Part A: 4 marks each

1. Calculate

\[(1 + 2 + 3 + 4 + 5)^2 - 3^2\]

(A) 26 (B) 46 (C) 144 (D) 216 (E) 49 284

**Solution 1:** Using order of operations we get

\[(1 + 2 + 3 + 4 + 5)^2 - 3^2 = 15^2 - 3^2 = 225 - 9 = 216\]

**Solution 2:** We can think of the number we are interested in graphically as the area of a 15 × 15 square with a 3 × 3 square removed (see figure on the left). We can slice off a 12 × 3 piece as in the central figure. We then relocate that slice to obtain a 18 × 12 rectangle with area 216 square units (see figure on the right).

![Graphical representation of the solution](image)

Answer: (D)
2. Each symbol in the equations below represents a different whole number.

\[
\begin{align*}
\text{✠} + \text{✠} &= ♠ \\
\text{✠} \times ♣ &= ♠ \\
♣ + ♣ + ♣ &= ♠
\end{align*}
\]

What number does ♠ represent?

(A) 6   (B) 10   (C) 2   (D) 20   (E) 8

**Solution:** Clearly if \( ✠ = ♣ = ♠ = 0 \), all three equations are satisfied. However, since each symbol must be different, this is not allowed. Also, if \( ✠ = 0 \) or \( ♣ = 0 \), then the second equation would make \( ♠ = 0 \), which is not allowed. Similarly, if \( ♠ = 0 \), then the first equation makes \( ✠ = 0 \) while the last equation makes \( ♣ = 0 \), which is not allowed. So we can assume that \( ✠, ♣, ♠ > 0 \).

Since \( ♣ + ♣ + ♣ = 3 \times ♣ \), the third equation can be rewritten as

\[ 3 \times ♣ = ♠. \]

This means that, from the second equation, we can conclude that \( ✠ = 3 \). Therefore the first equation yields

\[ ✠ + ✠ = 3 + 3 = 6 = ♠. \]

The last equation gives \( 3 \times ♣ = 6 \) which gives \( ♣ = 2 \). So all the equations are satisfied if \( ✠ = 3 \), \( ♣ = 2 \), and \( ♠ = 6 \).

**Answer:** (A)

3. Elizabeth walks at a constant speed. She walks \( \frac{4}{5} \) of the way home in 40 minutes. How long will it take for her to walk the rest of the way?

(A) 6 minutes   (B) 8 minutes   (C) 10 minutes   (D) 32 minutes   (E) 50 minutes

**Solution 1:** Since \( 1 - \frac{4}{5} = \frac{1}{5} \), Elizabeth has \( \frac{1}{5} \) of the trip left. Since she walks \( \frac{4}{5} \) of the way home in 40 minutes, then she walks \( \frac{4}{5} \div 4 = \frac{1}{5} \) of the way home in \( 40 \div 4 = 10 \) minutes.

**Solution 2:** If we draw a graph of Elizabeth’s distance verses time, it will be a line since she is travelling at a constant speed.
Extending the graph we see her total trip will take 50 minutes, which means she has 10 minutes left to walk.

**Answer:** (C)

4. In how many different ways can we pay $28 using $10 bills, $5 bills and $1 coins?

| (A) 8 | (B) 9 | (C) 10 | (D) 12 | (E) 14 |

**Solution:** Breaking it into cases based on the number of $10 bills used we get:

**Case 1:** two $10 bills
We have $28 − 2 × $10 = $8 left to pay. This can be paid using zero or one $5 bills and making up the rest in $1 coins, so there are two possibilities. (eight $1 coins; or one $5 bill and three $1 coins)

**Case 2:** one $10 bill
We have $28 − $10 = $18 left to pay. This can be paid using zero, one, two, or three $5 bills and making up the rest in $1 coins, so there are four possibilities. (eighteen $1 coins; one $5 bill and thirteen $1 coins; two $ bills and eight $1 coins; or three $5 bills and three $1 coins)

**Case 3:** no $10 bills
We have the full $28 left to pay. This can be paid using zero, one, two, three, four, or five $5 bills and making up the rest in $1 coins, so there are six possibilities. (twenty eight $1 coins; one $5 bill and twenty three $1 coins; two $ bills and eighteen $1 coins; three $5 bills and thirteen $1 coins; four $ bills and eight $1 coins; or five $5 bills and three $1 coins)

∴ There are 2 + 4 + 6 = 12 ways to pay $28 using $10 bills, $5 bills and $1 coins.

**Answer:** (D)
5. Jordana comes across a strange text where the letters have been replaced by numbers. She replaces each number with its corresponding letter (for example 1 = A, 2 = B and so on) but the message remains jumbled. Jordana suspects the numbers have been increased by some number $n$ (the same for each letter). The original message is:

```
24 12 9 7 5 18 5 8 5 14 5 29 13 23 5 6 13 22 8
5 23 23 19 7 13 5 24 9 8 27 13 24 12 24 12 9 13 18 8 13 11 9
18 19 25 23
17 29 24 12 19 16 19 11 13 7 5 16 10 13 11 25 22 9
27 13 23 5 15 9 8 14 5 15
```

Using her assumption, Jordana decreases each number by some number $n$ and gets the message:

```
The Canada Jay is a bird
associated with the Indigenous
mythological figure
Wisakedjak
```

What number $n$ did Jordana use to decode the message?

(A) 4   (B) 5   (C) 6   (D) 7   (E) 8

**Solution:** In the message the fifth word is “a” (The Canada Jay is a bird ...), which should have decoded as 1. However, in the message we see that it is coded as 5. Therefore, Jordana would have to subtract $5 - 1 = 4$ to decode her message. Checking, if we subtract 4 from each of the coded numbers, the first two coded words become

```
20 8 5 3 1 14 1 4 1 ... 
```

Using the following

```
A B C D E F G H I J K L M
1 2 3 4 5 6 7 8 9 10 11 12 13
N O P Q R S T U V W X Y Z
14 15 16 17 18 19 20 21 22 23 24 25 26
```

we see that we get

```
The Canada
```

and we see that we are correct.

**Answer:** (A)
Part B: 5 marks each

6. Hiroshi writes down a pattern of digits

\[123456789876543212345678987\ldots\]

If he continues his pattern, what is the 2021\(^{\text{st}}\) digit he writes down?

(A) 1  (B) 3  (C) 5  (D) 7  (E) 9

Solution 1: We see that the pattern 1234567898765432 is being repeated over and over. The pattern is 16 digits long. Since 2021 \div 16 = 126.3125, it means we have gone through the full pattern 126 times and part of another. Since 2021−126×16 = 5 the 2021\(^{\text{st}}\) digit will be the 5th number in our pattern which is 5.

Solution 2: Similar to the first solution, we see that the pattern is 16 digits long. Note that

\[2021 = 2000 + 21 = 16 \times 25 + 16 + 5\]

and hence 2021 has a remainder of 5 when divided by 16. Therefore the 2021\(^{\text{st}}\) digit will be the 5th number in our pattern which is 5.

Answer: (C)

7. Which is true about the number 2021 \times 2022 + 2022 \times 2023?

(A) It is odd.
(B) It is a perfect square.
(C) It can be divided by 2021 without remainder.
(D) It is a prime.
(E) It can be divided by 8 without remainder.

Solution 1: Since 2022 \times 2023 = 2023 \times 2022 we can think of 2021 \times 2022 + 2022 \times 2023 as 2021 groups of 2022 added to 2023 groups of 2022 which is 4044 groups of 2022 or

\[4044 \times 2022 = 2 \times 2022 \times 2022 = 2 \times 2022^2\]

which is even, not a perfect square, not a multiple of 2021, and not prime. Since 2022 = 2 \times 1011, our result can be written as

\[2 \times (2 \times 1011) \times (2 \times 1011) = 8 \times 1011^2\]

which can be divided by 8 without remainder.
Solution 2: Using some properties of addition and multiplication we can write

\[
2021 \times 2022 + 2022 \times 2023 = 2021 \times 2022 + 2023 \times 2022 \\
= (2021 + 2023) \times 2022 \\
= (2022^2 - 1 + 2022 + 1) \times 2022 \\
= 2 \times 2022^2
\]

and then proceed as in the first solution.

Answer: (E)

8. Zaria practiced ring toss by taking 15 tosses a day for 5 days. The graph below is her cumulative mean(average) number of tosses scored each day. That is, for example, the bar labelled 3 shows she had a mean of 6 tosses scored each day over the first 3 days.

Which day did Zaria score the fewest number of her tosses?

(A) day 1  (B) day 2  (C) day 3  (D) day 4  (E) day 5

Solution: Let \( s_1 \) represent the number of tosses scored on the first day and, similarly, \( s_2, s_3, s_4, s_5 \) represents the tosses scored on the second, third, fourth and fifth days, respectively.

From the graph, we know that \( s_1 = 5 \). Since the mean of the first two days is 7, then \( s_1 + s_2 = 2 \times 7 = 14 \). Since \( s_1 = 5 \), we can determine that \( s_2 = 9 \). Proceeding the same way we see

- \( s_1 + s_2 + s_3 = 3 \times 6 = 18 \), so \( s_3 = 4 \),
- \( s_1 + s_2 + s_3 + s_4 = 4 \times 8 = 32 \), so \( s_4 = 14 \),
- \( s_1 + s_2 + s_3 + s_4 + s_5 = 5 \times 7 = 35 \), so \( s_5 = 3 \),

So Zaria scored the fewest number of her tosses on the fifth day.
9. Numbers are *consecutive* when they follow their natural order. For example: $17, 18, 19$ are consecutive *whole numbers*; $11, 13, 17$ are consecutive *prime numbers* and $25, 36, 49$ are consecutive *square numbers*. When two consecutive *odd numbers* are multiplied together their product is 1443. What is the sum of the two numbers?

(A) 72  (B) 76  (C) 80  (D) 84  (E) 88

**Solution 1:** Since $30 \times 30 = 900$ and $40 \times 40 = 1600$, the two numbers are in the 30s. So we could have 31 and 33; 33 and 35; 35 and 37; or 37 and 39. However, the units’ digits of $33 \times 35$ and $35 \times 37$ would be 5, so it cannot be those. Since 1443 is closer to 1600 than 900, try $37 \times 39 = 1443$. Then, $37 + 39 = 76$.

**Solution 2:** We have a rectangle with consecutive odd numbers as sides, so we can say that it is 2 units higher than it is wide (see figure on the left).

Slice off a strip of width 1 from the top (green) and put it at the side of the figure to obtain a figure that is almost a square (see figure on the right).

The area of the figure is still 1443 square units, and it is 1 square unit short of being a perfect square; the sides of the square would therefore be $\sqrt{1444} = 38$ units, so our original two numbers were 37 and 39.

**Answer: (B)**
10. Hamza and his friends play in a one–on–one video game soccer tournament. Each plays one match against each other player. They agreed to award points as follows:

<table>
<thead>
<tr>
<th>Result</th>
<th>Win</th>
<th>Tie with goals</th>
<th>Tie without goals</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

After all the matches ended, Hamza had 10 points without losing any matches.

Which one of the additional pieces of information is sufficient, by itself, to find with certainty the number of people participating in the tournament?

(A) Hamza had at least one match in each of the first three categories.

(B) More than half of Hamza’s matches ended in ties with goals.

(C) Four of Hamza’s matches ended in either a win or a tie with goals.

(D) Hamza had the same number of wins and ties without goals.

(E) None of the above.

Solution: If we can figure out the number of games a person plays, then we know the number of people (since each person plays everyone else, if they play 6 games, there must be 7 people). Check the conditions one at a time:

(A)
If Hamza had at least one match in each of the other categories, we know he has been awarded at least 3 + 2 + 1 = 6 points. He needs 4 more points, but there are several ways to do that (another win and another tie without goals (5 games); two more ties with goals (5 games); or 4 more ties without goals(7 games)), so we cannot determine the number of people from this information.

(B) There are several ways this condition can be satisfied as well: 5 ties with goals(5 games); 4 ties with goals and 2 ties without goals(6 games); or 3 ties with goals, one win and one tie without goals(5 games), so we cannot determine the number of people from this information.

(C) There are two ways this condition can be satisfied: 2 wins and 2 ties with goals(4 games); 1 win, 3 ties with goals and 1 tie without goals(5 games); or 4 ties with goals and 2 ties without goals(6 games), so we cannot determine the number of people from this information.

(D) There are several ways this condition can be satisfied as well: 2 wins, 1 tie with goals and 2 ties without goals(5 games); 1 win, 3 ties with goals and 1 tie without goals(5 games); or 5 ties with goals(5 games). Even though we don’t know which
of these occurred, we know that 5 games were played so 6 people were participating.

Answer: (D)
Part C: 7 marks each

11. Sarah asked her friend Rina to solve the two similar puzzles shown below. The sum of any two circles that are beside each other is in the circle directly below both of them. For example, the first two circles in Puzzle 1 are 4 and 6, which add to 10 which is in the circle below them. A puzzle is solved when all circles have been filled with the correct numbers.

Puzzle 1

Puzzle 2

Which one of the following pieces of information, if given to Rina, would enable her to solve both puzzles?

(A) The number in circles labelled \(a\) and \(y\).
(B) The number in circles labelled \(b\) and \(z\).
(C) The number in circles labelled \(c\) and \(w\).
(D) The number in circles labelled \(d\) and \(x\).
(E) None of the above.

Solution 1: In Puzzle 1 we can calculate \(a = 7 + 9 = 16\) and then \(c = 13 + a = 13 + 16 = 29\). However, we cannot calculate \(b\) or \(d\). If we knew \(b\) we could calculate the empty circle above it, and \(d\), and then be able to fill in the rest of the puzzle. Similarly, if we knew \(d\) we could fill in the rest of the puzzle.

Similarly, in Puzzle 2 \(w\) and \(x\) could be calculated directly and if we knew either \(y\) or \(z\) we would be able to finish the puzzle.

\[\therefore\] If we knew \(b\) and \(z\) we would be able to complete both puzzles.

Solution 2: The entire triangle is determined if and only if the upper right circle is determined. In the first triangle, \(a\) and \(c\) are not in the “shadow” of the upper right, but \(b\) and \(d\) are. Similarly in the second triangle only \(y\) and \(z\) can affect the upper right. So the only answers can be (B) or (E).

Choose some numbers of \(b\) and \(z\) to see if we can finish the puzzle. Choosing \(b = 29\) and \(z = 50\), the rest of the puzzle can be filled in as shown below.
If we leave $b$ and $z$ as variables, we can complete the puzzles in terms of these variable.

Hence both puzzles can be completed if we know $b$ and $z$.

Answer: (B)
12. Jaytown has seven radio stations spread out throughout the town. If the regions covered by two radio stations overlap, they have to use a different frequency. But if the radio stations’s regions don’t overlap, they can use the same frequency. The picture below shows the regions covered each of the seven radio stations.

What’s the smallest number of frequencies Jaytown can have so that no two overlapping radio stations share the same frequency?

(A) 2  (B) 3  (C) 4  (D) 5  (E) 6

Solution: Since the regions of stations 1, 3 and 4 overlap together (as do 2, 5 and 6), at least 3 frequencies are needed. If we assign frequency A to stations 1, 5 and 7; frequency B to stations 2 and 3; and frequency C to stations 4 and 6 we see that the conditions are satisfied. Thus, the smallest number of frequencies is 3.

Answer: (B)
13. You have a collection of two types of squares, a smaller square and a larger square. When four of the larger squares are placed around a smaller square, as in the picture on the left, the total perimeter is 168 cm. When four of the smaller squares are placed around a larger square, as in the picture on the right, the total perimeter is 156 cm. What is the area of the smaller square?

(A) 121 cm$^2$  (B) 144 cm$^2$  (C) 169 cm$^2$  (D) 196 cm$^2$  (E) 225 cm$^2$

Solution: Let $\ell$ and $s$ represent the side lengths of the larger and smaller squares, respectively. Notice the “height” of the diagram on the left is $2\ell + s$ while the height of the top of the square on the left is $2\ell$. Therefore the length of the line segment at the top of the diagram (in blue) is $s$. Similarly on the right we find the corresponding length (red) to be $\ell$.

We can similarly determine the other sides around the perimeter. Labelling the lengths in the diagram we get the following.
Since we know the perimeters, we can write

\[ 8\ell + 4s = 168 \]
\[ 4\ell + 8s = 156 \]

Dividing both equations by 4 yields

\[ 2\ell + s = 42 \]
\[ \ell + 2s = 39 \]

Adding these two equations, then dividing by 3 yields

\[ 3\ell + 3s = 81 \Rightarrow \ell + s = 27. \]

If we subtract the equations we get

\[ \ell - s = 3. \]

So we are searching for two numbers that are three apart that add to 27. We can quickly deduce that \( \ell = 15 \) and \( s = 12 \), so the area of the small square is \( 12^2 = 144 \).

**Answer:** (B)

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14. James works for a courier company. He had to deliver 5 parcels. His motorcycle uses 3 litres of fuel to drive 45 km. The tank of the motorcycle holds 12 litres of fuel and was full when he started out.

The motorcycle used one quarter of the fuel for the delivery of the first parcel. For the second and third parcels together, the motorcycle used half of the remaining fuel.

James recognized that the fuel left in the tank after delivering the third parcel was only three fifths of what he needed to deliver the last two parcels. He filled up the tank of his motorcycle and then finished the deliveries.

How far did James travel to deliver the 5 parcels?

(A) 180 km  (B) 210 km  (C) 225 km  (D) 240 km  (E) 270 km

**Solution:** To deliver the first parcel, James consumed \( 12 \div 4 = 3 \) litres, so he has \( 12 - 3 = 9 \) litres left. To deliver the second and third parcels he consumed \( 9 \div 2 = 4.5 \) litres and has 4.5 litres left. Since he only has \( \frac{3}{4} \) of the fuel needed, for the remaining deliveries, he needs the extra \( \frac{2}{5} \) of the fuel which is \( \frac{2}{5} \) of the amount he already has. So, he needed \( (2 \div 3) \times 4.5 + 4.5 = 7.5 \) litres. Total fuel to deliver
the 5 parcels was \((3 + 4.5 + 7.5 = 15)\) litres. Then, the total distance he travelled was

\[
15 \times 45 \div 3 = 225\text{ km.}
\]

**Answer:** (C)

15. You have a number of \(2 \times 1\), \(1 \times 2\), and \(1 \times 1\) tiles and want to tile a \(2 \times 2\) square.

You notice that there are 7 different ways you could do the tiling as shown below.

How many different ways could you tile the \(2 \times 4\) rectangle, pictured below, using the tiles?

(A) 49   (B) 55   (C) 63   (D) 71   (E) 81

**Solution 1:** Draw a vertical line down the center of the \(2 \times 4\) rectangle. We count separately tilings which don’t cross or do cross this vertical line.

**Case 1:** No tile crosses vertical line.
We then have two separate \(2 \times 2\) squares which each can be tiled in 7 ways, so the total number of tilings of this type is \(7 \times 7 = 49\) ways.

**Case 2:** Both the upper and lower segment of vertical line crossed.
This can only happen if we have two \(1 \times 2\) tiles in the middle, as shown below.
We are left with two $2 \times 1$ regions, each of which can be tiled in 2 ways (a $2 \times 1$ tile or two $1 \times 1$ tiles), so the total number of tilings of this type is $2 \times 2 = 4$.

**Case 3:** Only one tile crosses the vertical line.  
If the tile that crosses is on top, we have the diagram below.

![Diagram](image)

Notice that we have to tile two “L” shaped regions (one in blue and its reverse in green). Each “L” can be tiled in 3 ways (the ones for the blue are shown below, there are similar ones for the green).

![L Shapes](image)

So the total number of tilings of this type is $3 \times 3 = 9$.

Similarly if the tile crosses on the bottom, we are left with two “L” shapes to tile which can be done in 9 ways.

∴ The total number of tilings is $49 + 4 + 9 + 9 = 71$.

**Solution 2:** We will proceed systematically looking at longer and longer rectangles.  
We know there are two ways to tile a $2 \times 1$ rectangle (show below) and we were shown in the problem that there are 7 ways to tile a $2 \times 2$ rectangle.

![Tile Patterns](image)

Next we will work on a $2 \times 3$ rectangle. We will look at 3 cases depending on which tile goes in the bottom left corner.

**Case 1:** A $2 \times 1$ in the bottom left corner.  
In this case, we are left with a $2 \times 2$ rectangle, which we know can be covered in 7 ways.

![Tile Patterns](image)

**Case 2:** A $1 \times 2$ in the bottom left corner.  
We now have two choices for what goes in the upper left corner.

If we put another $1 \times 2$ we are left with a $2 \times 1$ which can be tiled in 2 ways.
If we put a $1 \times 1$ we are left with an “L” which can be tiled in 3 ways.

So there are $2 + 3 = 5$ ways to deal with this case.

**Case 3:** A $1 \times 1$ in the bottom left corner.

We now have two choices for what goes in the upper left corner.

If we put another $1 \times 1$ we are left with a $2 \times 2$ square that can be tiled in 7 ways.

If we put a $1 \times 2$ we are left with an “L” which can be tiled in 3 ways.

So there are $7 + 3 = 10$ ways to deal with this case.

So there are $7 + 5 + 10 = 22$ ways to tile a $2 \times 3$ rectangle.

Finally on to the $2 \times 4$ rectangle. Proceeding similarly to the $2 \times 3$ rectangle, we again look at 3 cases:

**Case 1:** A $2 \times 1$ in the bottom left corner.

In this case, we are left with a $2 \times 3$ rectangle, which we know can be covered in 22 ways.

**Case 2:** A $1 \times 2$ in the bottom left corner.

We now have two choices for what goes in the upper left corner.

If we put another $1 \times 2$ we are left with a $2 \times 2$ which can be tiled in 7 ways.
If we put a $1 \times 1$, then beside it can either go another $1 \times 1$ and we are left with a $2 \times 2$ which can be tiled in 7 ways (below left) or we can put a $1 \times 2$ beside it and we are left with an “L” which can be tiled in 3 ways (below right).

So there are $7 + 7 + 3 = 17$ ways to deal with this case.

**Case 3:** A $1 \times 1$ in the bottom left corner.
We now have two choices for what goes in the upper left corner.

If we put another $1 \times 1$ we are left with a $2 \times 3$ rectangle that can be tiled in 22 ways.

If we put a $1 \times 2$ we are left with a thick “L”. If we put another $1 \times 1$ beside the first one, we are left with a $2 \times 2$ rectangle which can be tiled in 7 ways (below left). If we put a $1 \times 2$ beside the $1 \times 1$ we are left with an “L” which can be tiled in 3 ways (below right).

So there are $22 + 7 + 3 = 32$ ways to deal with this case.

So there are $22 + 17 + 32 = 71$ ways to tile a $2 \times 3$ rectangle.

**Answer:** (D)