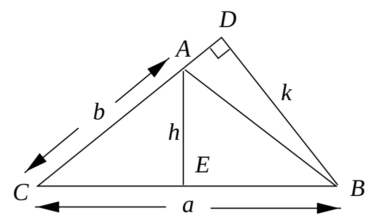
# Canadian Mathematical Olympiad
# 1970

PROBLEM 1

Find all number triples $(x, y, z)$ such that when any one of these numbers is added to the product of the other two, the result is 2.

PROBLEM 2

Given a triangle $ABC$ with angle $A$ obtuse and with altitudes of length $h$ and $k$ as shown in the diagram, prove that $a + h \geq b + k$. Find under what conditions $a + h = b + k$.




PROBLEM 3

A set of balls is given. Each ball is coloured red or blue, and there is at least one of each colour. Each ball weighs either 1 pound or 2 pounds, and there is at least one of each weight. Prove that there are 2 balls having different weights and different colours.

PROBLEM 4

a) Find all positive integers with initial digit 6 such that the integer formed by deleting this 6 is $1/25$ of the original integer.
b) Show that there is no integer such that deletion of the first digit produces a result which is $1/35$ of the original integer.

PROBLEM 5

A quadrilateral has one vertex on each side of a square of side-length 1. Show that the lengths $a$, $b$, $c$ and $d$ of the sides of the quadrilateral satisfy the inequalities

$$2 \leq a^2 + b^2 + c^2 + d^2 \leq 4.$$

PROBLEM 6

Given three non-collinear points $A$, $B$, $C$, construct a circle with centre $C$ such that the tangents from $A$ and $B$ to the circle are parallel.

PROBLEM 7

Show that from any five integers, not necessarily distinct, one can always choose three of these integers whose sum is divisible by 3.

PROBLEM 8

Consider all line segments of length 4 with one end-point on the line $y = x$ and the other end-point on the line $y = 2x$. Find the equation of the locus of the midpoints of these line segments.

### PROBLEM 9

Let $f(n)$ be the sum of the first $n$ terms of the sequence

$$0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, \dots .$$

a) Give a formula for $f(n)$.
b) Prove that $f(s+t) - f(s-t) = st$ where $s$ and $t$ are positive integers and $s > t$.

### PROBLEM 10

Given the polynomial

$$f(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a_n$$

with integral coefficients $a_1, a_2, \dots, a_n$, and given also that there exist four distinct integers $a$, $b$, $c$ and $d$ such that

$$f(a) = f(b) = f(c) = f(d) = 5,$$

show that there is no integer $k$ such that $f(k) = 8$.