

2021 Samuel Beatty Report
62nd International Mathematical Olympiad
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The International Mathematical Olympiad is the most prestigious math contest in the world. This was the 62nd IMO and the second remote IMO after 2020; which also had the same host country, Russia. 619 students participated in the 2021 IMO representing 107 countries from around the globe. Each country can send six participants to the IMO. This year, team Canada had 3 returning members and 3 new participants. It was Thomas Guo's 5th, Zixiang Zhou's 3rd and Eric Shen's 2nd IMO! The 3 new members were Warren Bei, Kevin Min and me.

Most of the strong performing countries at the IMO arrange multiple team selection tests to determine their final six members of the IMO team. The Canadian IMO team is chosen based on the following Olympiads: Canadian Mathematical Olympiad, Asian Pacific Mathematical Olympiad and the USA Mathematical Olympiad. The Canadian Open Mathematics Challenge acts as a qualifier for these Olympiads.

The Canadian Mathematical Society organizes two training camps each year for IMO training, both these camps were online this year due to Covid-19. The winter camp is held in January and nearly 15 students who are the most likely to make the IMO team participate each year. The other one is the summer camp which happens just before the IMO to train the IMO team.

This year's summer camp was a two-week-long enriched program. Days at the summer camp consisted of problem-solving sessions and lectures given by Alex Song (Leader), Dani Spivak (Deputy Leader), Robert Garbary (Observer), and previous IMO team members Victor Rong, Edgar Wang and Bill Huang. These wonderful lectures covered various topics in number theory, algebra, geometry and combinatorics, including functional equations, inequalities, projective geometry, linear algebra, probabilistic method, induction, polynomials and diophantine equations! Besides the lectures, we also wrote 4.5 hour-long IMO mock tests every other day. The problems of these mock tests were chosen by our trainers to simulate the IMO conditions and prepare us better for the actual IMO.

Due to Covid-19, this year's IMO was also held online and hosted by Russia, just like 2020. The opening ceremony happened on youtube and had videos of students from participating countries informing themselves. It also had laser dance with a continuous music performance going on!

Since IMO was online, students wrote the IMO in testing centers close to their homes instead of travelling to St.Petersburg. Four of the Canadian team members, including me, Thomas Guo, Eric Shen, Zixiang Zhou and one member of the Finnish IMO team, Tianyue

Sun, wrote the IMO at the Waterloo testing Center. We were proctored remotely by Russian proctors. Jen Nelson also acted as the IMO commissioner, who had to keep in touch with the Russian proctors during the exam. Kevin Min wrote the IMO in the US testing Center and Warren Bei took the IMO at his home in Vancouver and was remotely proctored by Victor Rong.

Since I was in Vancouver before the IMO, the CMS arranged my trip to Waterloo. Everything was very well planned by Sarah Watson and her colleagues and I was warmly welcomed in Waterloo by Jen Nelson from the CEMC! I arrived in Waterloo on the 15th of July and had 4 days before the IMO to explore the campus. The rooms in the residence were quite comfortable; we had our kitchen, showers and even a nice blackboard in the hallway! The rest of the team arrived at Waterloo on July 18th and were welcomed by Robert Garbary. This was the first time we met each other in person this year. After a delicious dinner at night, we all went to bed to mentally prepare for taking the IMO the next day!

And finally, on the morning of the 19th of July, the team members simultaneously started the first day of IMO 2021 at 8 am EDT, while Warren and Kevin had a harder job starting it at 5 am in their time zone (PDT)!

The first problem was combined in number theory and combinatorics. It asked to show we can pick two numbers from at least one of the sets made by breaking an initial set of integers into two sets such that their sum equals a perfect square. The main idea was to find 3 numbers such that the sum of any two of them is a perfect square. Though the main idea was not hard to find, the problem had many small details which made writing it up harder than usual. The rubric on this specific problem was also very hard, mentioning that points shall be deducted even for very small mathematical errors. This ended in Zixiang's one-point loss over problem 1, even though his small error was insignificant and could be fixed immediately. The rest of the team members received 7's for this problem. I was also very close to losing 1 point for this problem because of my math error, but I had fortunately done everything correctly in my scrap work; thanks to Alex Song and Dani Spivak during the coordination process, my score was saved for this problem!

The second problem was a hard inequality which contained the square roots of absolute values of some numbers. This scary-looking problem had a beautiful solution using Jensen's inequality which only Thomas came up with and earned him 7 points! We, later on, saw different solutions, some requiring undergrad-level math knowledge, but the fact that there was an elegant solution made it suitable for IMO. Other than Thomas, Kevin also received 1 point, but the rest of the Canadian team didn't get any points for this problem. Only 16 contestants received 7 points on it during the IMO. In comparison, the second problem of IMO 2020, which was also an inequality problem, had 138 solves, which clearly shows how hard this problem was, especially for its place as the second problem. Interestingly, the proposer of this problem is Calvin Deng, who was a member of the 2012 and 2013 Canadian IMO teams!

The third problem was another hard one, this time, geometry. It asked to prove 3 points are collinear and involved the circumcenters of some triangles. No one in the team managed to solve this problem during the contest. Eric had tried multiple approaches, one

of which involved an inversion and was in the same path of a certain solution in the rubric, so he got 2 points from this problem. Zixiang and I also received 1 point for proving some general statements in the problem. Only 15 participants received 7's for this problem (again, comparing with the third problem of IMO 2020 which had 42 full solves shows how hard it was).

And that was the end of a hard day 1. Other than Thomas, no one had fully solved two problems which initially made us stressed about our performance; but as we got news from other countries, we figured out this was one of the hardest days in the IMO's history, with problem 2 being especially hard, and the stats confirmed it later on!

It was lunchtime in Waterloo, so we had our lunch as we discussed the problems and hoped for a better day 2.

We also went out exploring the University of Waterloo's campus with Dani. Since he had previously studied at Waterloo, he was a great guide!

The second day had a better start, the first problem was an easy geometry problem which everyone got 7's on.

The second problem was a combinatorics problem asking to prove something while a given process is happening; the process involved a squirrel moving walnuts around a tree! One of the solutions was colouring the walnuts as the squirrel moves them, which was the solution for most of us. Interestingly most of us also used the colours white and red in our solution write-ups! The team did very well on this problem with everyone getting a 7. Indeed only Canada, China, Poland and Italy got full marks on this problem!

The last problem was a combination of Number Theory and Combinatorics. It asked for a lower bound on the number of elements of a set of integers, given that certain numbers are constructible as the sum of some elements of that set. Eric and Zixiang found beautiful solutions using linear algebra, and hence got a full mark on day 2!

So finally the second day of IMO was also over! We had a surprise guest though, Ian Vanderburgh (director of CEMC) met us at the testing site and gave us a short inspiring speech!

After having lunch, we went to take a couple of photos and have some fun after these two stressful days. Meanwhile, our leader and deputy leader were busy doing the very important process of coordination, which was different this year since everything was online.

We finally left the residence and had bubble tea in the UW Plaza. We played several board games and card games till late at night and even went out to get a pizza at 3 am! It was an unforgettable experience.

Finally, we all left Waterloo on the 21st of July and went back home, though IMO was not over yet. Back home, there were exciting various online events and lectures organized by the IMO host we could still attend.

As the results of coordination for each problem came one by one, we understood how well we did; but no one could even guess the final results. Finally, the results were uploaded to the IMO website on July 23rd; this was a very shocking moment for all of us.

The Canadian team made history! We were ranked fifth among 107 participating countries! This is Canada's best result ever (with a tie in 2012)! This time we also broke another record:

This was the first time all members of the team earned either silver or gold!

But this was not the last record broken by the Canadian team members; Warren Bei became the Youngest medalist in IMO 2021, he is also the 4th youngest silver medalist in IMO's history, earning his silver medal at the age of 13!

The award ceremony took place on July 24th and consisted of medal winners' announcements and several speeches.

We are all very happy and proud of the final results, which was not possible without the great efforts of our leader and deputy leader, Alex Song and Dani Spivak, along with our fantastic trainers during the camp!

Since three of our team members, Zixiang Zhou, Kevin Min and Warren Bei are still eligible for IMO 2022, we hope they perform even better for the next IMO!

Participating in the IMO has always been a great dream for me and I'm very glad that I had the chance to participate in such a wonderful event, which will leave us all an amazing experience. None of these were possible, especially in such hard conditions, without the help of CMS and very generous support from their sponsors including the University of Waterloo and the Samuel Beatty fund.