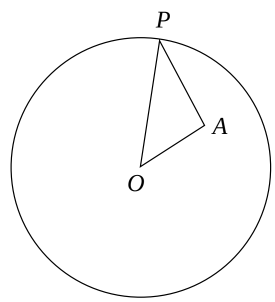
Canadian Mathematical Olympiad
1977

PROBLEM 1

If $f(x) = x^2 + x$, prove that the equation $4f(a) = f(b)$ has no solutions in positive integers a and b.

PROBLEM 2



Let O be the centre of a circle and A a fixed interior point of the circle different from O. Determine all points P on the circumference of the circle such that the angle OPA is a maximum.

PROBLEM 3

N is an integer whose representation in base b is 777. Find the smallest positive integer b for which N is the fourth power of an integer.

PROBLEM 4

Let

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

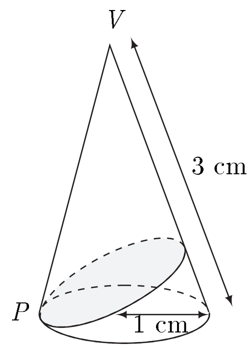
and

$$q(x) = b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0$$

be two polynomials with integer coefficients. Suppose that all the coefficients of the product $p(x) \cdot q(x)$ are even but not all of them are divisible by 4. Show that one of $p(x)$ and $q(x)$ has all even coefficients and the other has at least one odd coefficient.

PROBLEM 5

A right circular cone of base radius 1 cm and slant height 3 cm is given. P is a point on the circumference of the base and the shortest path from P around the cone and back to P is drawn (see diagram). What is the minimum distance from the vertex V to this path?



PROBLEM 6

Let $0 < u < 1$ and define

$$u_1 = 1 + u, \quad u_2 = \frac{1}{u_1} + u, \quad \ldots, \quad u_{n+1} = \frac{1}{u_n} + u, \quad n \geq 1.$$

Show that $u_n > 1$ for all values of $n = 1, 2, 3, \ldots$.

PROBLEM 7

A rectangular city is exactly m blocks long and n blocks wide (see diagram). A woman lives in the southwest corner of the city and works in the northeast corner. She walks to work each day but, on any given trip, she makes sure that her path does not include any intersection twice. Show that the number $f(m, n)$ of different paths she can take to work satisfies $f(m, n) \leq 2^{mn}$.