2020 Canadian Mathematical
Gray Jay Competition

Official Solutions

A competition of the Canadian Mathematical Society and supported by the Actuarial Profession.

Part A – 4 marks each

1. Calculate

$$20 - 19 + 18 - 17 + \cdots - 3 + 2 - 1$$

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12

Solution: Pairing the terms we get

$$(20 - 19) + (18 - 17) + \cdots + (4 - 3) + (2 - 1) = 1 + 1 + \cdots + 1 = 10$$

Answer: **(B)**

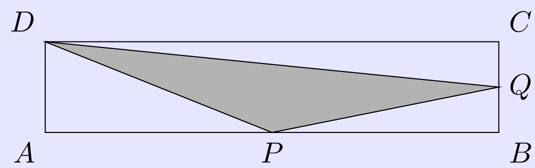
2. Jack drove his sister Jill from home to the mall and then he drove back home. After spending 3 hours at the mall, Jill called her brother at home asking him to pick her up. It took Jack 30 minutes to drive from home to the mall and 45 minutes to drive back home from the mall. How long did Jack spend at home waiting for his sister's call?

(A) 1 hour and 45 minutes.
(B) 2 hours and 15 minutes.
(C) 3 hours.
(D) 3 hours and 30 minutes.

Solution: He had to wait 3 hours, less the 45 minutes it took him to drive home, so 2 hours and 15 minutes.

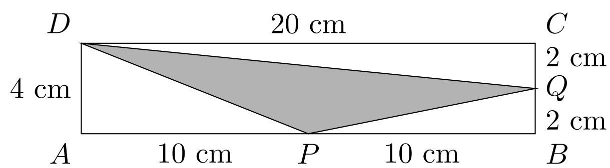
Answer: **(B)**

3. In rectangle $ABCD$, AB is 20 cm long and BC is 4 cm long. P is the midpoint of AB and Q is the midpoint of BC. What is the area of triangle DPQ?



(A) 20 cm^2 **(B)** 30 cm^2 **(C)** 40 cm^2 **(D)** 50 cm^2

Solution: Labelling the indicated lengths on the diagram we get:



The area of the rectangle is 4 cm $\times$ 20 cm $=$ 80 cm^2 and the areas of the triangles are

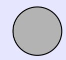
$$\text{Area of triangle } DAP = \frac{1}{2} \times 4 \text{ cm} \times 10 \text{ cm} = 20 \text{ cm}^2$$
$$\text{Area of triangle } QBP = \frac{1}{2} \times 2 \text{ cm} \times 10 \text{ cm} = 10 \text{ cm}^2$$
$$\text{Area of triangle } QCD = \frac{1}{2} \times 2 \text{ cm} \times 20 \text{ cm} = 20 \text{ cm}^2$$

Thus the area of triangle DPQ is 80 cm^2 $-$ (20 cm^2 $+$ 10 cm^2 $+$ 20 cm^2) $=$ 30 cm^2.

Answer: **(B)**

4. A teacher puts a square table next to a shaded circle drawn on the floor. She then divides the table surface into nine squares and numbers the squares as follows:

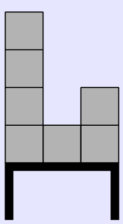
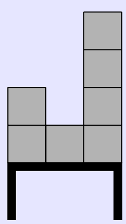
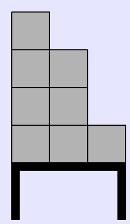
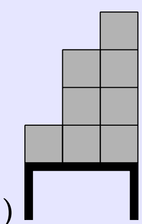
1	2	3
4	5	6
7	8	9



She then stacks equal-sized cubes on top of the numbered squares on the table as follows:

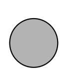
- Two cubes on square number 8.
- Three cubes on square number 2.
- One cube on square number 6.
- Four cubes on square number 1.

The teacher then asks Sam, a student, to stand on the shaded circle facing the cubes and asks him to draw what he sees. Which one of the following diagrams represents what Sam draws?

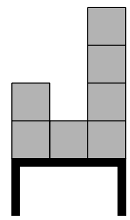
(A)  **(B)**  **(C)**  **(D)** 

Solution: Looking from Sam's point of view we see

7(0)	4(0)	1(4)
8(2)	5(0)	2(3)
9(0)	6(1)	3(0)



where the numbers in the brackets are the number of blocks on each square. Sam would see the top block on square 1 over the pile on square 2, so he would see



Answer: (B)

5. Martha got a piggy bank and started filling it with coins on January 1st. Every day, she puts as many dimes as the remainder of the division of the day of the month by 3. Also, on January 4 and every 4 days after she puts in 4 quarters. What is the probability that if she draws a coin on January 31st, after depositing the coins for the day, she will get a dime?

(A) $\frac{31}{59}$ **(B)** $\frac{1}{31}$ **(C)** $\frac{1}{28}$ **(D)** $\frac{61}{89}$

Solution: On the 1st she puts in 1 dime, the 2nd 2 dimes, the third none, and then the process repeats. At the end of the third day she will have put in 3 dimes. As such, on each day that is a multiple of 3 she will have exactly as many dimes as days. So on the 30th, she will have 30 dimes. On the 31st she will put in 1 more, so there will be 31 dimes in the piggy bank.

For quarters, $31 \div 4 = 7.75$, so she will deposit 4 quarters 7 times for a total of 28 quarters.

Thus she will have $31 + 28 = 59$ coins in the piggy bank, and the probability of drawing a dime will be $\frac{31}{59}$

Answer: (A)

Part B – 5 marks each

6. Alice and Bill play a game. They go to separate rooms, flip a coin and try to predict what the other person flipped. They win if at least one of them predicts correctly. They decide that Alice will always guess the same thing that she flips and Bill will always predict the opposite of what he flips. What percentage of the time should they win?

(A) 0% **(B)** 25% **(C)** 50% **(D)** 75% **(E)** 100%

Solution: Looking at each case we get the following:

Alice's flip	**Bill's flip**	**Alice's guess**	**Alice correct?**	**Bill's guess**	**Bill correct?**	**Win?**
heads	heads	heads	yes	tails	no	yes
heads	tails	heads	no	heads	yes	yes
tails	heads	tails	no	tails	yes	yes
tails	tails	tails	yes	heads	no	yes

So they win every time.

Another way to look at the problem is if Alice and Bill flip the same thing, Alice will guess correctly and if they flip different things, Bill will guess correctly. So one of them will always guess correctly.

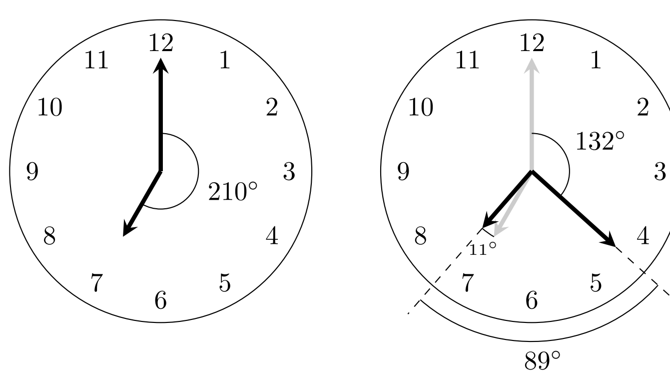
Answer: **(E)**

7. What is the smaller angle between the two hands of a clock when it shows exactly 7:22?

(A) 72° **(B)** 78° **(C)** 89° **(D)** 91° **(D)** 102°

Solution: At 7 : 00 the large angle between the hands is $\frac{7}{12} \times 360° = 210°$. Each hour the minute hand moves 360° and the hour hand moves $\frac{1}{12} \times 360° = 30°$. Thus in 22 minutes the minute hand has moved $\frac{22}{60} \times 360° = 132°$ and the hour hand has moved $\frac{22}{60} \times 30 = 11°$. Thus the new angle between the hands will be

$$210° - 132° + 11° = 89°.$$



Alternate solution: The hour hand travels $(7 + \frac{22}{60})(360°)/12 = 221°$ and the minute hand travels $(\frac{22}{60})(360°) = 132°$, so the angle between them is $221° - 132° = 89°$.

Answer: **(C)**

8. Anna writes down all numbers from 1 to 100. How many times did she write the digit 2?

(A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

Solution: The digit 2 appears in the units position 10 times $(2, 12, 22, \ldots, 92)$,and it appears in the tens digit 10 times $(20, 21, 22, \ldots, 29)$. Hence, the digit 2 appears a total of 20 times.

Answer: **(C)**

9. Adam is a cake supplier. He got an order to supply 26 large cakes and 800 cupcakes for a birthday party. However, he faced certain challenges. The kitchen was available only for 3 hours. Each chef could make either 2 large cakes or 35 cupcakes every hour. During any hour, chefs may work on cakes or cupcakes and then switch for later hour(s), if needed. Adam hired the minimum number of chefs needed to prepare the order. How many chefs did he hire?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 35 **(E)** 36

Solution: Since a chef can make 2 large cakes per hour, 26 cakes will require $26 \div 2 = 13$ hours of work. Similarly it will require $800 \div 35 \approx 22.86$ hours of work to make the cupcakes and so about 35.86 hours of work will be needed to prepare the order. Since each chef works for a 3 hour shift and $35.86 \div 3 \approx 11.95$, Adam will need to hire 12 chefs.

Answer: **(B)**

10. A bag contains red, blue, green and orange marbles. The red marbles represent $\frac{1}{3}$ of the total number of marbles, the blue marbles represent $\frac{1}{5}$ of the total number of marbles and the green marbles represent $\frac{2}{7}$ of the total number of marbles. If the bag contains as few marbles as possible, how many orange marbles are there?

(A) 11 **(B)** 13 **(C)** 19 **(D)** 34 **(E)** 35

Solution: The red, blue and green marbles account for

$$\frac{1}{3}+\frac{1}{5}+\frac{2}{7}=\frac{35}{105}+\frac{21}{105}+\frac{30}{105}=\frac{86}{105}$$

which is in lowest terms. Hence, the orange marbles must account for $1-\frac{86}{105}=\frac{19}{105}$ of the total marbles. Since we cannot have a part of a marble this means that the total number of marbles must be a multiple of 105. Hence the smallest amount of marbles we could have is 105, which means that 19 of them are orange.

Answer: **(C)**

Part C – 7 marks each

11. Alice types the fraction $\frac{30}{37}$ into an online calculator and it calculates the decimal form to thousands of decimal places. What is the sum of the first 2020 digits after the decimal?

(A) 6060 **(B)** 6061 **(C)** 6062 **(D)** 6063 **(E)** 6064 **(F)** 6065

Dividing, to convert to a decimal, we get $\frac{30}{37} = 0.810\ 810\ 810\ \cdots = 0.\overline{810}$ which is periodic with period 3. Dividing 2020 by 3 we get 673 with 1 remainder. This means 810 appeared 673 times and the 2020th digit would be 8, the start of the next period. Thus, the sum of the first 2020 digits after the decimal is

$$673 \times (8 + 1 + 0) + 8 = 6065$$

Answer: **(F)**

12. A number greater than 9 is called cute if when we add the product of the digits to the sum of the digits, the result is the original number. For example 29 is cute since $2 + 9 + 2 \times 9 = 29$, but 513 isn't cute since $5 + 1 + 3 + 5 \times 1 \times 3 \neq 513$. How many cute numbers are there?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 90 **(E)** 91 **(F)** Infinitely many.

Solution: We will look at this by cases:

Case 1: two-digit numbers
Consider the two digit number AB. The number is cute if $A \times B + A + B = 10 \times A + B$. Rearranging this equation we get $A \times B = 9 \times A$. This means that $B = 9$ and the numbers $19, 29, 39, \ldots, 99$ are all cute. So there are 9 cute two-digit numbers.

Case 2: three-digit numbers
Consider the three digit number ABC. The number is cute if
$A \times B \times C + A + B + C = 100 \times A + 10 \times B + C$. Rearranging this equation we get
$A \times B \times C = 99 \times A + 9 \times B$. Note that $B, C \leq 9$, so $B \times C \leq 81$. Hence
$A \times B \times C \leq 81 \times A < 99 \times A + 9B$. Therefore there are **no** cute three-digit numbers.

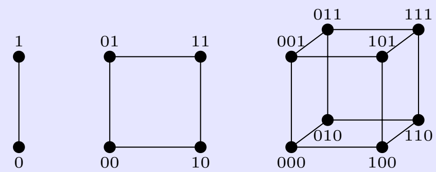
The argument can be extended to any number of digits greater than 2. If we have an n-digit number $AB \cdots N$, then it is cute if $A \times B \times \cdots \times N = 10^{n-1} \times A + \ldots$. After rearranging we get

$$A \times (B \times \cdots \times N) \leq A \times 9^{n-1} < (10^{n-1} - 1)A + \ldots$$

so there are no cute numbers with more than two digits. So there are only 9 cute numbers.

Answer: **(A)**

13. Gray codes are ways to assign a binary list of numbers to corners of a line segment, square or cube as shown. In every case, corners are joined if their Gray codes differ by exactly 1 digit.



Imagine a hypercube, the 4-dimensional version of a cube, with 16 corners. A Gray code is used to label its corners. Alien Alice stands on corner 1010 of the hypercube, Alien Bob stands on corner 1011 and Alien Charlie stands on 0010. If each alien can only move to a corner joined to theirs or remain at their starting corner, is it possible for them to all meet at the same corner?

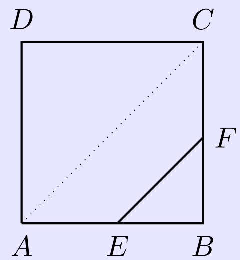
(A) All 3 can meet after each moves to a nearby corner.
(B) They can only meet if Alice doesn't move and the other 2 join her.
(C) They can only meet if Bob doesn't move and the other 2 join him.
(D) They can only meet if Charlie doesn't move and the other 2 join him.
(E) They cannot meet, they are too far apart.
(F) More than one of (B),(C),(D) is true.

Solution: By analogy, moving to a corner is switching 1 digit in the corner's gray code. There is no list abcd which has exactly 1 digit different with each of 1010, 1011, and 0010, so A is false. Likewise, Charlie can't make it from 0010 to 1011 in 1 move, so C is false: and likewise by symmetry D is false, since Bob can't make it to Charlie.

But it's possible for Alice to stand at 1010, and both aliens to come to her, since each has a 1-digit difference with her corner.

Answer: **(B)**

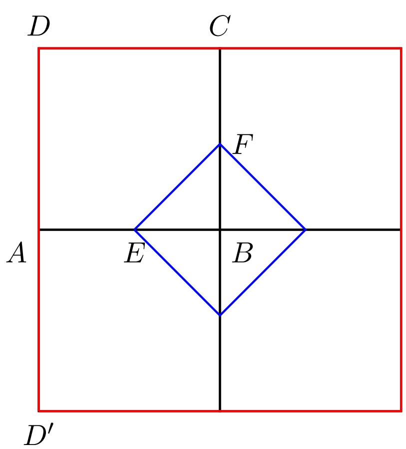
14. In the picture below, $ABCD$ is a square and EF is parallel to AC.



If the areas of $AEFCD$ and $ABCD$ are in a ratio of $8 : 9$, the ratio $EF : AB$ is

(A) $1 : 2$ **(B)** $1 : 3$ **(C)** $1 : 6$ **(D)** $2 : 3$ **(E)** $1 : 1$ **(F)** $3 : 4$

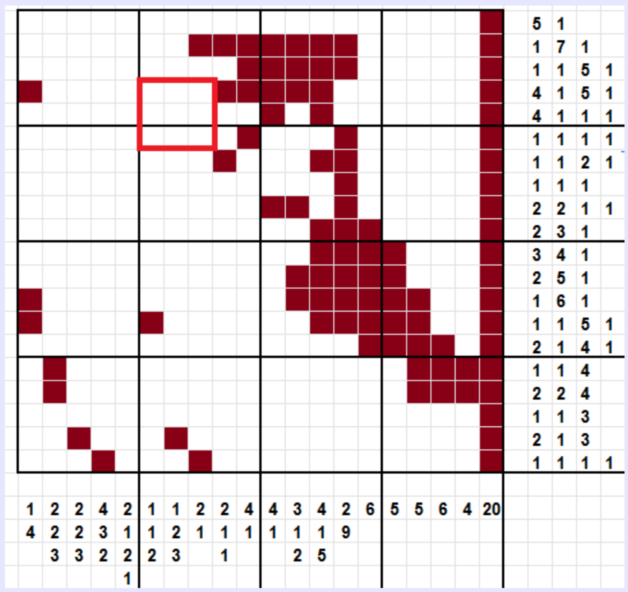
Solution: By symmetry



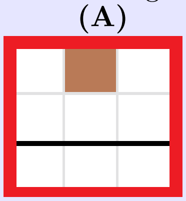
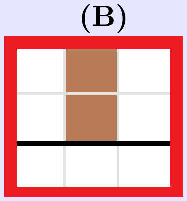
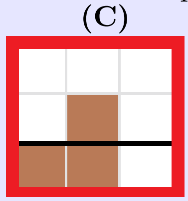
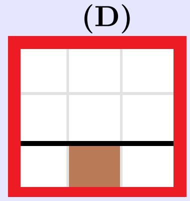
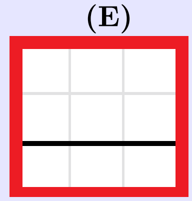
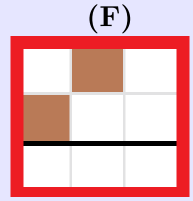
the blue and red squares have areas in ratio $1 : 9$. Therefore, the ratio $EF : DD'$ of their edges is $1 : 3$. This means that for AB, which is half of DD', $EF : AB = EF : \frac{1}{2}DD' = 1 : \frac{3}{2}$. So $EF : AB = 2 : 3$.

Answer: **(D)**

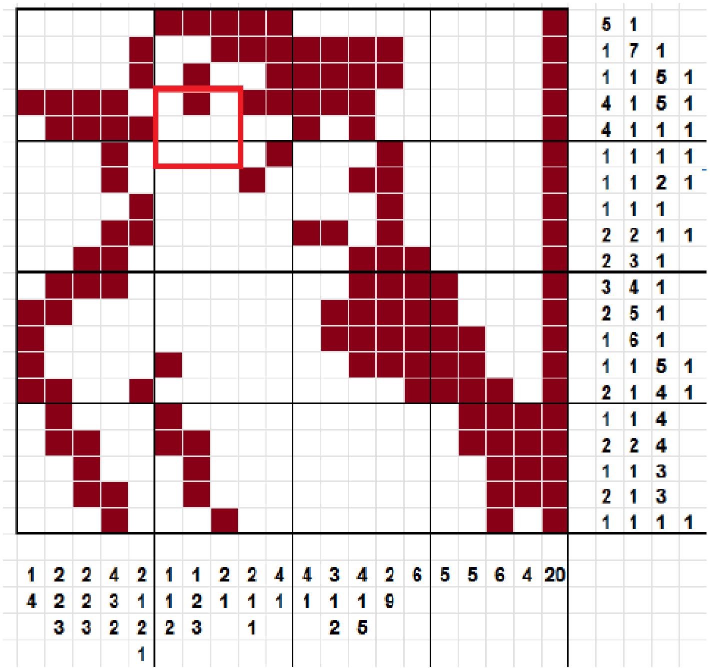
15. A partially completed nonogram is shown in the picture below. Numbers along the sides describe blocks of shaded squares in that row/column; the numbers themselves show how many touching shaded squares are in each block; at least one blank square must be between blocks in each row/column.



What configuration is in the indicated 3x3 square?

(A)

(B)

(C)

(D)

(E)

(F)


Solution: The completed diagram would be



Answer: **(A)**

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