

2018 Samuel Beatty Report

59th International Mathematical Olympiad in Cluj, Romania

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The International Math Olympiad, (IMO) is the most prestigious high school mathematics competition in the world. Each year, more than 100 countries across the globe send teams of six representatives to compete at the IMO. This year, the six members of Math Team Canada were Nicholas Sun, Thomas Guo, Victor Rong, William Zhao, Zixiang Zhou, and Howard Halim. The team is selected in May based on their performances on the Canadian Mathematical Olympiad (CMO), the Asian Pacific Math Olympiad (APMO), and the United States of America Math Olympiad (USAMO). Before writing these contests, student qualify by writing the Canadian Open Mathematics Challenge in November each year. IMO 2018 was held in Romania, the country where the contest originated in 1959.

Before going to Romania, the team attended a two-week training camp held at the Banff International Research Station (BIRS). To prepare for the contest, we listened to lectures given by 4 amazing trainers: James Rickards, Calvin Deng, Vince Chan, and Mike Pauliuk. James, Calvin, and Vince later accompanied us to Romania as our leader, deputy leader, and observer, respectively. These lectures covered a wide variety of subjects, such as number theoretic functional equations, inequalities, Euclidean geometry, and graph theory. The training

also included 6 mock Olympiads, meant to simulate the IMO exam.

In our free time, we played card games, basketball, Frisbee, a word game called *Contact*, and finished problem sets from the lectures. We also watched a movie, embarked on a nature scavenger hunt called *Geocaching*, and celebrated Canada Day by going to the town of Banff to see fireworks. It was certainly an eventful two weeks!



The accommodations at Banff were great. The rooms were very spacious and comfortable, and everyone enjoyed the delicious buffet. The scenery was nice, especially when we hiked up the nearby Tunnel Mountain.



On July 5th-6th, we travelled to Cluj, Romania. James had already arrived there a few days in advance, along with the leaders of the other countries, to select the problems that would appear on the contest.

The IMO opening ceremony was held the day before the contest. We listened to speeches from distinguished national authorities, such as the President and Vice Prime Minister of Romania. Following this was the parade of countries, where each team walked up on stage, carrying their country's flag.

The next two days were the competition days. Now, a brief digression on the format of the IMO: the students are given 6 problems, split across 2 days, with 4.5 hours each day to solve them. These 6 problems require lots of cleverness and ingenuity to solve, and solutions must be written up as rigorous proofs. Each question is scored out of 7 points, for a maximum total of 42 points in total. Given the difficulty of the contest, very few people achieve such high scores, with the average score this year being 15.5 out of 42. Problems 1 and 4 are meant to be easy (by IMO standards, at least), problems 2 and 5 are medium, and problems 3 and 6, the last question on each day, are very difficult and meant to challenge even the brightest young mathematicians in the world.

In recent IMOs, it has been common to have a geometry question as either problem 1 or 4. This year was no exception, and I'm glad that Canada's entire team was able to solve the geometry problem 1. Problem 2 was

harder, involving a recursively defined periodic sequence, and asked you to find, with proof, all possible values for the period of the sequence. Three of our team members solved this question, by looking at which terms in the sequence were positive and which ones were negative. Later, we discovered that there was a more elegant solution involving clever algebraic manipulations followed by an application of the Rearrangement Inequality. Problem 3 introduced a variant of Pascal's triangle called an "anti-Pascal triangle", where each number in the triangle is the difference of the two numbers below it, instead of being the sum of the numbers above. This question was considered to be the hardest on the exam, and only William was able to make non-trivial progress on it.

On day 2, Problem 4 was a game theory problem, where you had to find the outcome if both players played the game optimally. Like Problem 1, our entire team was able to solve this. Problem 5 was a number theory problem, involving an infinite sequence of positive integers satisfying a certain condition, namely that a sum of fractions of certain terms in the sequence was always an integer. It asked to prove that the sequence was eventually constant, which 5 of us were able to successfully do, and the sixth team member made some progress and earned partial marks. Problem 6 was a difficult geometry question, and the most common solution that contestants found involved an advanced geometry technique called inversion. Nobody on Canada's team solved it, although most of us earned 1 point for

constructing a pair of similar triangles, and I was able to get 4 points for performing an inversion followed by a few synthetic observations.

After the contest, the organizers took us on some excursions across Romania. We got to see some popular tourist attractions, including a mini underground amusement park deep inside an enormous salt mine. Meanwhile, our leader and deputy leader were busy with coordination, the IMO scoring process where each country's leaders negotiate with the problem coordinators over how many points each solution deserved.

In the evenings, we would socialize with the other students at our hotel, playing bughouse chess, mafia, Settlers of Catan, and card games. It was nice to be surrounded by a group of peers who were very diverse, yet similar in many ways. Some of us discussed the problems and solutions, and speculated about the medal cutoffs.

Soon, the anxious wait for our scores was over, and all the IMO results were posted online. Our team got 5 silver medals and one bronze medal (the top half of the contestants receive medals, and the ratio of the number of gold, silver, and bronze medals is 1 : 2 : 3). The winning team was team USA, with Russia in second place and China in third. Team Canada has a combined score of 156, putting us in 16th place out of 107 countries. This is definitely an improvement over our rank of 29th last year, and with 5 out of 6 team members still

eligible for next year's IMO, we hope to reach even better results next year!



The Closing Ceremony took place on July 13, where medals were awarded and more speeches were made. We gave out Canadian IMO pins, and received souvenirs from other countries in return. Then, we had a farewell banquet at the leaders' hotel. Unfortunately, IMO 2018 was now over, and we had to say goodbye to all the new friends we made, before flying back to Canada.

Thanks to all the leaders, trainers, IMO organizers, especially to the Canadian Mathematical Society, sponsors like the Samuel Beatty Fund, and everyone else who made this IMO possible. IMO 2018 was an unforgettable experience, and one of the best events of my life!