A Taste Of Mathematics

Aime–T–On les Mathématiques

Volume / Tome III

PROBLEMS FOR MATHEMATICS LEAGUES

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The ATOM series

The booklets in the series, **A Taste of Mathematics**, are published by the Canadian Mathematical Society (CMS). They are designed as enrichment materials for high school students with an interest in and aptitude for mathematics. Some booklets in the series will also cover the materials useful for mathematical competitions at national and international levels.

La collection ATOM

Publiés par la Société mathématique du Canada (SMC), les livrets de la collection Aime-t-on les mathématiques (ATOM) sont destinés au perfectionnement des étudiants du cycle secondaire qui manifestent un intérêt et des aptitudes pour les mathématiques. Certains livrets de la collection ATOM servent également de matériel de préparation aux concours de mathématiques sur l'échiquier national et international.

Contents

Foreword

This volume contains a selection of some of the problems that have been used in the Newfoundland and Labrador Senior Mathematics League, which is sponsored the the Newfoundland and Labrador Teachers Association Mathematics Special Interest Council. The support of many teachers and schools is gratefully acknowledged.

We also acknowledge with thanks the assistance from the staff of the Department of Mathematics and Statistics, especially Ros English, Wanda Heath, Menie Kavanagh and Leonce Morrissey, in the preparation of this material.

Many of the problems in the booklet admit several approaches. As opposed to our earlier 1995 book of problems, *Shaking Hands in Corner Brook*, available from the Waterloo Mathematics Foundation, this booklet contains no solutions, only answers. Also, the problems are arranged in the form in which we use them — in games. We hope that this will be of use to other groups running Mathematics Competitions.

While we have tried to make the text as correct as possible, some mathematical and typographical errors might remain, for which we accept full responsibility. We would be grateful to any reader drawing our attention to errors.

It is the hope of the Canadian Mathematical Society that this collection may find its way to many high school teachers, and to many high school students, including those who may have the talent, ambition and mathematical expertise to represent Canada internationally.

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History of the NLTA Senior Mathematics League

The Newfoundland and Labrador Senior Mathematics League began in 1987 as a competition amongst the high schools in the St. John's area. It has grown since then into a province-wide competition with many schools competing in local leagues in several districts all over the province. The same game takes place simultaneously in each place, with the schools competing at the district level. A provincial championship game (game 5 of each season) takes place towards the end of a school year, with the top schools from each district competing at a common site.

There have even been leagues outside Newfoundland and Labrador, using our games. We are happy to provide master copies of materials for a modest fee.

The NLTA Senior Math League stresses cooperative problem solving. Each participating school sends a team of four students, who will work together on each problem. Students may, if they wish, submit individual answers. However, to reward cooperative work, a bonus mark is given for a correct team answer.

A typical contest consists of ten questions and a relay. Unlike most mathematics competitions where the contestant receives all of the questions at once, the team receives only the first problem at the start.

Each team is seated around a table, and is given two copies of the first problem. Thus, everyone can read the problem easily. There is a time limit announced for each problem (usually between three and ten minutes). When time is called, the team hands one sheet to a proctor for marking.

While the contestants' answers are being marked, a solution to the problem is presented, usually using an overhead projector. A correct team answer gets five marks whereas an incorrect team answer gets zero marks. If the team members do not agree on the answer, each may submit an answer for one possible mark each. This results in much discussion and debate, especially if two team members arrive at different answers. Such debate is indeed encouraged. The next problem is now given to the teams, and the above sequence of events is repeated.

After ten problems, which usually increase in difficulty, and in time allotted, students do the relay question.

The relay question has four parts: the answer to part # 1 is an input to part # 2; the answer to part # 2 is an input to part # 3; and the answer to part # 3 is an input to part #4.

The relay question has fifteen minutes allotted, and up to ten bonus points over and above the five normal points. One point is awarded if only part # 1 is correct; two points are awarded if only both parts # 1 and # 2 are correct; three points are awarded if only all of parts # 1, # 2 and # 3 are correct; and five points are awarded if all four parts are correct.

If time has not been called when the team hands the relay answer sheet to the proctor, the proctor will say either “CORRECT” or “WRONG”. If the proctor answers “WRONG”, the relay answer sheet is returned to the team. No indication of the place of any error is given.

Bonus points, up to ten, are awarded for each minute (or part thereof) remaining in the first ten minutes allotted for the relay.

The theoretical maximum score consists of five points for each question plus five for the relay and ten time bonus points, giving a grand total of 65. In practice, a score of above 60 has yet to be achieved.

After the scores have been tallied, the day’s winners are announced unless there is a tie for first place. In this case, a tie breaker question is used to determine the contest’s winner. No points are awarded for winning a tie breaker.

The rules for the tie breaker are a little different. Essentially, the first team to solve the question wins. But to prevent silly guessing games, the following rule applies: a team that has offered an incorrect answer to a tie breaker may not offer any other answer for a subsequent period of one minute.

In some locations, it has been customary for the host (school or institution) to award a plaque to the winning team. Scores are cumulative over the league year, and at the end of the season (we have four contests), the top three schools receive awards. These are usually in the form of a plaque for the season. In the St. John’s district, the top team is awarded a trophy, which it holds for one year.

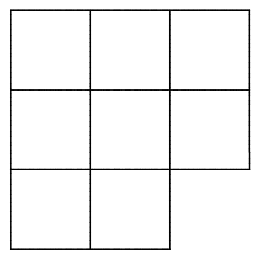
There is usually a “nutrition break” after the fifth question. Students and proctors are provided with a soft drink, juice, tea or coffee and a donut or muffin. This allows for some social interactions amongst the participants.

Attending an NLTA Math League contest is a very rewarding experience. They usually take place on Saturday mornings. It is very gratifying to be in a room full of young men and women, usually in equal numbers, doing mathematics, enjoying mathematics, and having fun.

1994–95 Game #1

1. Consider the points $A(2,7)$, $B(4,9)$ and $C(8,-3)$. Find the equation of the line AD where D is the mid-point of BC.

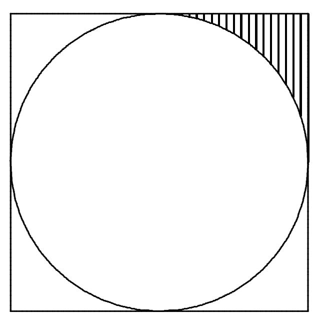
2. How many different rectangles appear in the following diagram? Remember that squares are rectangles, too!



3. Which of the following expressions represents the greatest quantity?

   (a) $(2^{111})^2$ (b) $2^{(4^4)}$ (c) $2^{(22^2)}$ (d) 8^{88}

4. A circle is inscribed in a square (as shown below). If the area of the hatched region is $(196-49\pi)$ cm^2, determine the area of the square in cm^2.



5. Assuming that x is a real number, $x \neq 0$ and $x \neq 3$, find all possible values of the following expression.

$$\frac{||x|-x|}{x} + \frac{||x-3|-(x-3)|}{x-3}.$$

6. Determine the value of a^2-b^2 given that $a^2+b^2=208$ and $2ab=192$. It is known that $a>b>0$.

7. A grocer weighed four fruits: a lemon, a pear, an apple, and a grapefruit. The following observations were made:

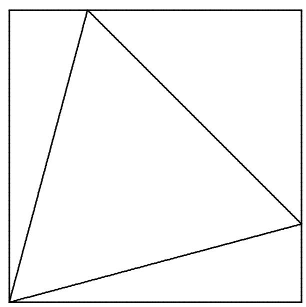
(a) the lemon weighed more than the pear,

(b) the combined weight of the pear and the apple was greater than the combined weight of the lemon and the grapefruit, and

(c) the combined weight of the lemon and the pear was equal to the combined weight of the apple and the grapefruit.

List the four fruits in order of weight from heaviest to lightest.

8. A square of unit area contains an equilateral triangle as shown. Calculate the length of the sides of the equilateral triangle. Your answer should be in the form $\sqrt{a} - \sqrt{b}$, where a and b are positive integers.



9. A three digit whole number, $5xy$, is as much greater than 500 as the three digit whole number $xy5$ is less than 500. Find x and y.

10. Let $K = 1^2 + 2^2 + 3^2 + 4^2 + \ldots + 1994^2$. What is the last digit of K?

Relay

R1 Six family members leave an airport in pairs on three different airplanes. Before leaving, each family member hugs all of the other family members except for the one taking the same plane. The total number of hugs is M.

Write the value of M in Box # 1 of the Relay Answer Sheet.

R2 Two classes wrote the same test. The first class had $2M$ students and the second class had N students who wrote the test. The class average of the first class was 75 and of the second class was 66.

If the overall average of all students who wrote the test was 70, determine the value of N.

Write the value of N in Box #2 of the Relay Answer Sheet.

R3 Manuel and Rita together can paint a building in N hours. If Manuel paints the building alone, it takes him 80 hours. Rita requires only P hours to paint the building alone. Determine the value of P.

Write the value of P in Box #3 of the Relay Answer Sheet.

R4 Given distinct integers $X < Y < Z$ such that:

$$\begin{aligned} XYZ &= P\,, \\ X+Y+Z &= -14\,, \end{aligned}$$

determine the value of $X + 2Y + 6Z$.

Write this value in Box #4 of the Relay Answer Sheet.

Tiebreaker

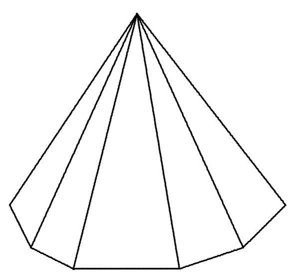
Let P, Q and R be three distinct digits. The sum of all possible three digit numbers that each use all three of these digits is 1554. Find the value of $P + Q + R$.

1994–95 Game #2

1. On 30 December 1959, the population of Corner Brook was 14% higher than it was in 1949. On 31 December 1959, 354 people left Corner Brook for Fort McMurray. On 1 January 1960, a further 6% of the remaining population left Corner Brook for Ontario. The result was a population 1178 higher than that of 1949.

   Find the 1949 population.

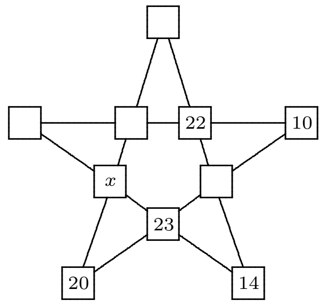
2. A pyramid has a 43 sided base (partially shown here).

   Calculate the difference between the number of edges and the number of vertices of this pyramid.

3. An AMPEC 486DSX Computer retails at $2996 in Markham, Ontario, where G.S.T. of 7% must be added to make the total consumer's price. In St. John's, Newfoundland, G.S.T. of 7% must be added to the price of an item, and then R.S.T. of 12% must be added to that cost to make the total consumer's price. What would be the retail price of the same computer in St. John's in order for the total consumer's price to be the same?

4. A **magic pentagram** has the same sum of 72 for the numbers in each row.



Determine the value of x.

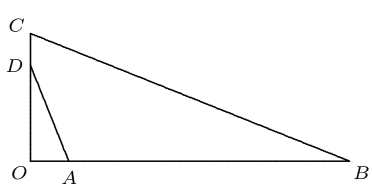
5. Find the exact value of

$$\frac{1}{\sqrt{10000}-\sqrt{9999}} - \frac{1}{\sqrt{9999}-\sqrt{9998}} + \frac{1}{\sqrt{9998}-\sqrt{9997}} - \dots - \frac{1}{\sqrt{9803}-\sqrt{9802}} + \frac{1}{\sqrt{9802}-\sqrt{9801}}.$$

6. If $x^3+2x^2+3x+4=0$, find the values of:

   (a) $x^4+2x^3+3x^2+4x$,

   (b) $x^5+3x^4+6x^3+9x^2+7x+8$.

7. 

The right triangles $\triangle OBC$ and $\triangle OAD$ are shown in the diagram on the left.

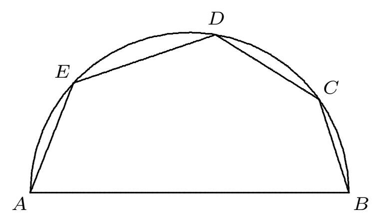
The angles of quadrilateral $ABCD$ are in arithmetic progression.

Calculate the smallest interior angle of the quadrilateral.

8. A group of friends did quite well on a Math test, and had an average mark of 75%. If an extra student was included in the group, the average would have been raised to 78%.

   Find the **maximum** number of people in the group of friends.

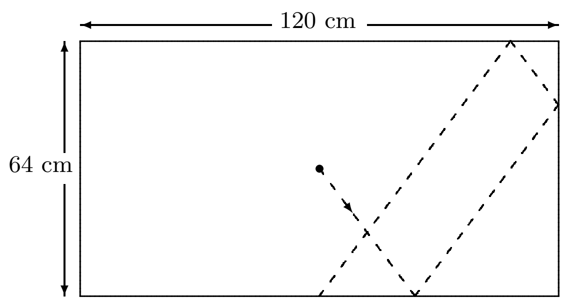
9. A pentagon $ABCDE$ is inscribed in a semi-circle with one side on the pentagon being the diameter AB



Calculate the value of $\angle BCD + \angle DEA$ in degree measure.

10. A pool table, measuring 64 cm by 120 cm has a ball at the centre spot as shown.

A three cushion path is shown to put the ball in a centre pocket.



Find the length of this three cushion path. (You may assume that all rebounds are perfect).

Relay

R1 Find A and B such that

$$\frac{x+53}{x^2+2x-15} = \frac{A}{x-3} - \frac{B}{x+5}$$

for all $x \neq 3,\ -5$.

(HINT: substitute appropriate values for x).

Let $C = A \times B$.

Write C in Box #1 of the Relay Answer Sheet.

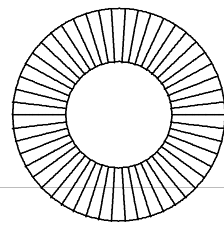
R2 A rectangular field is bounded on one side by a perfectly straight river. The longer side, parallel to the river, is of length $6x$, and the other two sides are of length y.

The length of fencing required to enclose the field is $C + 2$, and the area of the field is C.

Find x and y, and let $z = x + y$.

Write z in Box #2 of the Relay Answer Sheet.

R3 A ring has outer radius $2z$ and inner radius z.



Find $r = \dfrac{\text{Area of the Ring}}{\text{Total Perimeter of the Ring}}$.

Write r in Box #3 of the Relay Answer Sheet.

R4 A group of r friends eat at the Classical Cafe, and each spends a different positive prime number of dollars.

The total cost is 100 dollars.

Let M be the maximum possible value of the part of the total bill that any one of the group of friends could have paid.

Let m be the minimum possible value of the part of the total bill that any one of the group of friends could have paid.

Write $M - m$ in Box #4 of the Relay Answer Sheet.

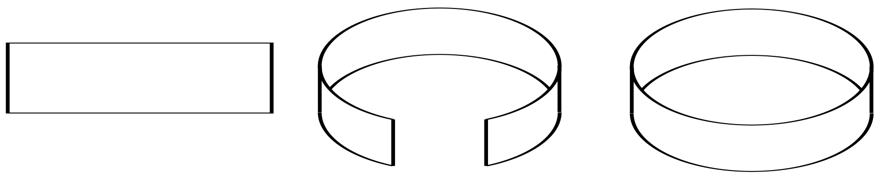
Tie Breaker

Solve the system

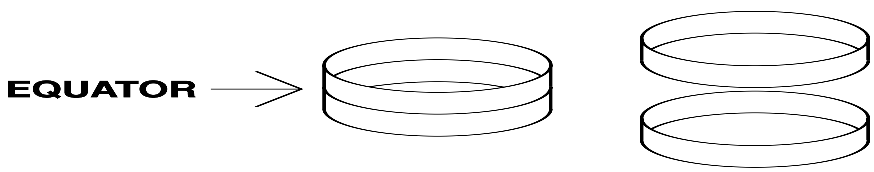
$$\begin{aligned} x^2 + xy + xz &= 20\,, \\ xy + y^2 + yz &= -10\,, \\ xz + yz + z^2 &= 15\,, \\ x - y + z &> 0\,. \end{aligned}$$

1994–95 Game #3

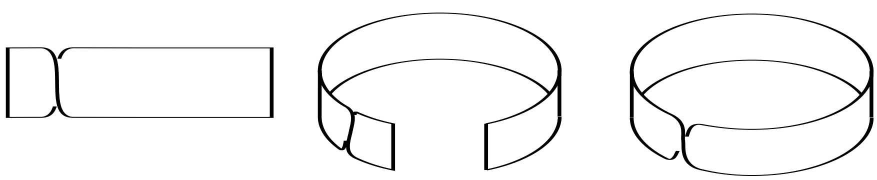
1. If a pair of opposite ends of a rectangular piece of paper are attached together, without any twists in the paper,



and the resulting band is cut with scissors around its equator, then the result is two pieces of paper:



If the original piece of paper is given a half–twist before the opposite ends are attached, then a similar process can be repeated.



The resulting band with a half-twist is known as a Möbius strip. How many pieces of paper are obtained after cutting around its equator with scissors?

2. If the point $(0, 4)$ is reflected in the line $y = -x$, the resulting point is then reflected in the line $x = 2$ and the new resulting point reflected in $y = -3$, what are the coordinates of the final resulting point?

3. Archaeologists excavating the ruins of an old building discovered an ancient piece of paper with the following message:

> “Think of a number. Add 2 to that number to obtain a new number. Multiply the new number by itself and subtract $\star$ to obtain another new number. If this new number is divided by a number that is 10 greater than the number that you first thought of, then the answer is a third new number. If 6 is added to this last number then you will obtain the number that you first thought of.”

The symbol $\star$ indicates a number on the document that has faded away over the centuries. What is the number $\star$?

4. A specially designed airplane, with many extra fuel tanks, leaves Gander International Airport and circumnavigates the globe; it travels by the great circle route that includes Gander, the North Pole and the South Pole. The plane flies through a fixed point X at a height of 1000 m over Gander and maintains this exact height for the entire journey, again passing through X at the end of the circular trip.

   Assuming that the earth is a sphere, how much longer than the earth's circumference is the portion of the plane's journey that starts and ends at the point X?

   Note: You can solve this problem without knowing either the radius or the circumference of the earth. You should give the exact value of your answer.

5. On the planet Xenon the year is 81 days long. This year is divided into 8 months named (surprisingly) **A**, **B**, **C**, **D**, **E**, **F**, **G** and **H**. Months **A** to **G** are 10 days long whilst month **H** is 11 days. The Xenonians have a five day week, the days being numbered **I**, **II**, **III**, **IV** and **V**. If the $7^{\mathbf{th}}$ day of the month **C** in their year 3571 is a day **IV**, what day of the week is the $1^{\mathbf{st}}$ day of month **G** in their year 3578?

6. Suppose that there are students taking each of the following combinations of courses:

   (a) French, Mathematics and Physics,

   (b) French, English and Mathematics,

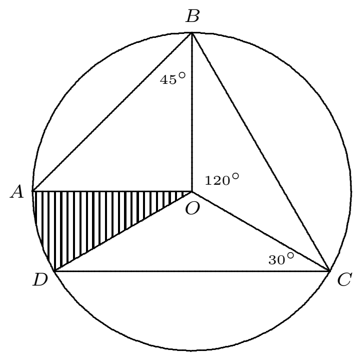
   (c) Chemistry, English and History,

   (d) Chemistry, French and English, and

   (e) Chemistry, English and Mathematics.

   Each course has one two–hour examination. Examinations for different courses can be scheduled at the same time if this does not cause conflicts for any students. What is the minimum number of two-hour periods required to set up examinations that meet these requirements?

7. Given that the circle below has centre O, radius 2, and that the angles between the line segments AB, BO, OC and CD are as indicated, find the exact value of the hatched area.



8. A manufacturing company imports basic widgets and upgrades them into super–deluxe widgets. The process involves four operations **A**, **B**, **C** and **D**. Although no two operations can be carried out simultaneously there is some flexibility concerning the order in which they can be carried out. However it is essential that operation **C** comes after operation **B**, and that operation **D** comes after both **B** and **C**. Operation **A** costs \$300. Operation **B** costs \$200 if carried out before **A** and \$300 otherwise. **C** costs \$500 if done before **A** and \$200 otherwise. **D** costs \$200 if done before **A** and \$400 otherwise.

   Give the least possible cost of the overall process, and the order in which the operations must be carried out to realize that minimum cost.

9. There are 5 Smarties in a box: 3 red, 1 green and 1 yellow. A blindfolded person randomly selects 2 of the Smarties and eats them. What is the percentage likelihood that the 3 remaining Smarties are 3 different colours?

10. Given that x, y and z are positive integers such that $x < y < z$,

$$x + y + z = 22\,,$$

and

$$xyz = 180\,,$$

find the value of $x^2 + y^2 - z^2$.

Relay

R1 Let A be the sum of the roots of the polynomial $x^2 - 13x - 300$. Determine A.

Write A in Box #1 of the Relay Answer Sheet.

R2 Consider the modified Fibonacci sequence

$$1,\ A+1,\ A+2,\ 2A+3,\ 3A+5, \ldots\ .$$

where the next term is found by adding together the two previous terms. If the $1995^{\textbf{th}}$ term is even take $B = 5$, if it is odd take $B = 4$,

Write the value of B in Box #2 of the Relay Answer Sheet.

R3 Find the value of C that satisfies

$$B + \frac{1}{1 + \left(\frac{1}{B-C}\right)} = C\,,$$

Write the value of C in Box #3 of the Relay Answer Sheet.

R4 If the perimeter of a triangle is 7 and the sides are of integer length, then the triangle is of one of two possible types, that is, it either has sides of length 1, 3 and 3 or it has sides of length 2, 2 and 3. There are no other possibilities.

Given that the perimeter of a triangle is of length $C + 5$ and the sides are of integer length, then the triangle is of one of D possible types. Determine D.

Write D in Box #4 of the Relay Answer Sheet.

Tiebreaker

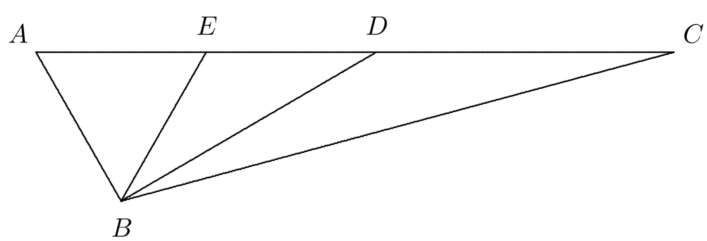
Factorize completely

$$x^3 - (3a+b)x^2 + a(2a+3b)x - 2a^2b\,.$$

1994–95 Game #4

1. Find the exact value of $\dfrac{3^{-2} + 2^{-3}}{-2}$.

2. Given that $\angle BAC = 7\angle ACB$, $AB = AE$, $BE = ED$ and $BD = DC$, find the measures (in degrees) of all the angles of $\triangle ABC$.



3. Pat's cat has had kittens. "How many?" asks Chris. "Three-quarters of their number plus three-quarters of a kitten!" says Pat. How many kittens did Pat's cat have?

4. If $\frac{2}{111}$ is expressed as a decimal, what digit occurs in the 101^{st} place?

5. Circle A, has circumference $\frac{7}{5}$ cm. Find the circumference of the circle B, which has radius 1 cm greater than the radius of circle A.

6. Given that $q^2 = -2$, evaluate $3q^4 + q^3 + 4q^2 + q + 1$.

7. Chris bought 2 boxes of cookies at \$1.09 per box and 2 bars of chocolate at 85¢ per bar. Pat bought 3 packages of apples and 6 bags of chips. The cashier told them that the total cost was \$10.50, but Chris and Pat disagreed.

   Who was correct, the cashier or Chris and Pat?

8. Find all solutions of the equation $\dfrac{3x+5}{4x^2-25} + \dfrac{1}{x+5} = 0$.

9. If a flagpole is 16 metres from the centre of a long straight hedge, how many trees can be planted so that each tree is exactly 20 metres from the flagpole and 4 metres from the centre line of the hedge?

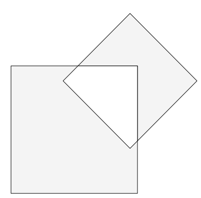
10. Given that x and y are integers, solve $x^2 + |y-10| = 1$.

Relay

R1 Let A be the least number of coins that can make up any amount from one cent to ninety-nine cents. (Fifty cent coins are not permitted.)

Write A in Box #1 of the Relay Answer Sheet.

R2 In the figure, $\frac{3}{4}$ of the smaller square is shaded and $\frac{A-4}{7}$ of the larger square is shaded. Let B be the number such that $B\times$ the shaded area of the smaller square $=$ the shaded area of the larger square.



Write the value of B in Box #2 of the Relay Answer Sheet.

R3 During a vacation on Frodlia, it rained $6B+1$ days. Strangely, when it rained, it was always either morning or afternoon. It never rained on both the morning and the afternoon of the same day.

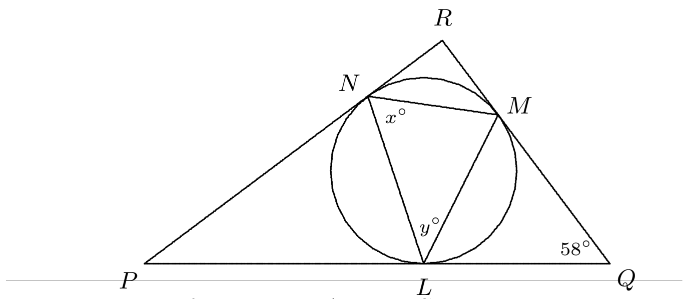
There were eleven fine mornings and twelve fine afternoons.

Let C denote the total number of days of the vacation in Frodlia.

Write the value of C in Box #3 of the Relay Answer Sheet.

R4 In the diagram, the sides of $\triangle PQR$ are tangent to the incircle at the points L, M and N as shown.

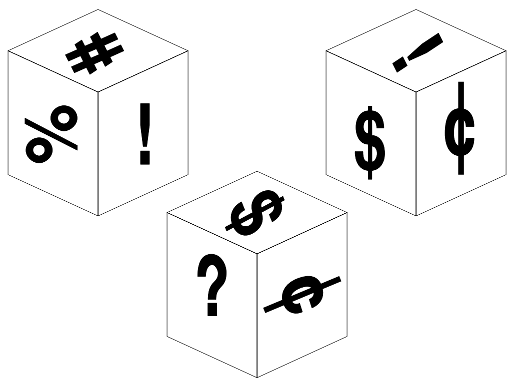
Given that $\angle RPQ = (2C+6)^\circ$, $\angle PQR = 58^\circ$, $\angle MNL = x^\circ$ and $\angle NLM = y^\circ$, determine $D = x + y$.



Write D in Box #4 of the Relay Answer Sheet.

Tiebreaker

Three views of the same cube are show below. All six sides are shown. When the % symbol is on top, what is on the bottom?

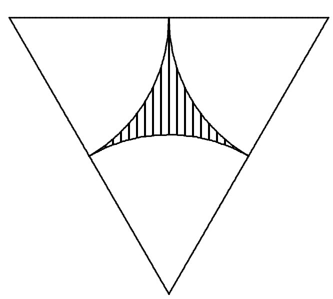


1994–95 Game #5

1. The repeating decimals $1.\dot{3}$ and $0.\dot{6}$ are multiplied together. The product equals another repeating decimal, namely, $0.\dot{n}$. What is the value of n?

   Calculators are not to be used with this question

2. Consider the equilateral triangle that is pictured below. The three curves shown are arcs of circles, centred at the vertices, with radius equal to one half the length of a side of the triangle. What fraction of the total area of this triangle is hatched?

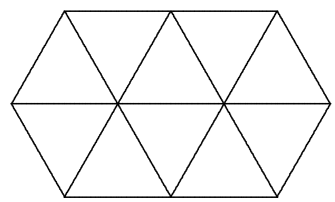


3. Arrange the following expressions in decreasing order, from largest to smallest.

$$2^{(3^{3!})} \qquad 2^{(3^3)!} \qquad 2^{(3!^3)} \qquad 3!^{(2^{3!})}$$

   You should recall that $n! = n(n-1)(n-2)...(2)(1)$. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

4. Determine the number of distinct quadrilaterals that appear in the following diagram of equilateral triangles.

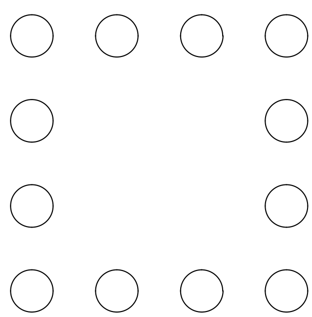


5. For which value or values of k, if any, does the quadratic equation in x,

$$(k-8)x^2 + kx - 1 = 0\,,$$

   have exactly one solution?

6. Each of the numbers 1, 2, 3, ..., 12 is to be placed in one of the circles. The sum of the entries along each side of the square equals 22. Determine the sum of the numbers that appear in the four corner circles.



7. A coffee shop has an unusual payment plan. The shop keeps a weekly tally for each customer. A customer's first coffee costs \$2.50; their second costs \$2.25; third \$2.00 and so on. The cost decreases by \$0.25 until the price reaches \$0.00. The remaining coffees that week are "free" for the customer.

   Ka Fene was a regular customer. In a two week period, Ka consumed a total of 25 coffees at this shop. Ka paid a total of \$26.00 for the two weeks' worth of coffee. How many "free" coffees did Ka get during this two week period?

8. Given that c and d are real numbers. Let f be the function determined by the rule :
$$f(x) = \frac{x}{cx+d}$$
for all real numbers x except $x = -d/c$, and that $f\big(f(x)\big) = x$ whenever $f\big(f(x)\big)$ is defined, determine all possible pairs of values for c and d that produce such a function.

9. Find the solution or solutions to the equation
$$4(4^x) + 127(2^x) = 32\,.$$

10. A four digit number is a multiple of 13. The sum of the four digits is 20. Each digit is a factor of 18. Find the largest number that satisfies all of these conditions. Digits may appear more than once.

Relay

R1 It is known that six hens lay twelve eggs in four days, and that J is the number of days that is required for two hens to lay four eggs. Assuming that all hens lay eggs at the same rate, find J.

Write the value of J in Box #1 of the Relay Answer Sheet.

R2 The lengths of the sides of a right angled triangle are K, $2K+J$, and $(3K-J)$ in increasing order.

Write the value of K in Box #2 of the Relay Answer Sheet.

R3 The positive integers are written in an array as indicated. The first number in a row equals the number of entries in that row. Numbers continue to be written according to this pattern. Each entry is in a row and a column. For example, 2 is in row 2 and column 1, whereas, 13 is in row 4 and column 6. Determine both the row, R, and column, C, in which you would find the number $(K^2 + 3K + 7)$.

1							
2	3						
4	5	6	7				
8	9	10	11	12	13	14	15
.	.	.	.	.	.	.	.

Write the value of $(R^2 + C^2)$ in Box #3 of the Relay Answer Sheet.

R4 A survey of $(R^2 + C^2)$ third year university students produced the following results concerning mathematics courses.

1. Exactly 50% of these students had taken a course in statistics.
2. Exactly 25% of the students surveyed had taken a statistics course and a calculus course but not an algebra course.
3. Exactly 25% of the students surveyed had taken an algebra course.

4. The number of students who had taken only statistics is one greater than the number of students who had taken both statistics and algebra.
5. Two students had taken algebra as their only math course.

Let L represent the number of students who took courses in both algebra and calculus but not in statistics. Given the above information, determine the value of L.

Write L in Box #4 of the Relay Answer Sheet.

Tiebreaker

A partition of a set is a way of dividing it up into a family of non-empty components. For example the set $\{1, 2, 3\}$ has five different possible partitions, including one (see (v) below) with just one component. The complete list is as follows.

(i) $\{1\}$, $\{2\}$, $\{3\}$;

(ii) $\{1, 2\}$, $\{3\}$;

(iii) $\{1, 3\}$, $\{2\}$;

(iv) $\{2, 3\}$, $\{1\}$;

(v) $\{1, 2, 3\}$.

What is the number of possible different partitions for a set with four elements?

1995–96 Game #1

1. A bottle and a cork together cost \$1.10.

   The bottle costs \$1.00 more than the cork.

   What does the cork cost?

2. On the planet **Metricon**, days coincide with **Earth** days. A year on **Metricon** consists of one hundred **Earth** days. Ages in **Metricon** are given in years and days.

   On the planet **Earth**, Ivo was born on 14 February 1979.

   How old would Ivo be on **Metricon**?

   (Note: this problem was set for 14 October 1995.)

3. A car dealer bought 85 cars, each at the same price.

   He sold 40 at a profit of 25%.

   He sold 25 at a profit of 20%.

   He sold the remainder at a profit of 10%.

   His total profit was \$255 000.

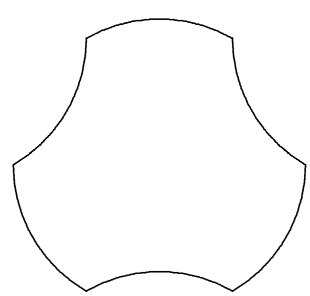
   What did each car cost the dealer?

4. On a "Trouter's tape measure", the first ten "centimetres" are each exactly one centimetre long, the second ten "centimetres" are each exactly one half of a centimetre long, the third ten "centimetres" are each exactly one third of a centimetre long, the fourth ten "centimetres" are each exactly one quarter of a centimetre long, and so on.

   A Trouter tells you that he caught a salmon that was one "metre" long.

   What was the true length (to nearest millimetre) of the salmon?

5. A **hexle** is constructed from a circle by reversing three non-intersecting arcs, each of which is one sixth of the circumference.

   If the radius of the circle is 1, find the (exact) area of the hexle.

6. Alice can do a job in 15 minutes, Bobby can do the same job in 18 minutes, and Chet can do the same job in 24 minutes.

   If all three work together, how long will the job take?

   Give your answer (in minutes and seconds) correct to the nearest second.

7. Because of a big mineral find, there was an increase of 1900% in the population of Noiseys Cove at the start of the prospecting season.

   At the end of the prospecting season, all the incomers departed.

   By what percentage did the season's population fall?

8. The cube of a number is equal to eleven times the square of the number less thirty one times the number plus twenty one.

   The cube of the number is also equal to fifteen times the square of the number less thirty eight times the number plus twenty four.

   Find the number.

9. The Librarian of Noiseys Cove High School wishes to arrange all the books on shelves so that there are exactly the same number on each shelf. (Some shelves may be empty).

   She tries to put 15 on each shelf: this does not work, for there are only three books on one of the shelves.

   She tries to put 16 on each shelf: this does not work, for there are only three books on one of the shelves.

   She tries to put 17 on each shelf: this does not work, for there are only three books on one of the shelves.

   She knows that the total number of books in the library is between twenty-five thousand and thirty thousand.

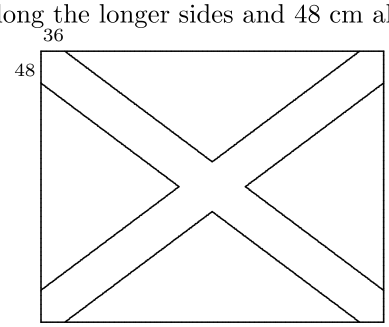
   After several tries, she discovers that she can indeed make a desired arrangement. (Some shelves are empty!)

   How many books are in the library?

10. A **St. Andrew's Cross** flag (a white cross on a blue field) is constructed as shown.

    The longer side is 516 cm and the shorter side is 408 cm.

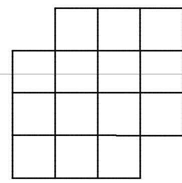
    The white cross is placed symmetrically, and the lengths showing at the corners are 36 cm along the longer sides and 48 cm along the shorter sides.



Find, correct to one decimal place, the percentage of the flag that is blue.

Relay

R1 Let A be the number of squares in this diagram.



Write A in Box #1 of the Relay Answer Sheet.

R2 A regular polygon of A sides is inscribed in a circle of radius B.

If the length of each side is 18.8 cm, find B correct to the nearest centimetre.

Write B in Box #2 of the Relay Answer Sheet.

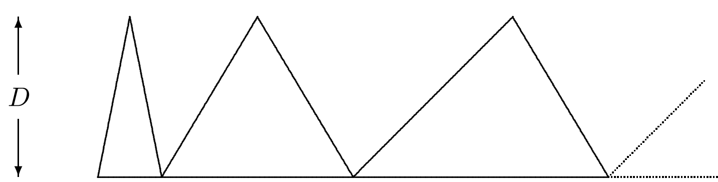
R3 Let C be the sum of the digits of B.

The quadratic equation $Bx^2 + Dx - C = 0$ has rational solutions.

Let D be the smallest positive whole number that satisfies this property.

Write D in Box #3 of the Relay Answer Sheet.

R4 A figure is made up from a sequence of E triangles of altitude D and bases 1, 2, 3, ….



If the area of the figure is 9933, find the value of E.

Write E in Box #4 of the Relay Answer Sheet.

Tiebreaker

In a group of 60 students, each student must take exactly two of Physics, Chemistry and Earth Sciences.

47 students take Physics or Chemistry, but not both.

24 students take Chemistry and Earth Sciences.

P students take Physics and Chemistry.

Find P.

1995–96 Game #2

1. A pizza restaurant sells small, medium, large, extra large, and super large pizzas. The diameters are 10, 12, 14, 15, and 17 inches, respectively, and they are divided into 6, 6, 8, 10, and 12 slices, respectively. Which pizza has the largest slices?

2. Find the smallest whole number n such that

$$1 + 2 + 3 + \ldots + n \; > \; 1000 \, .$$

3. A fair coin is tossed three times. What is the percentage likelihood that the number of "heads" obtained is less than three? Note: The statement that a coin is fair means that if it is tossed once, then there is a 50% likelihood of getting "heads" and a 50% likelihood of getting "tails".

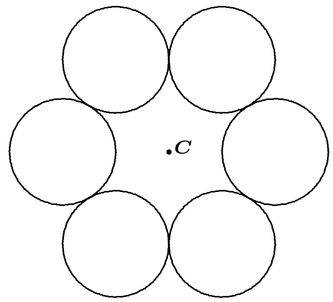
4. Rita's Rooster Restaurant sells chicken in boxes of four, seven, and thirteen chunklets. Find the largest number of chunklets that you cannot buy in a combination of Rita's boxes.

5. A referendum was held in Quebec and 49.4% of voters supported sovereignty. Assuming that francophones make up 82% of the Quebec population and that 59% of this group supported sovereignty, determine the percentage of non–francophones who supported sovereignty. You should give your answer to the nearest integer value.

6. The ferries "Joseph and Clara Smallwood" and "Caribou" sail between Port aux Basques, Newfoundland, and North Sydney, Nova Scotia. Assuming that the "Joseph and Clara Smallwood" leaves Port aux Basques at 11:30 am, and the "Caribou" leaves North Sydney at 4 p.m., how many nautical miles has the "Joseph and Clara Smallwood" covered when the boats pass each other?

   You should remember that Newfoundland time is a half-hour ahead of (that is, later than) Nova Scotia time. The distance from Port aux Basques to North Sydney is approximately 120 nautical miles, and the boats can be assumed to travel at 20 knots. Remember also that one knot is one nautical mile per hour.

7. Six circles, each of radius 1, are arranged so that they are touching and form a symmetric pattern about a centre point C, as illustrated.



Find the distance from the centre of any circle to the centre of the opposite circle.

Note: The arrangement is symmetric in the sense that if the diagram were to be rotated about C through any multiple of $60°$, then it would be unchanged.

8. Determine all integers n, with $100 < n < 200$, such that

$$16|(n^2 + 2n) \text{ and } 25|(n^2 - n).$$

Note: $a|b$ means that a is a factor of b, i.e. that a divides b.

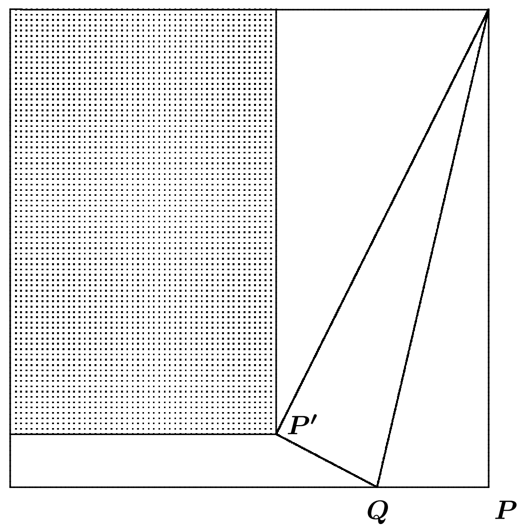
9. This question concerns the problem of overfishing in a region of the North Atlantic. You can assume that scientists have determined that if the net of a trawler has mesh size x cm $\times$ x cm, then the percentage of turbot entering the net that are caught is $(100 - \frac{3}{4}x^2 + 6x)$. For example, if the net mesh size was zero, then the percentage of turbot caught would be $100 - 0 - 0 = 100$.

   A trawler, that was suspected of using an illegal size net, dropped its net to the ocean floor before it was arrested. Scientists later estimated, from the proportions of different sized fish in the catch, that $96\frac{13}{16}$ % of turbot entering the net had been caught.

   This information was then given to a group of expert high school students who were able to determine the mesh size. What was the value of x?

10. A square piece of paper, with sides of length n, is folded along the line joining one vertex to a point Q, this last point being on a side of the square and at a distance 1 from the vertex P, as illustrated.

   So the vertex P is moved, after folding, to the position P'. The position of the point P' determines the shaded rectangle, as illustrated.



What fraction of the area of the original square is contained within the shaded rectangle? Express your answer in the form $p(n)/q(n)$, where $p(n)$ and $q(n)$ are polynomials in n, and each is factorized as much as possible.

Hint: the first part of your argument can use similar triangles, but first you must decide which triangles are similar!

Relay

R1 Let K be a digit (that is, an integer between 0 and 9, inclusive) such that the number $576\,3K6$ is divisible by 8. Then take A to be the sum of all such digits K.

Write the value of A in Box #1 of the Relay Answer Sheet.

R2 Let B be the sum of all real number values of x such that

$$\begin{aligned} x^2 + y &= 0\,, \\ Ax + 10y^2 &= 0\,, \end{aligned}$$

has at least one real solution.

Write the value of B in Box #2 of the Relay Answer Sheet.

R3 Karen, Kathy, Ken, Tammy, Tony, and Tracy are six friends. Let C be the number of ways in which a team with $B+3$ members can be selected from them, subject to the requirement that at least one member of the team has a name beginning with a 'T'.

Write the value of C in Box #3 of the Relay Answer Sheet.

R4 In a group of $5C$ high school students, 17 are left-handed, 10 have names beginning with 'T', and 35 are both right-handed and have names that do not begin with 'T'. What is the number of left-handed students whose names begin with 'T'?

Write your answer in Box #4 of the Relay Answer Sheet.

Tiebreaker

The following mathematical game was described in an old manuscript.

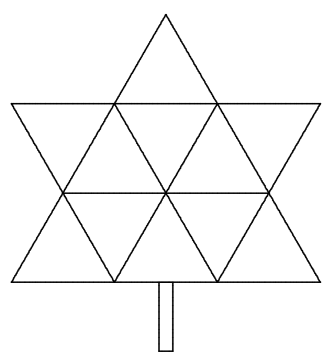
"Ask a friend to think of a number but to keep it secret from you. Tell the friend to add seven to that number, and to then multiply the answer by the number first thought of. Next get your friend to ...... , to then add nine and to finally take the square root of the last number obtained. Get your friend to tell you the last answer. You can surprise your friend as you now know the original number. You obtain this simply by subtracting three from the final answer."

Unfortunately acid has been spilled over the original manuscript and "......" indicates a step that can no longer be read. What was this missing step?

1995–96 Game #3

1. A customer who is about to withdraw \$400 from a bank knows that Teller A will supply the \$400 in twenty \$$x$ bills and ten \$$y$ bills, while teller B will supply the \$400 in ten \$$x$ bills and fifteen \$$y$ bills. Find x.

2. How many triangles are present in the 1967 Centennial Year symbol?



3. When tossed, a coin is as likely to come down "heads" as it is to come down "tails". If the coin is tossed three times, what is the percentage probability (likelihood) that the three outcomes are **not** all the same?

4. Find the range of values of the real number k for which the quadratic equation,

$$2x^2 - kx + 8k = 0\,,$$

has no real solutions.

5. I place two red marbles in one box, two blue marbles in another box and one of each colour of marble in a third box. I then close all three boxes. I have three labels, one for the contents of each box (the labels say "*two red*", "*two blue*" and "*one red, one blue*"), but while your back is turned I swap the labels so that all three boxes are incorrectly labelled. I allow you to reach into any box of your choice and take a marble out in an attempt to identify the true contents of each box. I allow you to withdraw as many marbles as you wish from whichever box(es) you wish. (Obviously five marbles is sufficient to identify all boxes correctly).

   What is the **minimum** number of marbles that you must take in order to be certain of the true contents of all three boxes?

6. Two circles are orthogonal if their tangents at a point of intersection are at right angles to each other. Let us assume that two circles C_1 and C_2 of radii 20 mm and 21 mm, respectively, are orthogonal. The line **segment** joining their two centres intersects both C_1 and C_2: we denote these points of intersection by A_1 and A_2, respectively. Determine the distance A_1A_2 in mm.

7. Given that m and n are integers with $m \geq 3$, $n \geq 3$ and

$$2n - mn + 2m \; > \; 0\,,$$

   determine all possible pairs (m, n) that satisfy the above conditions.

8. Determine the real number x or all real numbers x, if any exist, such that

$$-\sqrt{3}x \; = \; \sqrt{3 - \sqrt{4x^2 + 1}}\,.$$

9. On how many occasions in a (24 hour) day are the hands of a clock at right angles to each other?

10. A triple of positive integers (p, q, r) will be said to be "good" if:

    (i) $p \leq q \leq r$, and

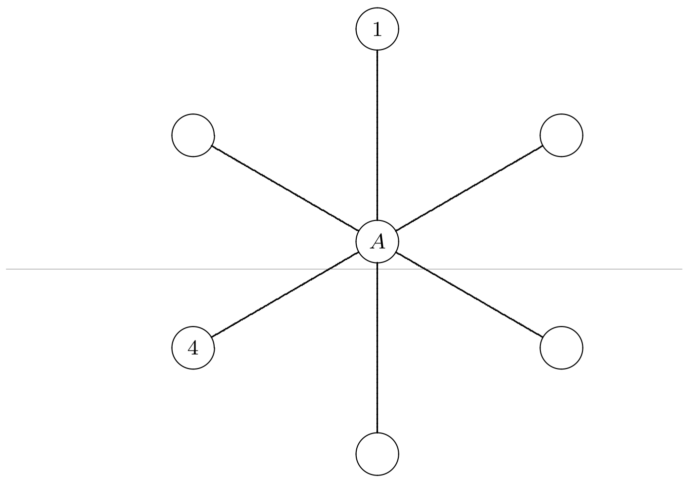
    (ii) $(1/p) + (1/q) + (1/r) = (11/18)$.

    Find five different good triples (p, q, r).

    In this question we use a different marking scheme: one triple correct scores 1 mark, 2 correct 2 marks, 3 correct 3 marks, 4 correct 4 marks and 5 correct 5 marks. Each incorrect answer loses a mark.

Relay

R1 In the magic snowflake, all three lines passing through the centre number have to add up to the same total. The integers 1 to 7 are each used once only. Find A.



Write A in Box #1 of the Relay Answer Sheet.

R2 The premier division of a sports league contains A teams. In a season each team plays each of the other teams in that division once at home and once at the other team's stadium. Let x be the number of matches that takes place in that division in a season and B the sum of the digits of x. Find B.

Write B in Box #2 of the Relay Answer Sheet.

R3 B is a digit in the six digit number $3B4\,500$. Find the smallest positive integer C that when multiplied with $3B4\,500$ gives a perfect cube.

Write the value of C in Box #3 of the Relay Answer Sheet.

R4 A lattice point is a point (a, b), where a and b are integers. Determine the number D of lattice points that are inside the triangle with vertices $(0, 0), (5, 0)$ and $(0, C + 5)$.

Write the value of D in Box #4 of the Relay Answer Sheet.

Tiebreaker

Determine all solutions to the equation $x^2 + 6x - 24 = 4|x|$.

1995–96 Game #4

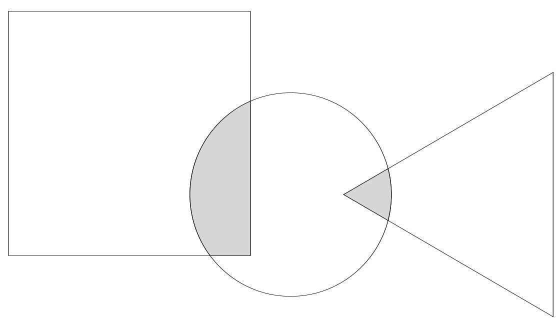
1. Janice, who is twelve years old, is four times as old as her brother. How old will Janice be when she is twice as old as her brother?

2. In this question you need to know that $3 \times 5 \times 7 = 105$. Imagine that you are given 105 small white cubes, all of which are the same size, and that you glue them together so that they form a box shaped block of length 7, width 5 and height 3. If the top and sides of the block are then painted red, how many of the small cubes are painted red on just two sides?

3. A year is (usually) a leap year if the number of the year is divisible by four. Exceptions (usually) occur when years are divisible by one hundred - in such cases no day is added. Exceptions to the exception rule occur if the year can be divided by four hundred - in that case a day is added.

   Assuming that the same system stays in place for the next few hundred years, how many days will be added between today (that is, 23 March 1996) and the one thousandth anniversary of John Cabot's visit to Newfoundland? (John Cabot sailed along the coast of Newfoundland in June 1497.)

4. The diagram shows a square with sides of length 2, a circle with radius $\frac{2}{\sqrt{\pi}}$ and an equilateral triangle with sides of length 2.



   If the area of the unshaded region of the square is 3.2 and that of the unshaded region of the circle is 2.6, determine the exact area of the unshaded region of the equilateral triangle.

5. An arrangement of the numbers 1, 2, 3, 4, 5, 6, 7, 8 will be said to be "*good*" if:

   {the first number} − {the second number} = 4,

   {the third number} − {the fourth number} = 3,

   {the fifth number} − {the sixth number} = 2, and

   {the seventh number} − {the eighth number} = 1.

   For example, 51634287 is such an arrangement. Find five other good arrangements.

   We will use a different marking scheme in this question. You will receive a number of marks equal to your number of correct answers, except that each incorrect answer loses a mark. In this case all members of a team must agree on the same set of answers.

6. A high school student remembers the quadratic formula incorrectly and thinks that the roots of $px^2 + qx + r$ are given by

$$\frac{-q \pm \sqrt{p^2 - 4qr}}{2p}.$$

Noticing that, if $p = q$, then

$$\frac{-q \pm \sqrt{q^2 - 4pr}}{2p} = \frac{-q \pm \sqrt{p^2 - 4qr}}{2p}\,,$$

the student's friend realizes that this incorrect formula can sometimes give the correct roots of the quadratic.

Find a value of p and a value of q such that p is 14 greater than q and for which the incorrect quadratic formula gives the correct roots of $px^2 + qx - 2$.

7. An operation $*$ is defined such that

$$a * b = 2a(a - b)\,,$$

where a and b are real numbers.

Suppose that m is the length of the hypotenuse of a right angled triangle with sides of length m, n and p. Given that m, n and p are integers, $m * n = 68$ and $n * m = -60$, determine the perimeter of this triangle.

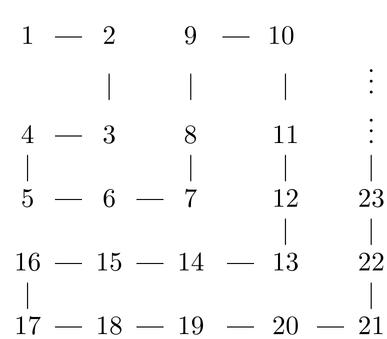
8. On December 8th an airline passenger travelled from St. John's to Vancouver, with a change of planes in Toronto. The passenger was scheduled to leave St. John's at 2:00 p.m. local time and leave Toronto at 5:00 p.m. local time. The flying time from St. John's to Toronto is 3 hours 24 minutes and from Toronto to Vancouver 4 hours 49 minutes.

However there was freezing rain in St. John's on December 8th and departure was delayed for de-icing the plane. The flight out of Toronto was delayed for other reasons. The passenger had exactly twenty-one minutes between his arrival and departure from Toronto. The passenger arrived in Vancouver at 7:49 p.m. local time.

How long was the delay in St. John's?

Note: Travelling from St. John's to Toronto you turn your watch back 1 hour and 30 minutes, from Toronto to Vancouver you turn it back 3 hours.

9. What is the $19^{\mathbf{th}}$ term in the sequence 4, 6, 14, 20, ... determined by a diagonal line in the following diagram:



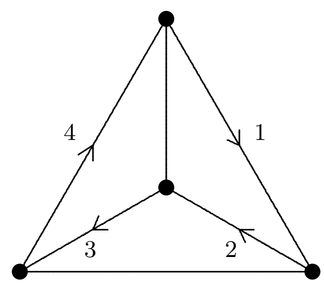
10. Let A, B, C and D represent digits; that is, members of the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Using the standard notation for representing natural numbers, $ABCD$ represents the number $1000A + 100B + 10C + D$ and we require that $A \neq 0$.

    Given that $ABCD \times 4 = DCBA$, determine the number $ABCD$.

    Hint: Start by asking yourself what is known about $4 \times A$ and $4 \times D$.

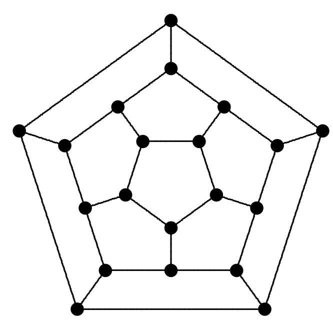
Relay

R1 The diagrams below are called graphs; the little solid circles in the graphs are called vertices and the line segments joining the vertices are called edges.



A Hamiltonian circuit is a route around the edges of a graph that starts and ends at the same vertex and visits each other vertex just once. For example the route 1234 is a Hamiltonian circuit in the following graph:

Some graphs have Hamiltonian circuits and others do not. Does the following graph have a Hamiltonian circuit?



If your answer is yes take Q to be 4, if your answer is no take Q to be 5.

Write your answer for Q in Box #1 of the Relay Answer Sheet.

R2 Let A represent the set of integers a for which $|a| < 1 + 5Q$. Let B represent the set of integers b for which $|b| < 1 + 2Q$. Let R denote the maximum possible value of $|a - b|$, where a is in A and b is in B.

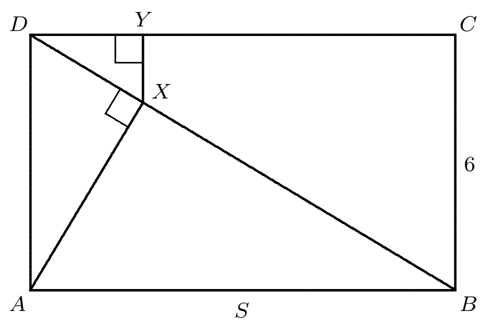
Write your answer for R in Box #2 of the Relay Answer Sheet.

R3 A pair of integers (a, b) will be said to be "onely" if $a < b$ and $(a^4 + b^4)$ has one as its last digit.

How many onely pairs exist if a and b are required to be selected from the set of integers 0, 1, 2, 3, 4, …, $R - 20$, $R - 19$?

Write your answer S in Box #3 of the Relay Answer Sheet.

R4 In the rectangle $ABCD$ with $AB = S$ and $BC = 6$, the foot of the perpendicular from A to DB will be denoted by X. The foot of the perpendicular from X to DC will be denoted by Y. Determine the exact value of the distance CY.



Write your answer in Box #4 of the Relay Answer Sheet.

Tiebreaker

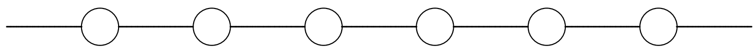
Let P denote a moving point in the xy–plane. Starting at position $(2, 1)$ the point moves to a new position by being reflected in the origin. From the new position P is reflected in the line $y = x$ and thus takes up a third position. From the third position P is rotated about the origin through 90° in a clockwise direction and takes up a fourth position. From the fourth position P is reflected in the line $y = 3$. What are the coordinates of the final resting place for P?

1995–96 Game #5

1. An ant takes a multi-stage walk on horizontal ground. The first stage is to walk 1 cm north, the second 2 cm east, the third 3 cm south, the fourth 4 cm west. The pattern of the distances is 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1, 2, 3, …; the pattern of directions N, E, S, W, N, E, S, W, N, E, S, W, ….

   How many stages have been covered when the ant first returns to its starting place?

2. A princess has six jewels: two diamonds, two emeralds and two rubies, and wants them to be arranged on a necklace as illustrated.



The princess is interested in numbers and wants an arrangement in which the two diamonds are separated by one other jewel, the two emeralds by two other jewels and the two rubies by three other jewels.

If this can be done state a possible order for the jewels; otherwise state that it is impossible.

3. Two trains are approaching each other on a straight line railway track. Both trains travel slowly at a steady speed of 10 km/hr. At the moment when the trains are 20 km apart a fly leaves the first train, flies to the second train, instantly reverses its motion and flies back to the first train, again reverses direction and flies back to the second train and so on. If the fly always travels at 40 km/hr, determine the total distance covered by the fly before the trains collide?

4. If $x > y$ are real numbers such that

   (i) $x + y = 3$ and

   (ii) $xy = -180$,

   determine $x - y$.

5. The pair of numbers $(2, 3)$ has the interesting property that:

$$2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}}\,,$$

   Also for $(3, 8)$ we have

$$3\sqrt{\frac{3}{8}} = \sqrt{3\frac{3}{8}}\,.$$

   Find three other pairs of positive integers (a, b) satisfying the same property with b between 50 and 100.

6. A committee of three students is to be set up to discuss the high school mathematics curriculum. Seven students are considered suitable, four of whom like mathematics and three of whom do not.

   In how many ways can the committee be selected if it is required that at least one committee member should like mathematics and at least one should not?

7. A space station is to be constructed from modules that are made from a rubber like material. The size and shape of each module are determined by an associated pair of index numbers. So there are $(3, 5)$–modules, $(4, 4)$–modules and so on. The order of the index numbers is important, so a $(2, 6)$–module is not the same as a $(6, 2)$–module.

Modules can be attached together according to the following rule: any (m,n)–module A can be linked up with any (n,p)–module B to form an (m,p)–module AB, for any choice of numbers m, n and p.

A Canadian astronaut discovers that modules can be turned inside out, and that in doing so an (m,n)–module A is transformed into an (n,m)–module A^T, for all choices of m and n.

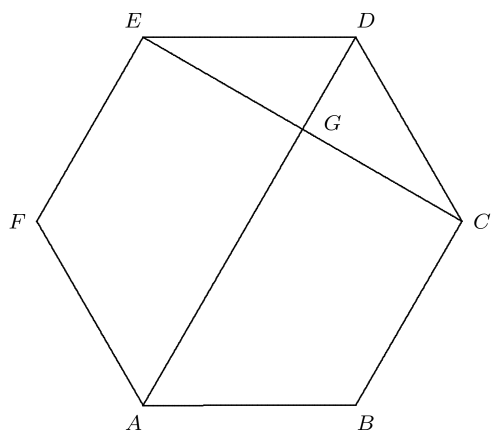
The astronaut takes a $(5,7)$–module AB, disconnects the two units, turns the module A inside out and forms a new compound module $C(A^T)$. Given that C is a $(3,4)$–module, what are the index numbers of B?

8. Given that x and y are positive real numbers such that:

$$(2x)^{y^2+3} = 8x^3 \times 2^{(y^2+3y)},$$

determine x^y.

9. Given a regular hexagon $ABCDEF$ with sides of length 2. Let AD intersect CE at G, determine the exact value of the area of the quadrilateral $ABCG$.



10. Let f be the function

$$f(x) = \frac{x(2-x)}{(x-1)^2},$$

where x is any real number except 1, and g be the function

$$g(x) = \frac{x+2}{2x+1},$$

where x is any real number except $-\frac{1}{2}$.

Solve the equation $9f(g(x)) = -8$, listing all solutions.

Relay

R1 Consider a rectangle 5 cm by 20 cm, divided into 100 squares, each 1 cm by 1 cm, as indicated:

If V denotes the number of vertices •, E denotes the number of edges (that is, the number of horizontal or vertical 1 cm line segments that start and end with a •) and S the number of 1 cm^2 squares, determine the value of $A = V - E + S$.

Write A in Box #1 of the Relay Answer Sheet.

R2 A *prime pair* is a pair of positive integers of the form $(n, n+2)$, both n and $n+2$ being prime numbers. For example $(11, 13)$ is a prime pair but $(9, 11)$ and $(13, 15)$ are not.

Find the smallest prime pair $(B, B+2)$ for which B is greater than $(A+70)$.

Write B in Box #2 of the Relay Answer Sheet.

R3 If 15 hens lay $(B-47)$ eggs in 6 days, determine the number of days C that are required for 12 hens to lay 36 eggs. You may assume that all hens are laying eggs at the same rate.

Write C in Box #3 of the Relay Answer Sheet.

R4 A cylindrical pipe of diameter $\frac{4C}{5}$ cm, length 1 metre and with sealed ends lies in a horizontal position and is filled to a depth of 1 cm with water. Determine the exact volume D of water in the pipe in cm^3. Express your answer in the form

$$\frac{a\pi + b\sqrt{3}}{c},$$

where a, b and c are integers.

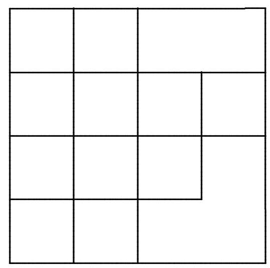
Write D in Box #4 of the Relay Answer Sheet.

Tiebreaker

Which real number exceeds its square by the largest amount?

1996–97 Game #1

1. How many squares appear in the diagram?



2. The perimeter of a rectangle is 100 cm. The area of this rectangle is 609 cm^2. Find the length and width (in cm) of the rectangle.

3. Balls numbered 11, 12, 13, ... 97, 98, 99 are placed in a drum. A game is played in which you get to select one of the balls randomly. You are a winner in this game if the number drawn has either digits whose sum ends in 5 (for example, 96 has sum 15) or digits whose product ends in 5 (for example, 75 has product 35). How many winning numbers are there?

4. Given that $(3^x)(9^y) = 27$ and $\dfrac{9^x}{81^y} = 1$, determine the value of $16^x + 16^y$.

5. Each letter represents a different digit. Find the values of A, B, and C that make the following multiplication true.

$$\begin{array}{r} AB \\ \times AB \\ \hline BCB \end{array}$$

6. A visiting team from North Bay participates in a hockey tournament with teams from Newfoundland and Labrador. The results (wins, losses, ties) of the tournament, a series of games in which each team plays against each other team once, are summarized below:

	Wins	Losses	Ties
Labrador City	2	0	1
North Bay	1	1	1
Gander	1	2	0
Corner Brook	0	1	2

Using the results, determine the outcome of each game. Write the name of the winning team in the space provided. If the game ended in a tie, write "Tie" in that space.

	Winner
Gander vs. Corner Brook	________________
Gander vs. Labrador City	________________
Corner Brook vs. North Bay	________________
North Bay vs. Labrador City	________________
North Bay vs. Gander	________________
Labrador City vs. Corner Brook	________________

7. A line passes through the point $(1, 4)$. The line and the two axes make a triangle of area 9 in the first quadrant. Determine all possible points where the line could meet the x–axis.

8. Consider a three-digit number with the following properties:

   (a) If its second and third digits are switched, its value would increase by 27.

   (b) Alternatively, if its first and third digits are switched, its value would decrease by 99.

   By how much would the value of the original number decrease if the first and second digits are switched?

9. Find all real values x such that:

$$2(2 - x^2) = -\sqrt{13x^2 - 9}\,.$$

10. Assign values to the letters of the alphabet as shown:

$A = 1$	$F = 6$	$K = 11$	$Q = 17$	$V = 22$
$B = 2$	$G = 7$	$L = 12$	$R = 18$	$W = 23$
$C = 3$	$H = 8$	$M = 13$	$S = 19$	$X = 24$
$D = 4$	$I = 9$	$N = 14$	$T = 20$	$Y = 25$
$E = 5$	$J = 10$	$O = 15$	$U = 21$	$Z = 26$
		$P = 16$		

The challenge is to find an English word with a value as close to 100 as possible. The value of a word is the sum of the numbers corresponding to the letters.

For example, the value of "LEAGUE" would be

$$12 + 5 + 1 + 7 + 21 + 5 = 51.$$

Note that the English word must be one that would appear in a standard dictionary. That is, personal names and places are not permitted.

Write your best word only as your answer. You will get 5 points for a word with a value of 100. However, good efforts will also be rewarded as follows:

4 points for 99 or 101,

3 points for 98 or 102,

2 points for 97 or 103, and

1 point for 96 or 104.

Relay

R1 A and B are unequal single digit numbers such that $A^B = B^A$.

Write the value of $(A+B)$ in Box #1 of the Relay Answer Sheet.

R2 The mathematical operation "!" is called factorial. In general,

$$n! = (n-1)(n-2)\ldots(3)(2)(1)\,.$$

For example, $20! = 20 \times 19 \times 18 \times \ldots \times 3 \times 2 \times 1$.

The product of $(A+B)! \times 7! = C!$ where C is another integer.

Write the value of C in Box #2 of the Relay Answer Sheet.

R3 The time showing on a clock is C minutes after 2 o'clock. The measure of the small angle formed by the hands of the clock at this time is D degrees.

Write the value of D in Box #3 of the Relay Answer Sheet.

R4 A lattice point is a point (x, y) where x and y are both integers. How many lattice points lie in the interior of the circle $x^2 + y^2 = D^2$? Points along the circumference are not to be counted.

Write your answer in Box #4 of the Relay Answer Sheet.

Tiebreaker

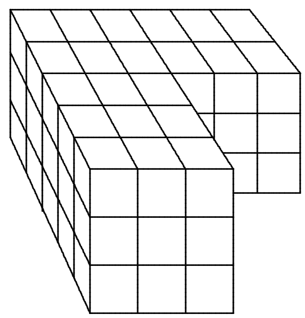
A student was about to take the final test of the year in a certain math course. The teacher stated that a mark of 90% would raise the student's overall average to 84%, whereas a mark of 72% would lower the student's overall average to 81%. Assuming that all tests in the year were equally weighted, how many math tests were there altogether in that year?

1996–97 Game #2

1. We define a prime string to be a sequence of numbers in which each adjacent pair sums to a prime. For example, 5, 2, 1, 4, 3 is a prime string because $5 + 2 = 7$, $2 + 1 = 3$, $1 + 4 = 5$, and $4 + 3 = 7$. All of the sums are primes.

   Create a prime string using each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 once.

2. An L–shaped block is constructed out of 63 identical white cubes, as illustrated.

   If the whole exterior of the block (including top and bottom) is painted red, how many of the 63 cubes will have exactly two red faces?

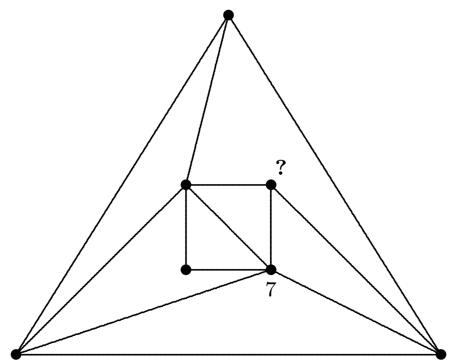
3. The manager of a shoe store could not decide on the price of a pair of boots. The original price was increased by 10%, then decreased by 10%, then increased by 20% and then decreased by 20%. Express the final price as a fraction of the original price. Your answer should be given in the lowest possible terms; that is, cancelled as much as possible.

4. Timbits are sold in packages of 20, 45, and 65. What is the largest multiple of 5 timbits that cannot be exactly obtained by ordering packages?

5. The calendar will be said to *fall the same way* in two years if every date falls on the same day of the week for both of the years; that is, both January 1st days are the same day of the week, both January 2nd days are the same day of the week, and so on through to December 31st. What was the first year of this century to fall the same way as 1996?

   Note: This is not essential information but some might find it helpful to know that 1996 started on a Monday.

6. The numbers 1 to 7 are to be used to label the small solid circles in the diagram below.



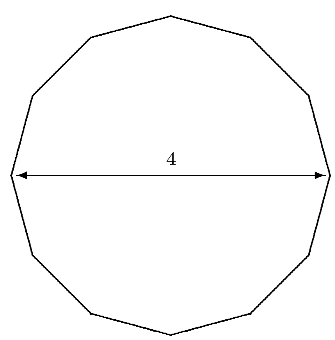
This should be done in such a way that the solid circles corresponding to consecutive numbers are never joined by a line segment. Given that the circle 7 is as indicated, find the number that corresponds to the solid circle with a ? symbol.

7. Find all values of k (if any) for which the quadratic equation in x:

$$x^2 + (2k+6)x - (k+3) = 0\,,$$

has two identical solutions.

8. A dodecagon or duodecagon is a polygon with twelve sides. If the distance between opposite vertices of a regular dodecagon is 4 units, determine its area in square units.



9. A group of extraterrestrials, from outside the Solar System, decided to set up a base on Earth. They selected a site on the bed of the Atlantic Ocean. When the base was fully operational 234 visitors were living there. Of these

    144 had blue hair,

    126 had yellow eyes,

108 had red noses,

72 had blue hair and yellow eyes,

54 had yellow eyes and red noses and

45 had blue hair, yellow eyes and red noses.

All 234 visitors had at least one of blue hair, yellow eyes or red noses.

How many had blue hair and red noses?

10. In the song "*The Twelve Days of Christmas*" the true love gave a total of 364 presents, from one partridge on all twelve days to twelve lords a-leaping on the last day only. The last verse of the song is:

On the twelfth day of Christmas my true love sent to me:
Twelve lords a-leaping,
Eleven ladies dancing,
Ten pipers piping,
Nine drummers drumming,
Eight maids a-milking,
Seven swans a-swimming,
Six geese a-laying,
Five golden rings,
Four calling birds,
Three French hens,
Two turtle doves,
And a partridge in a pear tree.

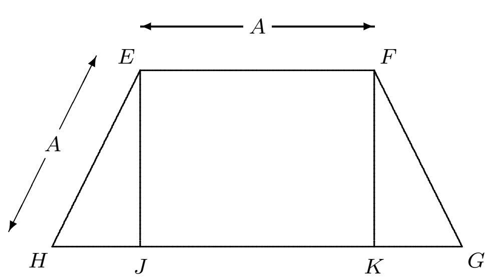
Over the twelve days of Christmas, two of these twelve types of gifts are given by the true love more than any other. Which gifts are they **and** how many of each of them is given?

Relay

R1 The line $Ax+3y = 7$ passes through the point $(A, -3)$ in the fourth quadrant.

Write the value of A in Box #1 of the Relay Answer Sheet.

R2 The area of the trapezoid $EFGH$ pictured below, when expressed in the simplest terms, is $B\sqrt{C}$ square units. You should assume that $\angle EHG = 60°$, $HJ = KG$ and $EF = EH = A$.



Write the value of $B+C$ in Box #2 of the Relay Answer Sheet.

R3 Consider the set of numbers

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}\,.$$

A game requires that you keep on selecting numbers from this set until your collection contains two different numbers with total $(B+C)$. Note that, once a number has been selected, it is no longer available for future use. The game then ends. The maximum possible number of selections that could occur in this game is D.

Write the value of D in Box #3 of the Relay Answer Sheet.

R4 Given that

$$\begin{aligned} x^2+y^2+z^2 &= D\,, \\ xyz &= 0\,, \\ x+y+z &= 0\,. \end{aligned}$$

Determine the value of $(x^4+y^4+z^4)$.

Write this value in Box #4 of the Relay Answer Sheet.

Tiebreaker

Find the exact value of the positive real number x that satisfies

$$x^2+30\sqrt{2}=43\,.$$

1996–97 Game #3

1. What number must 189 be divided by to obtain a quotient of 17 and have a remainder of 2?

2. A man died and left one-third of his estate to his wife. The remainder was divided into two equal portions. One portion was divided equally amongst his three daughters and the other portion was divided equally amongst his four sons.

   If a daughter's share was \$ 20 000 more than a son's share, what was the total value of the estate?

3. The lines through the point $(6,4)$ that are parallel to the line $4x+y=10$, and perpendicular to the line $3x+y=17$, make a quadrilateral with the positive parts of the axes.

   Find the area of this quadrilateral.

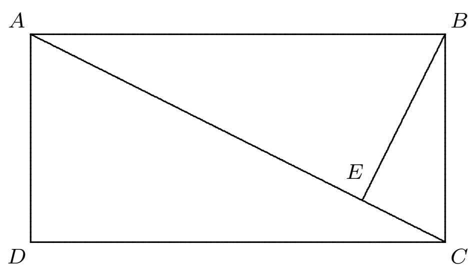
4. Chris and Pat are at diametrically opposite sides of an impenetrable wood with perimeter 1350 metres. They start to walk around the wood in the same direction.

   Chris walks at a rate of 110 metres in 2 minutes, while Pat walks at a rate of 170 metres in 3 minutes.

   How many times will Pat go round the wood before catching up with Chris?

5. The rectangle $ABCD$ is twice as long as it is broad. The diagonal AC is drawn, and a perpendicular is dropped from B to AC, meeting it at E.

   Calculate the ratio $AE : EC$.



6. Find all solutions of the equation

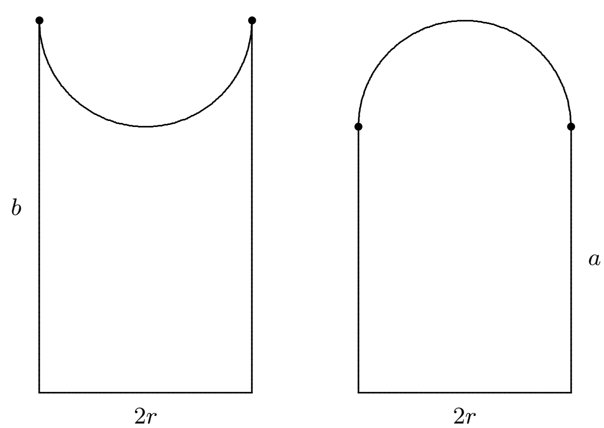
$$x = \sqrt{1 + \sqrt{13 - x^2}}\,.$$

   NOTE: "$\sqrt{\ }$" means the positive value.

7. A vessel containing 500 litres of pure vinegar has 50 litres withdrawn, and is then filled up with water. The resulting combination is thoroughly mixed. This process is repeated four more times.

   Calculate the quantity of water in the resulting mixture.

8. There are two ways that a rectangular window frame can be topped with a semi-circle:



The straight line sides are of lengths a and b as shown, and the semi-circles have radius r.

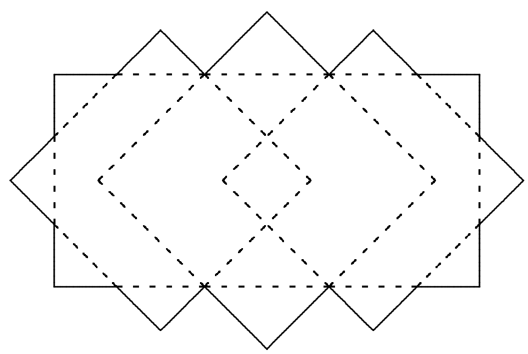
If both windows are of the same area, find $b - a$ in terms of r.

9. In a football team, Andy and Bernie are guards, and Chuck and Dave are tackles. Their total weight is 500 kilos.

   Each player weighs an integer number of kilos, and no two players have the same weight. In fact, their weights are in the same order as their names are given. No player weighs more than 3 kilos more than the next player alphabetically.

   What is the average of all the possible positive differences in weight between Andy and Dave?

10. An ancient church form was devised starting with a doubled square. Each square was then rotated one half of a right angle, and a third (larger) square, with sides parallel to the rotated squares, was constructed as shown. The church form was the outer (solid) lines.



    If the original square was of side 100 paces, find the area of the third (larger) square — an exact answer is required.

Relay

R1 The sum of two numbers is 24. The product of the two numbers is 128.

Find a, the smaller of the two numbers.

Write a in Box #1 of the Relay Answer Sheet.

R2 Old style English money had 12 pennies to the shilling and 20 shillings to the pound. At one time in history, the exchange rate was that the dollar was worth 4 shillings and 2 pence.

Calculate the equivalent in dollars for $(a-3)$ pounds. Call this number b.

Write b in Box #2 of the Relay Answer Sheet.

R3 A square of side $b/2$, $ABCD$, has centre O. The line segment OA is extended outside the square to E, so that $AE = AB$.

Find BE^2 in the form $p\sqrt{q} + r$.

Let $c = \sqrt{p} + \frac{1}{q}\left(\frac{r}{p}\right)^2$.

Write c in Box #3 of the Relay Answer Sheet.

R4 The equation $x^3 - 500x^2 + 588x = c$ has roots α, β and γ.

Calculate $d = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

Write d in Box #4 of the Relay Answer Sheet.

Tiebreaker

An S–column is a column of three numbers, with the bottom number always being twice the top number.

An S–operation on an S–column consists of creating a new S–column, where the top number is the sum of the top and middle numbers in the old S–column, and the middle number is the sum of the middle and bottom numbers in the old S–column.

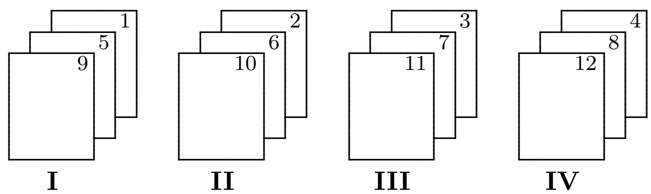
Start with the S–column $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, and perform 9 successive S–operations.

In the $10^{\textbf{th}}$ S–column, calculate the difference between the larger and the smaller of the ratios of the middle number to the top number, and the bottom number to the middle number. (Exact answer please.)

1996–97 Game #4

1. A population starts with ten mice and triples every month. How many complete months will have passed at the moment when the population first exceeds one million?

2. A deck of twelve cards numbered 1 to 12 are face up and in order, so 1 is on the topside of the top card, 2 on the topside of the card below it, and so on. The cards are dealt into 4 piles of 3, as shown



The piles are then consolidated with pile **III** being placed on top of pile **IV**, then pile **II** on top of pile **III**, and then pile **I** on top of pile **II**. So we again have a deck of 12 cards, but they are in an order different from the original order.

Starting with the original order 1, 2, ..., 12, how many times do we have to go through this dealing and consolidation process before we return to a pile in the original order?

3. 27% of the population of Oldlostland have travelled by airplane in the last year, and 19% have travelled by boat in that time. These figures include the 9% of the population who have used both forms of transportation in the last year. What percentage of the population have used neither of these modes of transportation in that time period?

4. Given that $w + x + y + z = 3$ and that $w^2 + x^2 + y^2 + z^2 = 7$, determine the value of $wx + wy + wz + xy + yz + zx$.

5. A geometric sequence consists of numbers

$$a, ar, ar^2, ar^3, \dots ,$$

where a and r are fixed real numbers. For example,

$$3, 6, 12, 24, \dots$$

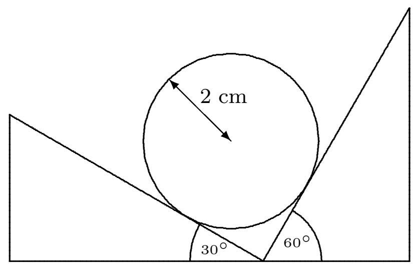
is the geometric sequence with $a = 3$ and $r = 2$.

Find n such that $24-n$, $44-n$, $144-n$ are the first three terms of a geometric sequence.

6. Five fair coins are to be tossed simultaneously onto a flat table top and the result observed. What is the percentage likelihood that the number of heads will be an integer power of 3; that is, equal to 3^n for some integer n?

7. Determine $\sqrt{79 - 20\sqrt{3}}$. Remember that $\sqrt{\ }$ refers to the positive square root only. You might like to try looking for a solution of the form $a + b\sqrt{3}$, where a and b are integers.

8. Consider a ball of radius 2 cm that rests on two inclined planes that meet at ground level and have slopes of 30° and 60°, respectively.



Determine the height of the centre of the ball above ground level.

Hint: $\sin(A+B) = \sin A \cos B + \cos A \sin B$ may be used.

9. Let k be a real number and f be the function defined by

$$f(x) = \frac{x+k}{1-x},$$

where $x \neq 1$. Given that

$$ff(x) = f\big(f(x)\big) = \frac{2x-3}{2-x},$$

whenever these expressions are defined, determine k.

10. This problem concerns a branch of logic, namely the algebra of propositions.

We use letters p, q, r, s and t to denote propositions. There are several operations that produce new propositions from old propositions.

(i) for any proposition p there is a new proposition $\sim p$ known as the *negation of* p.

(ii) for any pair of propositions p and q there are new propositions $p \wedge q$ known as p *and* q, as well as $p \vee q$ known as p *or* q.

There is also a special proposition c known as *contradiction*.

These operations satisfy several rules including, for all p, q and r:

$$\begin{aligned} p \wedge (q \vee r) &= (p \wedge q) \vee (p \wedge r), \\ \sim(p \wedge q) &= (\sim p) \vee (\sim q), \\ \sim(\sim p) &= p, \\ p \wedge (\sim p) &= c, \\ c \vee p &= p. \end{aligned}$$

Determine which of the propositions $s \vee t$, $(\sim s) \vee t$, $s \vee (\sim t)$, $s \wedge t$, $(\sim s) \wedge t$ or $s \wedge (\sim t)$ is equivalent to

$$s \wedge (\sim(s \wedge \sim t)).$$

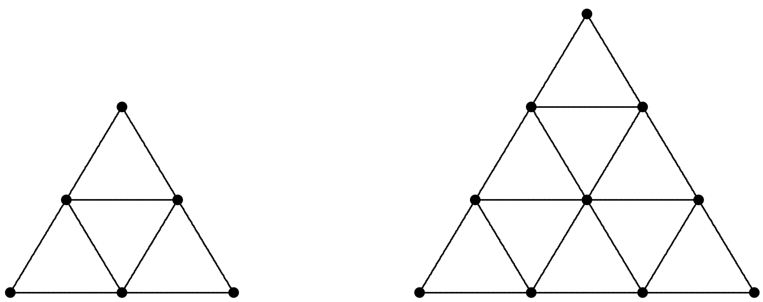
Relay

R1 In the diagrams we have

(i) a 2–triangle of triangles, and

(ii) a 3–triangle of triangles.

Extending this idea in the obvious way we have the concept of an n–triangle of triangles, for every positive integer n.



(i) (ii)

Let S be a 6–triangle of triangles, V be the number of vertices • in S, E the number of edges •——• in S, and T the number of small triangles in S.

Write the value of $p = V - E + T$ in Box #1 of the Relay Answer Sheet.

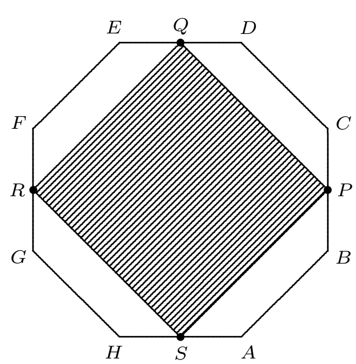
Note: •——•——• counts as two edges, etc.

R2 You may recall that $a^2 + b^2 = c^2$ has just one integer solution with $0 < a < b < c < 10$, namely $a = 3$, $b = 4$ and $c = 5$.

Find an integer solution to $e^2 + f^2 + p = g^2$ where $0 < e < f < g < 10$.

Write the value of $q = 2g - f - 2e$ in Box #2 of the Relay Answer Sheet.

R3 Let $ABCDEFGH$ be a regular octagon with sides of length q. The mid-points of the sides BC, DE, FG and HA will be denoted by P, Q, R and S, respectively

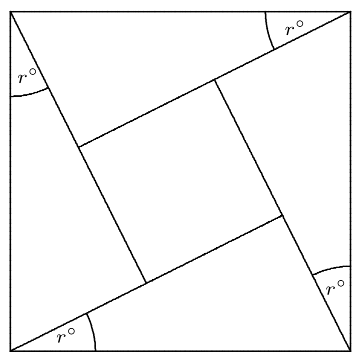


Determine the area of the unshaded portion of the octagon in the form $a + b\sqrt{2}$, where a and b are integers.

Write the value of $r = 5(a + b)$ in Box #3 of the Relay Answer Sheet.

R4 Given that the two figures that appear to be squares in the diagram below really are squares, and that the angles so marked are each r°, determine the value of:

$$\frac{\text{area of smaller square}}{\text{area of larger square}}$$



Write your answer in Box #4 of the Relay Answer Sheet.

Tiebreaker

The calendar will be said to *fall the same way* in two years if every date falls on the same day of the week for both of the years; that is, both January 1st days are the same day of the week, both January 2nd days are the same day of the week, and so on through to December 31st.

What was the first year of this century to fall the same way as 1997?

Notes: (i) This is not essential information but you might find it helpful to know that 1997 started on a Wednesday.

(ii) The corresponding problem for 1996 appears in Game #2 of 1996–97. However 1996 was a leap year and 1997 is not, and there are significant differences between the two cases.

1996–97 Game #5

1. All A–blocks are identical, all B–blocks are identical, all C–blocks are identical. Seven A–blocks weigh the same amount as four B–blocks, and two A–blocks the same amount as a B–block and a C–block. How many C–blocks weigh the same amount as an A–block?

2. Three students X, Y and Z took a test. The average score of X and Y was 81, that of Y and Z was 78 and that of X and Z was 89. Who had the highest mark and what was it?

3. Patricia's mother pays a hired help to do the weekly washing and ironing at a rate of 25 cents per hour. Formerly she used a 575 watt electric iron with which the help averaged 6 hours per week to do the ironing. She has now bought an electric iron that requires 1875 watts and the help completes the ironing in 2 hours. Assuming that the cost of electricity is 50 cents per kilowatt-hour, how many weeks would be required for the saving to pay for an electric iron that cost \$34?

4. The Smith family has two children and at least one of them is a girl. What is the exact probability that both are girls?

   Notes:

   (i) You should assume that boys and girls are equally likely in general.

   (ii) Remember that the probability of an event is a number p such that $0 \le p \le 1$, thus if $p = \dfrac{1}{4}$ there is a 25% likelihood that the event will occur.

5. Find integers a and b such that

$$(a + b\sqrt{3})^3 = 135 - 78\sqrt{3}\,.$$

6. Given that m and n are integers ≥ 4, determine the number of pairs (m, n) that satisfy the equation

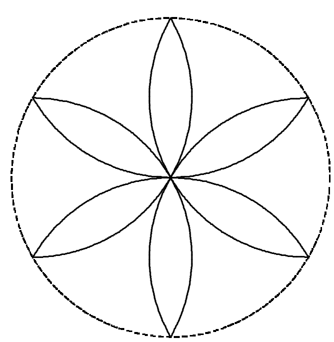
$$3m - mn + 3n > 0\,.$$

7. Given that $x^3 + px^2 + qx + r$ has roots a, b and c, determine the value of

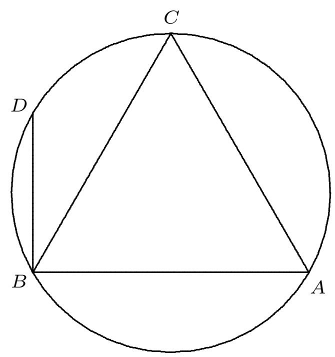
$$a^2b^2 + b^2c^2 + c^2a^2$$

   in terms of p, q and r.

8. The "flower" in the diagram below is constructed from arcs of circles of radius 1. Determine the area that is covered by the "petals".

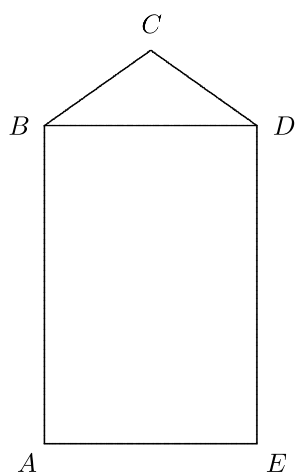


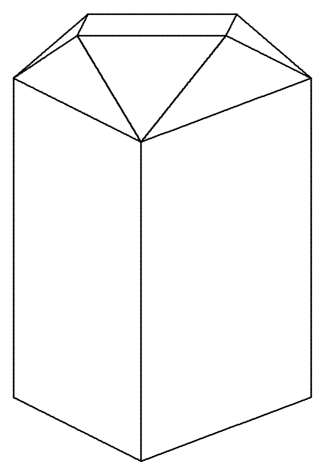
9. An equilateral triangle ABC is inscribed in a circle. The line through B, perpendicular to AB, meets the circle again at D.



Calculate the exact value of AB/CD.

10. A building with a square base has four flat vertical sides, each in the form of a pentagon. We show typical side and corner views.





The roof consists of four triangular pieces and a horizontal, flat top in the centre. Given that $AE = BD = 10$ metres, $AB = ED = 15$ metres, and that $\angle CBD = \angle BDC = 45°$, calculate the exact area of the roof in square metres.

Relay

R1 An Egyptian fraction is a method of representing a fraction such that it is a sum of positive fractions, all of whose numerators are 1. For example

$$\frac{3}{5} = \frac{1}{2} + \frac{1}{10} \quad \text{or} \quad \frac{11}{16} = \frac{1}{2} + \frac{1}{8} + \frac{1}{16}\,.$$

It is required that the denominators to the right of the = sign in such expressions should all be different and in increasing order of size.

The length of an Egyptian fraction is the number of terms, thus the length of $\frac{1}{2}+\frac{1}{8}+\frac{1}{16}$ is three.

Find the shortest possible ways of representing $\frac{2}{3}$ as Egyptian fractions that start with $\frac{1}{2}$ and start with $\frac{1}{3}$

Let A be the sum of all of the denominators to the right of the equals signs in these representations.

Write A in Box #1 of the Relay Answer Sheet.

R2 You are given that there are A people. One third of them are left handed and six have no hair. Two of the left handed people have no hair.

Let B be the number of people who are hairy and right handed.

Write B in Box #2 of the Relay Answer Sheet.

R3 A rectangular barn is 10 metres by 12 metres. A goat is attached to one corner on the outside of the barn by a rope B metres long.

The goat can graze in an area of $C\pi$ square metres.

Write C in Box #3 of the Relay Answer Sheet.

R4 Given number C, find **all** the prime factors of C, and then construct the polynomial with these numbers as its roots.

Let $3D$ be the coefficient of x^2 in this polynomial.

Write the integer D in Box #4 of the Relay Answer Sheet.

Note A repeated prime factor should be repeated amongst the roots of the polynomial. Thus if C were $45 = 3 \times 3 \times 5$ then the associated polynomial would be $(x-3)(x-3)(x-5)$.

Tiebreaker

A line through the point $(3, 2)$ intersects the x–axis at A and the y–axis at B. Given that O is the origin and that the area of triangle OAB is 13.5, determine **two different** possible values for the x–coordinate of the point A.

1994–95 Answers

Q.	Game #1	Game #2	Game #3	Game #4	Game #5
1.	$y=-x+9$	21 100	1	$\frac{-17}{144}$	8
2.	27	42	$(8,-6)$	$\angle ACB=12°$, $\angle CAB=84°$ $\angle ABC=84°$	$\frac{2\sqrt{3}-\pi}{2\sqrt{3}}$
3.	(c) $2^{\left(22^2\right)}$	\$2675	64	3	$3!^{\left(2^{3!}\right)} < 2^{\left(3!^3\right)} < 2^{\left(3^{3!}\right)} < 2^{\left(3^3\right)!}$
4.	784	15	2000π metres	1	33
5.	-4, -2 and 0	199	**V**	$2\pi+\frac{7}{5}$	4 and -8
6.	80	0 and 4	4	$5-i\sqrt{2}$ or $5+i\sqrt{2}$	10
7.	$a>l>p>g$	$22.5°$	$2+\sqrt{3}+\frac{2\pi}{3}$	Chris and Pat	8
8.	$\sqrt{6}-\sqrt{2}$	7	\$1100 for $BACD$	$x=0$ or $x=-\frac{20}{7}$	$c=0$ and $d=1$ or $c=$ any value and $d=-1$
9.	$x=4$ and $y=5$	$270°$	30	3	-2
10.	5	200 cm	-200	$(x,y)\in\{(0,9),(0,11),(1,10),(-1,10)\}$	9191
R1.	12	42	13	10	4
R2.	30	8	4	2	10
R3.	48	4	6	18	$R=8$ and $C=10$
R4.	0	76	4	111	19
TB.	7	$x=4$, $y=-2$ and $z=3$	$(x-a)$ $(x-2a)$ $(x-b)$	¢	15

1995–96 Answers

Q.	Game #1	Game #2	Game #3	Game #4	Game #5
1.	5 ¢	The large pizza	10	18	12
2.	61	45	17	24	RDEDRE or ERDEDR
3.	$15 000	87.5%	75%	122	40 km
4.	29.29 cm	10	$0 < k < 64$	$\sqrt{3} - 0.6$	27
5.	$3\sqrt{3}/2$	6%	1	51748632, 62415387, 73856421, 84637521, 84523176	$(8, 63)$, $(9, 80)$, and $(10, 99)$
6.	6 min 6 sec	110	12 mm	$p = 11$ and $q = -3$	30
7.	95%	4	$(3, 3)$, $(3, 4)$, $(3, 5)$, $(4, 3)$ and $(5, 3)$	40	$(4, 7)$
8.	1	126, 150 and 176	$-2/3$	1 hr 45 min	8
9.	28 563	8.5 cm	44	382	$5\sqrt{3}/2$
10.	65.9%	$\frac{(n-1)^3(n+1)}{(n^2+1)^2}$	$(2, 18, 18)$, $(3, 6, 9)$, $(2, 12, 36)$, $(3, 4, 36)$ and $(4, 4, 9)$	2178	$-\frac{2}{7}$ or $-\frac{4}{5}$
R1.	23	10	7	4	1
R2.	69	-1	6	28	101
R3.	22	12	2	8	5
R4.	42	2	12	5.12	$\frac{400\pi - 300\sqrt{3}}{3}$ cm^3
TB.	13	Subtract original number	4 and -12	$(-2, 5)$	$\frac{1}{2}$

1996–97 Answers

Q.	Game #1	Game #2	Game #3	Game #4	Game #5
1.	19	Check individual solutions	11	10	4
2.	29 cm × 21 cm	29	\$720 999	3	X had 92
3.	18	$\frac{594}{625}$	20	63%	40
4.	72	115	17	1	$\frac{1}{3}$
5.	$A = 2$, $B = 6$, and $C = 7$	1912	4 : 1	19	$a = 3$ and $b = -2$
6.	In order: Gander, Lab. City, Tie, Lab. City, North Bay, Tie.	5	2	46.875%	20
7.	$(3, 0)$ and $\left(\frac{3}{2}, 0\right)$	-3 and -4	204.755	$-2 + 5\sqrt{3}$	$q^2 - 2pr$
8.	360	12 sq. units	$\pi r/2$	$1 + \sqrt{3}$	$2\pi - 3\sqrt{3}$
9.	$\frac{5}{2}$ or $-\frac{5}{2}$	63	6	-3	$\sqrt{3}$
10.	Check word by word	42 geese-a-laying and 42 swans-a-swimming	$55\,000 - 30\,000\sqrt{2}$	$s \wedge t$	$50\left(1+\sqrt{3}\right)$ m^2
R1.	6	4	8	1	27
R2.	10	15	24	2	14
R3.	5	8	14	30	152
R4.	69	32	42	$\frac{2-\sqrt{3}}{2}$	42
TB.	6	$5 - 3\sqrt{2}$	$\frac{1}{7\,997\,214}$	1902	9 or $\frac{9}{2}$

ATOM

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