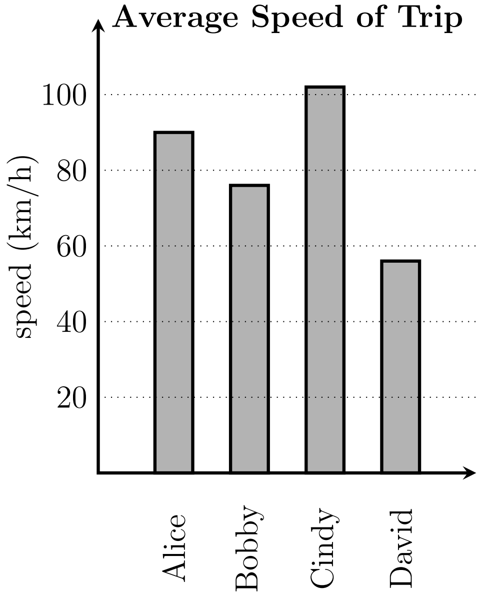
2020 Gray Jay Practice Problems – SOLUTIONS

1. (Part A, 4 marks) Alice, Bobby, Cindy, and David all work at the same location. They will each drive from their work to the same conference. Below is a graph of their average speeds for the trip.



If they all left at the same time, who arrives at the conference first?

(A) Alice **(B)** Bobby **(C)** Cindy **(D)** David

Answer: (C)

Solution: From the graph we see that Cindy is going the fastest, hence it will take her the shortest amount of time to arrive.

2. (Part B, 5 marks) Bob is a 6^{th} grade student. His marks in his subjects at the end of the first and second semester of the academic year 2018/2019 were as follows:

Subject	Art	English	Math	Science
First semester	79	75	...	...
Second semester	74	70	78	78

His average mark at the end of the second semester was 5 marks less than his average mark at the end of the first semester, and, at the end of the first semester, his mark in mathematics was 6 marks higher than his mark in science.

What was the mark achieved by Bob in science at the end of the first semester?

(A) 60 **(B)** 70 **(C)** 76 **(D)** 80 **(E)** 83

Answer: (D)

Solution:

The average mark for the second semester is $(74 + 70 + 78 + 78) \div 4 = 75$.
Therefore, average mark for the first semester is $75 + 5 = 80$.
The total marks of the first semester is then $80 \times 4 = 320$, which means total marks in science and math is $320 - (79 + 75) = 166$.
Since the math mark was 6 more than the science mark, then $166 - 6 = 160$ is twice the science mark.
Therefore, science mark is $160 \div 2 = 80$.

3. (Part C, 7 marks) Find the smallest n for which the numbers $1^2, 2^2, 3^2, 4^2, \ldots, n^2$ can be split into two groups with the same sum.

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8 **(F)** 9

Answer: (D)

Solution: Note that if S is the common sum, then $1^2 + 2^2 + ... + n^2 = 2S$ must be even so that it can be broken into two equal sums. Working through cases we get:

Case 1: $n = 1$, which is clearly impossible.

Case 2: $n = 2$, $1^2 + 2^2 = 5$, which isn't even, so this case is impossible.

Case 3: $n = 3$, note $3^2 = 9 > 2^2 + 1^2$, so this case is impossible.

Case 4: $n = 4$, note $4^2 = 16 > 3^2 + 2^2 + 1^2$, so this case is impossible.

Case 5: $n = 5$, $1^2+2^2+3^2+4^2+5^2 = 55$, which isn't even, so this case is impossible.

Case 6: $n = 6$, $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$, which isn't even, so this case is impossible.

The smallest potential number which could work is 7, which works:

$$1^2 + 2^2 + 4^2 + 7^2 = 1 + 4 + 16 + 49 = 70$$
$$3^2 + 5^2 + 6^2 = 9 + 25 + 36 = 70$$