, giving the required number as $\boxed{180}$.

7. I have seven coins totaling to 49 cents. How many dimes do I have?

Solution. Clearly, I must have four 1 cent coins. Thus, three coins must make 45 cents.

The only way to get this is from $25 + 10 + 10$.

Hence, I must have $\boxed{2}$ dimes.

8. I live at Dotheboys Hall, where the clock runs at the correct rate, but does not show the correct time.

I left to deliver a parcel to Dothegirls Hall and noticed that the time on the clock was 2 : 17.

When I got to Dothegirls Hall, their clock (which is always correct) showed 1 : 37.

I delivered the parcel instantly, turned round and was home (taking the same amount of time for the return journey as for the outward journey) and saw that the clock at Dotheboys Hall indicated 3 : 01.

What was the correct time of my return?

Solution. The total journey took 44 minutes. Hence, one way took 22 minutes.

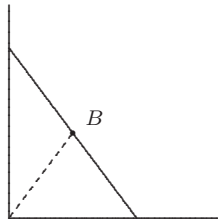
Hence I returned at 22 minutes after 1 : 37; that is, at 1 : 59.

9. A straight ladder 4 metres long, rests against a vertical wall. The bottom of the ladder slides along the ground while the top of the ladder remains against the wall until the ladder is lying on the ground.

A bug is on the rung that is exactly in the middle of the ladder.

Determine the **exact** distance that this bug travels.

Solution. Look at a diagram for an intermediate position.



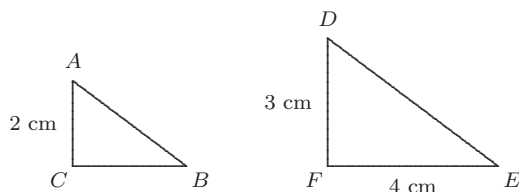
The trick here is to know that the the mid-point of the hypotenuse (of length $2z$) of a right triangle is distance z from the point where the right angle is. In other words, the circle, centred at the mid-point of the hypotenuse, passing through the ends of the hypotenuse, also passes through the third point of the right triangle.

Thus, the bug is always the same distance from the point where the wall meets the ground; that is, half of the length of the ladder; that is, 2 metres.

Therefore, the bug travels along an arc of a circle that is one quarter of a full circle, radius 2 metres. This distance is then $\frac{1}{4} \times 2 \times \pi \times 2 = \pi$ metres.

10. Given that triangle ABC is similar to triangle DEF , that $\angle ACB = \angle DFE = 90^\circ$, that $AC = 2$ cm, $DF = 3$ cm, and $FE = 4$ cm, determine the exact length of AB .

Solution. Draw a diagram!



By Pythagoras' Theorem, $DE = 5$ cm.

Since the triangles are similar, their sides are in proportion.

Thus, $\frac{AB}{AC} = \frac{DE}{DF} = \frac{5}{3}$. But $AC = 2$ cm.

Hence, $AB = 2 \times \frac{5}{3} = \boxed{\frac{10}{3} \text{ cm}} = \boxed{3\frac{1}{3} \text{ cm}}$.

NOTE: since an EXACT answer is required, any decimal representation must be $3.333\ldots$ or equivalent, indicating that the decimal goes on for ever.

11. The perfect prime number associated with a positive composite whole number is calculated as follows: find all the proper factors of the number and add them up; if that number is a prime number, then that is the answer; if it is not a prime number, repeat the process with that number, and repeat until you get a prime number.

For example, if we start with 15, the proper factors are 1, 3 and 5, with sum 9, which is not a prime. Now, the proper factors of 9 are 1 and 3, with sum 4, which is not a prime. The proper factors of 4 are 1 and 2 with sum 3 which is a prime. Hence, the perfect prime number associated with 15 is 3.

Determine the perfect prime number associated with 63.

Solution. $63 = 3^2 \times 7$, so that the proper factors are 1, 3, 7, 9 and 21, with sum 41, which is a prime number.

Hence, the perfect prime number associated with 63 is $\boxed{41}$.

ATOM

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