Canadian Mathematical Olympiad
1978

PROBLEM 1

Let n be an integer. If the tens digit of n^2 is 7, what is the units digit of n^2?

PROBLEM 2

Find all pairs a, b of positive integers satisfying the equation $2a^2 = 3b^3$.

PROBLEM 3

Determine the largest real number z such that

$$\begin{aligned} x + y + z &= 5 \\ xy + yz + xz &= 3 \end{aligned}$$

and x, y are also real.

PROBLEM 4

The sides AD and BC of a convex quadrilateral $ABCD$ are extended to meet at E. Let H and G be the midpoints of BD and AC, respectively. Find the ratio of the area of the triangle EHG to that of the quadrilateral $ABCD$.

PROBLEM 5

Eve and Odette play a game on a 3×3 checkerboard, with black checkers and white checkers. The rules are as follows:

I. They play alternately.
II. A turn consists of placing one checker on an unoccupied square of the board.
III. In her turn, a player may select either a white checker or a black checker and need not always use the same colour.
IV. When the board is full, Eve obtains one point for every row, column or diagonal that has an even number of black checkers, and Odette obtains one point for every row, column or diagonal that has an odd number of black checkers.
V. The player obtaining at least five of the eight points WINS.

(a) Is a 4–4 tie possible? Explain.
(b) Describe a winning strategy for the girl who is first to play.

PROBLEM 6

Sketch the graph of $x^3 + xy + y^3 = 3$.