Canadian Junior Mathematical Olympiad 2022

A competition of the Canadian Mathematical Society.

Official Problem Set

P1. Let ABC be an acute angled triangle with circumcircle Γ. The perpendicular from A to BC intersects Γ at D, and the perpendicular from B to AC intersects Γ at E. Prove that if $|AB| = |DE|$, then $\angle ACB = 60°$.

P2. You have an infinite stack of T-shaped tetrominoes (composed of four squares of side length 1), and an $n \times n$ board. You are allowed to place some tetrominoes on the board, possibly rotated, as long as no two tetrominoes overlap and no tetrominoes extend off the board. For which values of n can you cover the entire board?
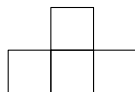

Figure 1: T-shaped tetromino

P3. Assume that real numbers a and b satisfy

$$ab + \sqrt{ab + 1} + \sqrt{a^2 + b} \cdot \sqrt{b^2 + a} = 0.$$

Find, with proof, the value of

$$a\sqrt{b^2 + a} + b\sqrt{a^2 + b}.$$

P4. Let $d(k)$ denote the number of positive integer divisors of k. For example, $d(6) = 4$ since 6 has 4 positive divisors, namely, 1, 2, 3, and 6. Prove that for all positive integers n,

$$d(1) + d(3) + d(5) + \cdots + d(2n - 1) \le d(2) + d(4) + d(6) + \cdots + d(2n).$$

P5. Let $n \geq 2$ be an integer. Initially, the number 1 is written n times on a board. Every minute, Vishal picks two numbers written on the board, say a and b, erases them, and writes either $a + b$ or $\min\{a^2, b^2\}$. After $n - 1$ minutes there is one number left on the board. Let the largest possible value for this final number be $f(n)$. Prove that

$$2^{n/3} \ < \ f(n) \ \leq \ 3^{n/3}.$$

Important!
Please do not discuss this problem set online until 24 hours after the CJMO finishes!