2020 Gray Jay Practice Problems – SOLUTIONS

1. (Part A, 4 marks) Alice, Bobby, Cindy, and David all work at the same location. They will each drive from their work to the same conference. Below is a graph of their average speeds for the trip.

If they all left at the same time, who arrives at the conference first?

(A) Alice  (B) Bobby  (C) Cindy  (D) David

Answer: (C)

Solution: From the graph we see that Cindy is going the fastest, hence it will take her the shortest amount of time to arrive.

2. (Part B, 5 marks) Bob is a 6th grade student. His marks in his subjects at the end of the first and second semester of the academic year 2018/2019 were as follows:

<table>
<thead>
<tr>
<th>Subject</th>
<th>Art</th>
<th>English</th>
<th>Math</th>
<th>Science</th>
</tr>
</thead>
<tbody>
<tr>
<td>First semester</td>
<td>79</td>
<td>75</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Second semester</td>
<td>74</td>
<td>70</td>
<td>78</td>
<td>78</td>
</tr>
</tbody>
</table>

His average mark at the end of the second semester was 5 marks less than his average mark at the end of the first semester, and, at the end of the first semester, his mark in mathematics was 6 marks higher than his mark in science.

What was the mark achieved by Bob in science at the end of the first semester?

(A) 60  (B) 70  (C) 76  (D) 80  (E) 83

Answer: (D)

Solution:
The average mark for the second semester is $(74 + 70 + 78 + 78) ÷ 4 = 75$.
Therefore, average mark for the first semester is $75 + 5 = 80$.
The total marks of the first semester is then $80 \times 4 = 320$, which means total marks in science and math is $320 – (79 + 75) = 166$.
Since the math mark was 6 more than the science mark, then $166 – 6 = 160$ is twice the science mark.
Therefore, science mark is $160 ÷ 2 = 80$.

3. (Part C, 7 marks) Find the smallest $n$ for which the numbers $1^2, 2^2, 3^2, 4^2, \ldots, n^2$ can be split into two groups with the same sum.

(A) 4  (B) 5  (C) 6  (D) 7  (E) 8  (F) 9

Answer: (D)

Solution: Note that if $S$ is the common sum, then $1^2 + 2^2 + \ldots + n^2 = 2S$ must be even so that it can be broken into two equal sums. Working through cases we get:

Case 1: $n = 1$, which is clearly impossible.
Case 2: $n = 2$, $1^2 + 2^2 = 5$, which isn’t even, so this case is impossible.
Case 3: $n = 3$, note $3^2 = 9 > 2^2 + 1^2$, so this case is impossible.
Case 4: $n = 4$, note $4^2 = 16 > 3^2 + 2^2 + 1^2$, so this case is impossible.
Case 5: $n = 5$, $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$, which isn’t even, so this case is impossible.
Case 6: $n = 6$, $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$, which isn’t even, so this case is impossible.

The smallest potential number which could work is 7, which works:

$$1^2 + 2^2 + 4^2 + 7^2 = 1 + 4 + 16 + 49 = 70$$
$$3^2 + 5^2 + 6^2 = 9 + 25 + 36 = 70$$