

CMS

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## DU BUREAU DE LA PRÉSIDENTE ÉLUE



*Christiane Rousseau*

*(see page 7 for English translation)*

Le paysage mathématique s'est considérablement enrichi au pays ces dernières années. La communauté mathématique a maintenant trois instituts mathématiques de premier calibre se concertant dans la programmation des années thématiques. Les contacts se sont accentués et les liens resserrés entre les branches des sciences mathématiques qui s'enrichissent mutuellement. La recherche multi-disciplinaire prend un essor sans précédent en particulier du côté de la biologie mathématique ou informatique. L'arrivée de BIRS favorisera dans le futur l'organisation de rencontres de recherche permettant aux idées et aux théorèmes de germer. Les contacts et collaborations entre la communauté mathématique universitaire et les chercheurs industriels se sont multipliés, favorisant par le fait

même le placement des futurs docteurs et boursiers postdoctoraux dans les entreprises actives en recherche et développement. D'autre part beaucoup de départements se retrouvent à un point tournant et renouvellent ou renouvelleront une partie importante de leur corps professoral d'ici quelques années alors que de nombreux collègues prendront leur retraite dans un avenir proche.

La communauté mathématique canadienne a fait de grands efforts ces dernières années pour redorer l'image des mathématiques auprès du grand public et pour nouer des liens entre les différents ordres d'enseignement.

L'enseignement et la formation de personnel hautement qualifié demeurent au cœur de notre mission. Nos départements réunissant professeurs et étudiants sont un milieu de vie tout autant qu'un milieu de formation. Nous sommes heureux d'accueillir ces jeunes qui décident de venir étudier en mathématiques et en sciences mathématiques et de travailler avec eux à leur préparer un avenir et à leur ouvrir le monde. Notre mission de formation ne s'arrête pas à la formation de chercheurs. Plusieurs de nos membres sont activement impliqués dans la formation des maîtres au secondaire ou encore à l'élémentaire. En même temps que nous travaillons à faire reconnaître l'importance de notre discipline au niveau des organismes subventionnaires nous assistons à l'érosion

*(voir PRÉSIDENTE—page 28)*

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## EDITORIAL



*Peter Fillmore*

An interesting aspect of mathematical culture is the periodical, such as this one, not primarily devoted to research. Such publications appear to be flourishing, both in number and in quality, and in recent months the *Notes* has reprinted excellent articles from several of them. This month we are pleased to present reviews of a novel—from the Australian Mathematical Society's *Gazette*—and a musical—from MSRI's *Emissary*. It is perhaps surprising that such things are done well, but (as one might say, with Dr. Johnson) even more surprising that they are done at all. Is this an indication that our efforts at outreach are beginning to bear fruit?

However that may be, it is clear that much remains to be done. The same issue of *Emissary* carries a report of a media workshop which took place at MSRI last summer. The reporters and mathematicians present had rather different priorities, and the point was made that "the media doesn't see promoting mathematics as one of its responsibilities". The corollary is that if we want media coverage, it's up to us to make our research interesting and intelligible to reporters and their readers.

Those of you who may be tempted to try your hand at this are invited to keep the *Notes* in mind as a possible showcase for your efforts!

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Les périodiques comme celui-ci, qui ne se consacrent pas principalement à la recherche, sont un aspect intéressant de la culture mathématique. Ce type de publication est en pleine effervescence, à la fois en nombre et en qualité. Au cours des derniers mois, on a d'ailleurs repris dans les *Notes* d'excellents articles issus de telles sources. Ce mois-ci, il nous fait plaisir de vous présenter des critiques d'un roman (de la *Gazette* de la Société mathématique d'Australie) et d'une comédie musicale (du bulletin *Emissary* du MSRI). On s'étonnera sans doute que de tels articles soient bien faits, mais (comme certains le diront dans le cas du professeur Johnson) encore davantage que l'on rédige même ce genre d'article. Serait-ce un indice que nos efforts de sensibilisation et de vulgarisation commencent à porter fruit?

Quoi qu'il en soit, il est clair que la partie est encore loin d'être gagnée. Dans le même numéro de *Emissary*, on peut lire le compte rendu d'un atelier sur les médias tenu au MSRI l'été dernier. Les journalistes et mathématiciens présents avaient des priorités pour le moins divergentes, et il est ressorti que « les médias ne considéraient pas la promotion des mathématiques comme l'une de leurs priorités ». La morale de cette histoire : si nous voulons que les médias parlent de nous, c'est à nous de rendre nos recherches intéressantes et surtout intelligibles pour les journalistes et leurs lecteurs. Si vous avez envie d'écrire des articles grand public, songez à publier le fruit de vos efforts dans les *Notes*!

## 2001 CMS JEFFERY-WILLIAMS PRIZE LECTURE

### Mahler’s measure, hyperbolic geometry and the dilogarithm

David W. Boyd, University of British Columbia



*David Boyd*

*The author was sponsored in part by a grant from NSERC.*

This is a short description of my Jeffery–Williams lecture given in June, 2001 at the CMS Summer meeting at the University of Saskatchewan.

I wish to thank the Canadian Mathematical Society for the honour of being chosen as the Jeffery–Williams lecturer for 2001. I also wish to thank my good friend and collaborator Fernando Rodriguez Villegas without whose help and encouragement this work would not have been done.

#### Mahler’s Measure

Given a non-zero polynomial  $P \in \mathbb{Z}[x_1, \dots, x_n]$  the *logarithmic Mahler measure* is defined as

$$m(P) = \int_0^1 \cdots \int_0^1 \log |P(e^{2\pi i\theta_1}, \dots, e^{2\pi i\theta_n})| d\theta_1 \cdots d\theta_n \tag{1}$$

By using Jensen’s formula, the one variable Mahler measure has a simple expression in terms of the roots of the polynomial and so has been the object of study for many years. In 1934, Lehmer [L] asked (in essence) whether 0 is a limit point of  $L_1 = \{m(P(x)) : P \in \mathbb{Z}[x]\}$ . The author’s 1979 Coxeter–James lecture [Bo1], was devoted to showing that this question leads naturally to the study of  $L_n = \{m(P(x_1, \dots, x_n)) : P \in \mathbb{Z}[x_1, \dots, x_n]\}$  and to the large (but countable) set  $\mathbb{L} = \cup_{n=1}^\infty L_n$ . Indeed, the conjecture made there that  $\mathbb{L}$  is closed would imply that 0 is *not* a limit point of  $\mathbb{L}$  and hence give a negative answer to Lehmer’s question (or more positively, a *positive* answer to Lehmer’s conjecture).

In that lecture, we also presented Chris Smyth’s marvelous formula [Sm]

$$m(1 + x + y) = \frac{3\sqrt{3}}{4\pi} L(\chi_{-3}, 2), \tag{2}$$

where

$$L(\chi_{-3}, s) = 1 - \frac{1}{2^s} + \frac{1}{4^s} - \frac{1}{5^s} + \dots = \sum_{n=1}^\infty \left(\frac{-3}{n}\right) \frac{1}{n^s}.$$

This result strongly suggests that  $L_1 \subsetneq L_2$ . Probably  $L_1 \subsetneq L_2 \subsetneq L_3 \subsetneq \dots$ , but as far as is known these could all be equalities!

#### A conjecture of Chinburg

In 1984, Chinburg asked whether (2) could be generalized to other conductors, i.e. for each negative discriminant  $-f$  is there is a polynomial  $P_f(x, y) \in \mathbb{Z}[x, y]$  and a non-zero  $r_f \in \mathbb{Q}$  such that

$$m(P_f) = r_f d_f? \tag{3}$$

Here we have used the abbreviation

$$d_f = \frac{f\sqrt{f}}{4\pi} L(\chi_{-f}, 2).$$

More generally, given a number field  $F$  of degree  $n$  with  $r_2 = 1$  (i.e. with a single pair of complex embeddings), a natural generalization of  $d_f$  is

$$Z_F = \frac{3|\text{disc}(F)|^{3/2} \zeta_F(2)}{2^{2n-3} \pi^{2n-1}},$$

$\zeta_F$  denoting the Dedekind zeta function of  $F$ . Then  $d_f = Z_{\mathbb{Q}(\sqrt{-f})}$ . One can generalize Chinburg’s question and ask: can one find  $P_F \in \mathbb{Z}[x, y]$  and rational  $r_F > 0$  so that

$$m(P_F) = r_F Z_F? \tag{4}$$

Ray [R] constructed examples to settle (3) for  $f = 3, 4, 7, 8, 20$  and  $24$ , the example for 7 being particularly remarkable. Recently, using quite different methods, the author and Rodriguez Villegas settled (3) for

$$f = 3, 4, 7, 8, 11, 15, 20, 24, 35, 39, 55, 84$$

and also (4) for certain quartic fields. More recently, we have extended this to include  $f = 19, 40$  and  $120$ . The ideas described here so far have settled only one additional case of

(3), namely  $f = 23$ , but suggest that the answer to the general question lies in a connection with hyperbolic geometry (which seems in fact to have been Chinburg’s original motivation). These methods also settle (4) for certain cubic and quartic fields that the methods of [BRV] don’t handle.

**Hyperbolic 3-manifolds**

The quantities  $d_f$  and  $Z_F$  appear naturally in the expressions for the volumes of certain hyperbolic 3-manifolds. Using the upper half-space model of hyperbolic 3-space, we define  $\mathbb{H}^3 = \{(x, y, t) : x, y, t \in \mathbb{R}, t > 0\}$ , with the metric given by  $ds^2 = (dx^2 + dy^2 + dt^2)/t^2$ . Then the boundary of  $\mathbb{H}^3$  is  $\partial\mathbb{H}^3 = \hat{\mathbb{C}} = \{x + iy\} \cup \infty$ . [EGM]. The orientation preserving isometries of  $\mathbb{H}^3$  are given by elements of  $PSL(2, \mathbb{C})$ . If  $\Gamma \subset PSL(2, \mathbb{C})$  is discrete and torsion free then  $X = \mathbb{H}^3/\Gamma$  is a hyperbolic manifold.

To picture this, think of a hyperbolic polyhedron  $D$  (the Dirichlet domain of  $\Gamma$ ) with certain faces identified by isometries of  $\mathbb{H}^3$ . In general  $D$  will have some vertices at infinity, i.e. on  $\partial\mathbb{H}^3$ , and these will give rise to *cusps* of the manifold. If one uses the Klein model, which distorts angles but for which hyperbolic lines and planes are portions of Euclidean lines and planes, then this takes on a comforting familiarity.

For example, if one takes  $\Gamma = PSL(2, O_f)$ , where  $O_f$  is the set of integers of  $\mathbb{Q}(\sqrt{-f})$ , the Bianchi group, then one obtains the *Bianchi manifold*  $B_f = \mathbb{H}^3/PSL(2, O_f)$ . A classical formula of Humbert (1919) gives us

$$vol(\mathbb{H}^3/PSL(2, O_f)) = \pi d_f/6. \tag{5}$$

Thus one could answer Chinburg’s question if one could find  $P \in \mathbb{Z}[x, y]$  with  $2\pi m(P(x, y)) = vol(\mathbb{H}^3/PSL(2, O_f))$ . Or by using a subgroup of  $PSL(2, O_f)$  of index  $k$  one might hope for a  $P$  with  $m(P) = kd_f/12$ . But perhaps it is not yet clear that this really simplifies the problem!

For  $F$  a number field with  $r_2 = 1$ , Borel constructed discrete  $\Gamma_F \subset PSL(2, \mathbb{C})$  such that  $vol(\mathbb{H}^3/\Gamma_F) = \pi Z_F/6$  so one can approach the question (4) in the same way.

**Ideal Triangulations and the Dilogarithm**

Given a hyperbolic manifold  $X = \mathbb{H}^3/\Gamma$ , one can decompose

$$X = \bigcup_{j=1}^n \Delta(z_j), \tag{6}$$

where the  $\Delta(z_j)$  are *ideal tetrahedra*, i.e. tetrahedra in  $\mathbb{H}^3$  with all their vertices at infinity (on  $\partial\mathbb{H}^3$ ). Here the  $z_j$  are certain complex numbers with  $Im(z_j) > 0$  called the *shapes* of the  $\Delta(z_j)$ . To explain the meaning of the shape, we observe that each ideal tetrahedron is isometric to one with

its vertices at  $0, 1, z, \infty$  for some  $z$  with  $Im(z) > 0$ . Here  $z$  is the cross-ratio of the vertices of the tetrahedron. Thus the volume of the tetrahedron depends only on the single parameter  $z$ . In fact  $vol(\Delta(z)) = D(z)$ , [NZ] where  $D(z)$  is the Bloch-Wigner dilogarithm defined by

$$D(z) = Im(Li_2(z)) + arg(1 - z) \log |z|.$$

Here  $Li_2(z)$  is the classical Euler dilogarithm, the analytic continuation of

$$Li_2(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2}, \quad |z| < 1.$$

**The Gluing Equations**

The  $z_j$  in (6) satisfy a system of polynomial equations known as the *gluing equations*:

$$\prod_{j=1}^n z_j^{c_{i,j}} (1 - z_j)^{d_{i,j}} = \pm 1. \tag{7}$$

Here  $1 \leq i \leq n + 2c$ , where  $c$  denotes the number of cusps of the manifold, and the  $c_{i,j}, d_{i,j}$  are certain integers that depend on the combinatorics of the triangulation. Solving (7) for the shapes  $z_j$ , one then obtains

$$vol(X) = \sum_{j=1}^n D(z_j). \tag{8}$$

By Mostow’s rigidity theorem, the equations (7) have essentially only one solution (the geometric solution) with  $Im(z_j) > 0$  for all  $j$ . However they may very well have other solutions for which some of the  $Im(z_j) \leq 0$  and these also play a role in what follows.

**A-polynomials of Hyperbolic 3-manifolds**

Thurston had the idea of introducing some flexibility into (7) by “deforming” one of the cusps. That is, we replace the 1 in the right member of (say) the last pair of equations (7) by  $M^2, L^2$ , respectively, where  $M$  and  $L$  refer to “meridian” and “longitude”. We denote this new set of equations by (7 $\dagger$ ). Geometrically this means that we are considering triangulations that fit together properly around all finite edges and also at all but one cusp. The quantities  $L$  and  $M$  measure the extent to which this cusp is deformed.

Eliminating the other variables from (7 $\dagger$ ), one obtains a polynomial  $A(L, M)$  called the *A-polynomial*. (Although the “A” is used in homage to Alexander, this is *not* an Alexander polynomial.) For the smooth points of the variety

(see *JW-LECTURE – page 26*)

## Fermat's Last Tango in Berkeley

by David Hoffman, MSRI

*This article first appeared in the Fall 2001 issue of Emissary, the newsletter of the Mathematical Sciences Research Institute, Berkeley.*

Last July the screen version of *Fermat's Last Tango* had its world premiere before an audience of about 500 in the Roda Theatre, Berkeley Rep's dazzling new playhouse. *Fermat's Last Tango*, a musical drama based on the recent proof of Fermat's Last Theorem by Andrew Wiles was produced by the York Theatre Company and ran Off Broadway in late 2000 and early 2001. Under the direction of Arthur Jaffe, the Clay Mathematics Institute (CMI) taped a live performance, from which a professionally edited version of the show was produced.



I can hear your questions now as I write this article, weeks or months before you read it: A musical about Wiles' proof of Fermat's Last Theorem? What? Are you serious? True, it's hard to imagine what such a production could be like. I certainly had no idea what to expect the first time I saw it. So let me tell you something about the musical.

In *Fermat's Last Tango*, Daniel Keane (Wiles) earns overnight acclaim when, after many years of solitary labor in his attic, he announces his proof of Fermat's Last Theorem. Elation soon gives way to doubt when a problem arises in the proof. In the musical, Fermat—who is portrayed as rather mean-spirited—drops in on Keane soon after the announcement. Fermat is not happy about this turn of events. He takes Keane on an unsettling journey to the *Aftermath*, inhabited by Pythagoras, Euclid, Newton and Gauss. (Try to imagine the

apotheosis of a Ph.D. oral examination staged by the Marx Brothers.) There Keane is taunted, tested, then told that his proof contains a gap – “a big fat hole”, sings Fermat.

Consumed by the obsession to fill in the gap, Keane – in crisis – retreats again to his attic. He is haunted by Fermat, who has a personal interest in his failure, and challenged by his wife, who, while supportive, wants him to appreciate her curves, too. Her number, *Math Widow*, produced cries of recognition from a few spouses in the audience seated next to some of the many mathematicians present. The three of them perform the title tango in what may be the play's most moving scene.

Except for Keane, the Wiles character, all mathematicians involved are mentioned by name, including Ken Ribet who was on hand for the Q&A session after the screening. With one or two exceptions, the authors get the history of the subject fairly straight (and, yes, there are lines in which *Taniyama-Shimura* is sung). Of course, the *Aftermath*, the relationship between Keane and his wife, and the character of Fermat are fictional creations, and there are some campy moments, but something of the spirit of mathematical struggle does come through.

The occasion for the screening was the Clay Mathematics Institute Summer School on the Global Theory of Minimal Surfaces, of which I was the Director. This was the Summer School's public event, and it was jointly presented by CMI and MSRI. The evening began with a few minutes of minimal-surface animations to music, after which Arthur Jaffe, President of CMI, provided some background for the achievement of Wiles and the creation of this musical fantasy. Then an interview with Wiles (taped in Paris on the occasion of the CMI Millennium Event in 2000) was shown. It made concrete for the general public the nature of mathematics research and the atmosphere in which it is done. Introducing Wiles himself to the audience served as a counterpoint to the sometimes-frenetic dramatic conflict that followed in the musical. In the interview, Wiles is poised and lucid, magisterial in a quiet, calming way. The masterful editing of the interview and the setting (a vaulted room in the College de France) reinforced this impression.

Wiles, at one point in the interview, says:

... We feel a real part of history. Mathematics has been going on for thousands of years and we still use results that were proved a thousand or two thousand years ago. And so we feel part of an enterprise that's really been going on for a long time and will go on, we believe, forever. There are very few other enterprises where you feel this permanence. Countries come and go, political

systems come and go, but somehow mathematics has a permanence that almost nothing else has.

Wiles was thinking about Greek mathematics, but there is another enduring human activity whose tradition is the Western World begins in Greece: theatre. That night we saw these two traditions come together briefly, in an attempt to understand the human component and cultural meaning of a great

intellectual achievement. It was a special evening.

*The Clay Mathematics Institute is offering a VHS tape as well as a (higher quality) DVD of Fermat's Last Tango to the general public at cost. The package includes a video recording of the play, a video of an interview with Andrew Wiles in May 2000, and a pamphlet both about the production and about the history of Fermat's Last Theorem. See [www.claymath.org/events/fermatstango.htm](http://www.claymath.org/events/fermatstango.htm) for information about ordering.*

## FROM THE INSTITUTES

### PIMS Activities

#### PIMS Thematic Programme on Asymptotic Geometric Analysis

[www.pims.math.ca/aga](http://www.pims.math.ca/aga)

Conference on Non-Commutative Phenomena and Random Matrices, August 6-9, PIMS-UBC

Conference on Banach Spaces, August 12-15, PIMS-UBC

#### PIMS Thematic Programme on Selected Topics in Mathematical & Industrial Statistics

[www.pims.math.ca/stats](http://www.pims.math.ca/stats)

Workshop on the Role of Statistical Modeling in the 21st Century, May 4-6, SFU

3rd MITACS AGM: Statistics for Large Scale Industrial Modeling, May 23-25, UBC

International Conference on Robust Statistics, May 12-18, PIMS-UBC

Workshop on Design and Analysis of Experiments, July 14-18, Coast Plaza Suites Hotel, Vancouver

Workshop on Filtering Theory and Applications, July 25-30, Edmonton and Jasper

#### PIMS Scientific Activities

1st PIMS School of Mathematical Biology for Senior Undergraduates, May 11-19, University of Alberta

3rd Pacific Northwest PDE Conference, May 15-18, Washington State University

MSRI Summer Graduate School, June 17-28, PIMS-SFU

2nd Canadian Conference on Nonlinear Solid Mechanics, June 19-23, SFU

Symposium on Aperiodic Order, Dynamical Systems, Operator Algebras and Topology, August 4-8, U. Victoria

Numerical Analysis Potlatch 2002, September 5, U. Victoria

Cascade Topology Conference, November 2-3, UBC

PIMS PDF Meeting, November 30-December 1

#### PIMS Industrial Activities

3rd IAM-CSC-PIMS Senior Undergraduate Math Modelling Workshop, February 16-17, UBC and SFU

5th PIMS Graduate Industrial Mathematics Modelling Camp, May 18-23, SFU

6th PIMS Industrial Problem Solving Workshop, May 27-31, UBC

PIMS-MITACS Summer School on Applications of Computational Geometry, June 10-14, SFU

PIMS-MITACS Workshop on Facility Location Problems, June 16-18, SFU

4th PIMS School of Environmental and Industrial Fluid Dynamics, July 28-August 9, University of Alberta

#### DEADLINES FOR PIMS OPPORTUNITIES

March 16: Applications to the National Programme Committee

March 16: Applications to the Spring Competition of the PIMS Scientific Review Panel

October 15: Applications to the National Programme Committee

October 15: Applications to the Fall Competition of the PIMS Scientific Review Panel

October 15: Nominations for PIMS Prizes

October 15: Submissions for BIRS 2004 programme

#### PIMS Education Activities

Alberta High School Mathematics Competition, Part I of the 2002-2003 Season, November 19

Math Fair, March 21, University of Alberta

Greater Regional Vancouver Science Fair, April 4-6, UBC

Calgary Youth Science Fair 2002, April 10-13

Changing the Culture V, April 26, Harbour Centre, SFU

PIMS Elementary Grades Math Contest, May 25, UBC

Summer Math Camp for Grades 8-10, August 14-21, U. Alberta

## FROM THE PRESIDENT-ELECT'S DESK

*(see page 1 for the French version)*

The mathematical landscape has expanded significantly in recent years. We now have three first-class mathematical institutes working together to organize theme years. Contacts and cooperation between the various branches of mathematical science have grown stronger, to their mutual benefit. Multidisciplinary research in mathematics is enjoying unprecedented growth, particularly in biology and informatics. The opening of the BIRS promises to lead to more research-oriented meetings in the future, fostering the development of new ideas and theorems. Contact and cooperation between the academic community and industrial researchers is increasing, making it easier to place future PhDs and post-doctorate fellows with companies active in R & D. In addition, many departments are at an important juncture, in that they are or will soon be replacing a significant part of their faculty staff as many colleagues reach retirement over the next few years.

The Canadian mathematical community has tried hard in recent years to rehabilitate the public image of mathematicians and to forge ties with mathematics teachers at all levels. Education and training highly qualified personnel remain one of the cornerstones of our mission. Our departments, where professors and students interact, offer as much a way of life as a learning environment. It is always a pleasure to welcome young people interested in studying math and the mathematical sciences and to work with them to prepare them for the future and for the real world. Our educational mission is not limited to researchers. Many of our members are actively involved in training secondary and even elementary school teachers. Yet even as we work to promote the importance of our discipline to granting agencies, we face the erosion of math courses in technical and many pre-university programs. Our discipline continues to receive bad press and to be perceived as elitist and a cause of drop-outs.

It is against this backdrop that the Canadian Mathematical Society has decided to launch a series of national forums on mathematical education. The first of these will be held at UQAM (Université de Québec à Montréal) on May 16-18, 2003. It will be followed by a second forum in Ontario in 2004. These forums will continue the tradition begun by the first national forum on mathematical education, which took place in Quebec City in May 1995 with Katherine Heinrich as Chair. These forums will bring together about 200 people from all provinces and territories in Canada, representing the different groups with interest in or impact on mathematical education in middle or high school. They will include school and college professors, university teachers of mathematics or mathematical education, school administrators, provincial ministries of education, student advisers, publishing firms, participants from outside Canada, provincial teachers' associations, etc.

The CMS hopes that these forums will foster fruitful contacts that will promote the teaching of mathematics and further the recognition of the discipline and its importance across Canada. To that end, it will invite the broadest possible range of organizations to work as partners on this initiative, including institutes of mathematics, provincial mathematics associations, associations connected with math sciences and mathematical education, granting agencies, government departments of education, etc.

The first forum will provide an opportunity for participants to discuss objectives and best practices at the national level. Issues will be identified for subgroups to investigate in preparation for the second forum. Their findings will be published both electronically and in print for the widest possible distribution. I, along with George Bluman of UBC's Department of Mathematics, will be the scientific chairs for the event. We are very grateful to UQAM for agreeing to look after local organization. The local organizing committee will be chaired by Louis Charbonneau with the able assistance of Pierre Bouchard and Manzour Ahmad.

Here are some of the main themes we hope to see addressed at the forum:

- mathematics in the schools: how to make the subject interesting
- mathematics and team building
- teacher training in mathematics at the elementary and secondary levels

Here are some other upcoming events. The Summer 2002 meeting of the Canadian Mathematical Society will take place in beautiful Quebec City on June 15-17, 2002. We already have 158 guest speakers, but organizers are launching a call for short presentations—which will not compete with the special sessions! Accordingly, we urge you to send your graduate students to the meeting and to encourage them to give a talk. It will be an excellent opportunity for them to make contacts and to add participation in a major scientific conference to their résumés.

The next Canadian Undergraduate Mathematics Conference will be held at the University of Calgary on July 2-7, 2002. It's not too early to alert your undergraduate students to the event and to encourage some of them to present. You might even suggest that they propose your department as host for the next congress. They will make you proud!

To conclude, a joint meeting of the Canadian Mathematical Society, the Canadian Applied and Industrial Mathematics

Society, the Société mathématique de France and the Société française de mathématiques appliquées has already been announced for July 2004 in Toulouse, France. We hope to organize a satellite meeting of mathematics societies from various North African French-speaking countries (Morocco, Tunisia, etc.). For some of us it will an opportunity to renew ties with former grad students!

I would also like to take this opportunity to tell you about the Independent University of Moscow (IUM). This small university, founded in 1991, has taken up the mission to train the next generation of mathematics researchers in the best traditions of the old school of Russian mathematics. The IUM now offers a program for foreign students called "Math in Moscow." Participants are invited to spend a semester in Moscow, during which they will also take English courses.

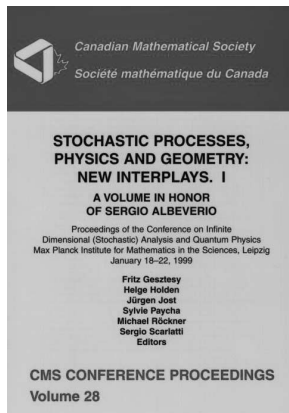
Tuition is US \$3500 a semester. Details on this program may be found at <[www.mccme.ru/mathinmoscow/](http://www.mccme.ru/mathinmoscow/)>. The IUM has entered into cooperation agreements with several institutions, including the École Normale Supérieure. In 1998, the AMS published a series of research articles by IUM professors and grad students ("Mathematics at the Independent University of Moscow," AMS Translations, Series 2, Vol. **185**, 1998). The IUM has just launched the "Moscow Mathematical Journal," a new periodical featuring research and expository papers of the very highest quality. The on-line version of the journal is at <[www.ams.org/distribution/mmj/](http://www.ams.org/distribution/mmj/)>. The subscription cost is very reasonable: only US \$150 for institutions and US \$75 for individuals. I encourage you to add this journal to your library collections and to promote the "Math in Moscow" program among your grad students.

## Stochastic Processes, Physics and Geometry

Book Review by Vojkan Jaksic, McGill University

### Stochastic Processes, Physics and Geometry: New Interplays. I and II

Fritz Gesztesy, Helge Holden and  
Jurgen Jost, Editors  
CMS Conference Proceedings 28 and  
29, AMS 2000  
x+333 pp and xii+647 pp



Sergio Albeverio's 60th birthday was celebrated by an international conference organized in Leipzig during January 18-22, 1999, and these books present articles that originated from invited talks at this conference. The

theme of the conference, "Infinite Dimensional (Stochastic) Analysis and Quantum Physics", was chosen to reflect the wide range of Sergio's scientific interests. Indeed, the seventy eight articles contained in these volumes deal with a wide range of subjects in mathematical physics involving probability, operator theory, spectral theory, statistical physics, integrable systems, quantum field theory, PDE's, stochastic analysis, and many others. Most of the articles contain original results and their scope and breadth mark the ultimate success of the conference.

The first volume begins with a preface by the editors, a brief curriculum vitae of Sergio Albeverio, and a touching opening address by Philippe Blanchard describing the life and personality of Sergio. They are followed by twenty one articles, which include:

- Growth and saturation in random media (G. Ben Arous and A.F. Ramirez)
- Probability and nonlinear analysis (E.B. Dynkin)
- The classical massive Thirring system revisited (V.Z. Enolskii, F.

Gesztesy, and H. Holden)

- Spectral theory of sparse potentials (D. Hundertmark and W. Kirsch)
- A Feynman-Kac formula for unbounded semigroups (B. Simon), to mention just a few highlights.

The other 57 articles are contained in the impressive second volume. We mention a few:

- Correlation effects in the trapping problem: General approach and rigorous results (L.V. Bogachev, S.A. Molchanov, Yu. A. Makhnovskii, and A.M. Berezkhovskii)
- Loop groups factorization of biorthogonal wavelet basis (S. Borac and R. Seiler)
- Irregular spectral asymptotics (J. Bruning)
- Perron theory for positive maps and semigroups on von Neumann algebras (F. Cipriani)
- Point interactions in a tube (P. Exner)
- The analytic quantum information manifold (R.F. Streater).

These volumes contain a wealth of useful information and are pleasure to read. They belong on the shelves of any working mathematical physicist.



## Mathematical History Through Stamps

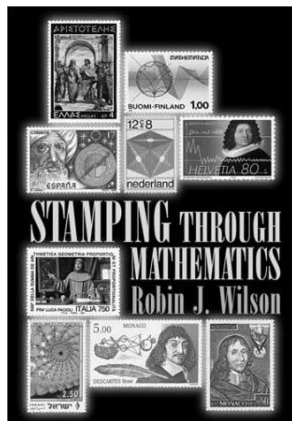
Book Review by Joerg Richstein, Dalhousie University

### Stamping Through Mathematics

by Robin Wilson

Springer-Verlag, New York, 2000

viii + 126pp



There are probably many reasons to give up stamp collecting. The lack of electronic stamps in the internet age or the discovery that the presumably most valuable stamp in your collection is actually missing one time are only two examples.

The most common reason is more likely to be the depressing experience that one will never be able to bring a collection even close to completion. But this of course depends on the initial decision of what type of stamps your album should contain. Specialising in butterflies or flowers probably doesn't really help, so a more specific selection of subject-matter must be made.

Robin J. Wilson's book *Stamping through Mathematics* is a printed form of such a specific collection: The history of mathematics presented by means of stamps from all over the world.

The book is divided into 55 topics, each consisting of two pages. On the right hand sides, between four and nine

mint stamps related to the current topic are shown on a black background. The text on the left pages gives information related to the historical topic (but no information about the stamps).

As Wilson explains in the introduction, he doesn't view his book as an ordinary history book. He lets the stamps dictate the story. Consequently, some historical topics and/or figures are missing due to the lack of corresponding stamps while others seem more important than they really are.

This "dictation" draws an interesting picture of which mathematical subjects or fields were of significant importance for the general public throughout the history (of stamps). Of course some areas are almost impossible to visualize whereas others are easy to illustrate. Countries proud of their mathematicians are likely to place them on a stamp eventually. On the other hand, even very deep and important theorems from not so publicly popular areas or their discoverers will never be seen on a postcard, letter or parcel.

Starting with ancient mathematics, Robin Wilson walks through 5000 years of mathematical history, visiting Egypt, Greece, China, India and even the Mayas and Incas. Developments in Islamic mathematics, the middle ages and the rebirth of European mathematics are described. Wilson includes mathematically related inventions such as calendars, maps and globes on his way. Considering that this could probably be viewed as a rather open definition of what directly belongs to mathematics, it is a bit surprising that there are only 389 stamps in this collection.

It is interesting to note that there is only one Canadian stamp among them (and that this is one of those only marginally related to mathematics; it shows a map of the British Empire as

of Christmas 1898) and although one cannot complain about Canada Post as far as the beauty of its stamps is concerned (it is the reviewer's strong opinion that Canadian stamps are among the nicest in the world), the appreciation of Canadian mathematics on a stamp is definitely overdue.

The book closes with a few non-historical topics about mathematical stamps that did not fit in any of the preceding sections. They include mathematical games, education, shapes, mathematics and nature and more.

Unfortunately, the exact philatelic references can only be found in the appendix. Giving them directly on each page would have made it easier to correctly reference a stamp.

Although being somewhat transient, the inclusion of some world wide web URL's related to mathematical stamps would have been desirable. Quite a few such pages offering images in good quality for easy download do exist.

Robin Wilson solved the difficult task of finding a suitable classification of mathematics related stamps in a very nice and interesting way. His long experience with the subject makes this printed stamp collection valuable for both stamp-collecting mathematicians and philatelists with mathematical interests. There is probably no classification that will ever satisfy everyone but I enjoyed reading the book and admired its collection of nice mathematical stamps. And maybe I will reorganize my old stamps into a new collection and try to fill in some of the gaps.

*Editors' note: Perhaps the CMS should investigate the possibility of honouring an individual (J.C.Fields or H.S.M.Coxeter come to mind) on a Canadian stamp.*

## AWARDS / PRIX

### Le 2001 prix Adrien-Pouliot du SMC

### The 2001 CMS Adrien Pouliot Award



*George Bluman*

For more than three decades, George Bluman has exercised extraordinary leadership and invested a great deal of energy in promoting sound mathematics education and student interest in mathematics in the province of British Columbia. On a national scale, he chaired the Education Committee of the Canadian Mathematical Society from 1978 to 1981, and has been a member of problems committees for the University of Waterloo contests.

Dr. Bluman has been presenting seminars for high school students since 1969, having given more than 140 talks and workshops to students and teachers over this time. He has developed a very successful workshop format and has mobilized a team of faculty, graduate and undergraduate students to give presentations to the schools. Eight grants from the Ministry of Education in BC have supported these programs.

In 1979, he introduced the Gauss and Euclid contests to British Columbia; the Euclid is now written in 175 schools and marked within the province. In the early 1990s, he introduced the challenge examination at the University of British Columbia, whereby talented mathematics students can qualify for advanced credit and gain access to advanced courses. He is a coauthor of a problem book for Grade 12 students.

At the University level, Dr. Bluman's influence has been felt, recently through his role as chair of the mathematics department at UBC. He was a founding member of the Institute of Applied Mathematics which provides an interdisciplinary graduate program and continues to attract excellent students.

Between 1989 and 1991, he established and helped develop an offcampus four-year degree program at Cariboo and at Okanagan colleges. Under his guidance, his mathematics department has developed a new course under the title of "Mathematics Demonstrations". Its goal is to show students how to communicate mathematics to school pupils through designing and presenting stimulating problems. He has published, with Springer, a Problems Book for first year calculus.

In the public domain, he has provided encouragement and support for teachers through his collaboration with the British Columbia Association of Mathematics Teachers, and has been frequently consulted by and given commentary in the print and broadcast media. The Society is proud to recognize the passion, insight and energy of one of its most distinguished members.

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Depuis plus de trente ans, George Bluman joue un rôle de leader extraordinaire en s'attachant à promouvoir un enseignement des mathématiques de qualité et à susciter l'intérêt des étudiants pour les mathématiques en Colombie-Britannique, tâche à laquelle il consacre d'ailleurs beaucoup d'énergie. À l'échelle nationale, il a été président du Comité d'éducation de la SMC de 1978 à 1981, et il est membre du comité responsable de choisir les problèmes pour les concours mathématiques de l'Université de Waterloo depuis de nombreuses années.

Depuis 1969, le professeur Bluman a donné plus de 140 conférences et ateliers à des étudiants et à des enseignants dans les écoles secondaires. Il a conçu un format d'atelier qui fonctionne à merveille, où il fait appel à une équipe de professeurs et d'étudiants de tous les cycles, qui font des présentations dans les écoles. Le ministère de l'Éducation de la Colombie-Britannique lui a déjà accordé huit subventions pour mener à bien ces programmes.

En 1979, il a lancé les concours Gauss et Euclid en Colombie-Britannique. Chaque année, 175 écoles participent au concours Euclid, qui est corrigé en entier dans la province. Au début des années 1990, il a lancé un concours-examen de calcul à l'Université de la Colombie-Britannique, dont les résultats permettent à des étudiants doués d'obtenir des crédits et de s'inscrire à des cours de niveau avancé. Il est en outre coauteur d'un recueil de problèmes pour élèves de 12e année.

À l'Université de la Colombie-Britannique, le professeur Bluman fait sa marque depuis quelque temps à titre de directeur du département de mathématiques. Il est également un des membres fondateurs de l'Institut de mathématiques appliquées de cet établissement, qui offre un programme interdisciplinaire de deuxième cycle et attire toujours des étudiants exceptionnels. De 1989 à 1991, il a contribué à la fondation d'un programme hors campus de quatre ans au University College of the Cariboo et à l'Okanagan University College.

Sous sa direction, le département de mathématiques a créé un nouveau cours intitulé «Mathematics Demonstrations», dont l'objectif consiste à montrer aux étudiants comment enseigner les mathématiques à des élèves par la conception et l'utilisation de problèmes stimulants. Il a aussi publié, chez Springer, un recueil de problèmes pour un cours de calcul de première année.

Du côté public, il a toujours épaulé les enseignants par son travail auprès de l'Association des enseignants de mathématiques de Colombie-Britannique. Il est en outre souvent consulté et commenté par la presse écrite et électronique. La Société est fière de souligner la passion, la perspicacité et l'énergie de l'un de ses membres les plus éminents.

### **The CMS 2001 Distinguished Service Award Le Prix de la SMC pour service méritoire 2001**



*James Timourian*

Dr. James Timourian obtained his undergraduate degree from Hamilton College (New York) and his doctoral degree from Syracuse University. He has been a member of the Department of Mathematical Sciences at the University of Alberta since 1969 and was the Associate Chair (Undergraduate) for the Mathematical Sciences from 1994 to 1998.

His research areas are singularity theory, elliptic operators and partial differential equations. He has long been interested in undergraduate education, especially first-year and honours calculus and, in 1996, organized a very successful in-service program for Alberta high-school calculus teachers.

James Timourian has made many contributions to mathematics in Alberta. Nationally he has served on and chaired many CMS committees, the Natural Sciences and Engineering Research Council's Grant Selection Committee, and was the Chair of the Canadian National Committee for the International Mathematical Union. He is currently the Chair of the CMS Endowment Grants Committee. Dr. Timourian is a successful entrepreneur in areas such as bio-technology and is a noted philanthropist.

The CMS is delighted that the 2001 Distinguished Service Award is given to Dr. Timourian to honour his extensive and diverse contributions to Canadian mathematics.

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James Timourian a obtenu son baccalauréat du Collège Hamilton (New York) et son doctorat de l'Université de Syracuse. Il est membre du département de sciences mathématiques de l'Université de l'Alberta depuis 1969, et il a été titulaire associé (premier cycle) de la chaire de sciences mathématiques de 1994 à 1998.

Ses recherches portent principalement sur la théorie des singularités, les opérateurs elliptiques et les équations aux dérivées partielles. Il s'intéresse depuis longtemps à l'enseignement au premier cycle universitaire, particulièrement à l'enseignement du calcul en première année et dans le cadre d'une spécialisation. Il a d'ailleurs organisé, en 1996, un programme très réussi de formation en cours de service destiné aux enseignants de calcul des écoles secondaires albertaines.

En Alberta, le professeur Timourian a beaucoup fait pour la cause des mathématiques. Sur la scène nationale, il a été membre ou président de nombreux comités de la SMC et du Comité de sélection des subventions du CRSNG, en plus d'avoir présidé le comité canadien de l'Union mathématique internationale. Il assume en ce moment la présidence du Comité d'attribution des bourses du fond de dotation de la SMC. Entrepreneur prospère dans des domaines comme la biotechnologie, James Timourian est en outre un philanthrope reconnu.

La SMC est ravie d'accorder son Prix pour service méritoire 2001 au professeur Timourian et ainsi de souligner sa contribution exceptionnelle et diversifiée à la communauté mathématique canadienne.

### **ICIAM Prizes**

The International Congress on Industrial and Applied Mathematics (ICIAM) is held every four years and is the most important general meeting, worldwide, for applied mathematicians. The Congress covers the full spectrum of research topics in applied mathematics and its industrial applications.

The Congress celebrates and describes the contributions of applied mathematics – as an intellectual creation in its own right, as a foundation stone of technological development, and as an indispensable collaborative partner for other scientific disciplines. These aspects of applied mathematics have held true since the dawn of civilisation. They will remain just as important in the new millennium.

The Congress is held under the auspices of the International Council for Industrial and Applied Mathematics, an international body consisting of approximately 20 professional applied mathematical societies. Previous meetings have been held in

Paris, 1987

Washington, 1991

Hamburg, 1995

Edinburgh, 1999

and the next, ICIAM 2003, will be held in Sydney, Australia. At ICIAM 1999 four new prizes were awarded for the first time.

The **Lagrange Prize**, recognizing individuals who have made exceptional contributions to applied mathematics throughout their careers, was awarded to the late Jacques-Louis Lions of the Collège de France.

The **Collatz Prize**, for outstanding work on industrial and applied mathematics by an individual under the age of 42, was presented to Stefan Müller of the Max Planck Institut für Mathematik in den Naturwissenschaften.

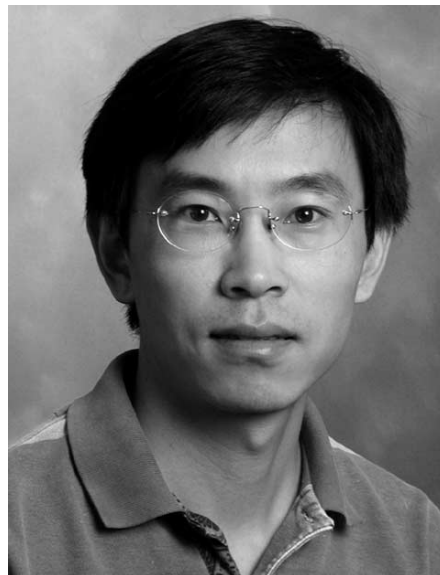
The **Pioneer Prize**, given in recognition of pioneering work in introducing applied mathematical methods and scientific computing techniques to an industrial area or a new problem area, was shared by Ronald Coifman (Yale) and Helmut Neunzert (Universität Kaiserslautern).

The **Maxwell Prize**, for originality in applied mathematics, was won by Grigory Barenblatt of UC Berkeley and Cambridge University.

For further information about ICIAM, go to [www.iciam.org](http://www.iciam.org).

### André Aisenstadt Prize

The CRM has announced that the André Aisenstadt Prize for 2001 has been won by Jingyi Chin of UBC. The prize has been awarded annually since 1991 by the CRM for outstanding work by an individual less than 7 years from the PhD.



*Jingyi Chin*

Jingyi Chen received his PhD degree at Stanford University under Richard Schoen. He held an NSF Post Doctoral Fellowship at MIT before joining UBC in 1997. He was awarded a Sloan Fellowship in 1999. Jingyi has made fundamental contributions in many of the most active areas of differential geometry and partial differential equations, including harmonic maps, where he proved a conjecture of Wood, the theory of minimal surfaces, gauge theory, hyper-Kähler manifolds, Ricci flows, and mirror symmetry.

## LETTER TO THE EDITORS

At the recent CMS meeting in Toronto I spent most of a day attending a session on mathematics education, organized by Pat Rogers (Windsor) and Walter Whiteley (York). It was incredibly inspiring and I can't recall a time where I've learned so much in one day. One of the key issues we discussed was the need for teacher development within each university mathematics department: we talked about why this was important and how we could facilitate this.

I find it interesting that a lot of research is very collaborative, but for the most part, teaching is very individualized. We rarely share lecture notes, attend each other's classes to learn new teaching techniques, discuss various forms of pedagogy, talk about alternative forms of assessment, etc. In short, all of us (including myself) pretty much do "our own thing", and we don't get a chance to share ideas and learn from one another.

So I am proposing a new seminar series in our depart-

ment, where we ask questions and share ideas in a very informal setting. Instead of calling this a formal "seminar series", I'd much rather refer to it as a "Math/Stats Education Study Group". The two assumptions I make about this study group are that we all have something to contribute, and that we all have something to learn. Some themes we could discuss are: teaching large first year classes, different presentation styles, teaching multi-section classes, learning styles, incorporating technology into our classes, and opportunities for further professional development (e.g. CMS and CMESG meetings).

Once the term is over, I plan to submit a reflection/report to the Education Notes column, on what we did, what worked, what could have been better, etc. Perhaps others would be interested in trying this experiment too.

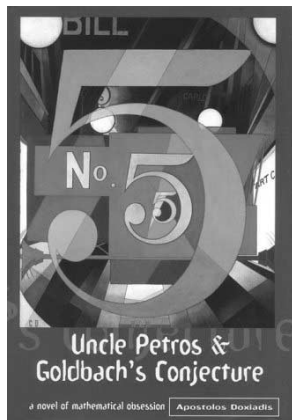
*Richard Hoshino (Dalhousie University)*

## Petros vs. the Queen of Number Theory

Book Review by Warren Brisley, University of Newcastle, Australia

### Uncle Petros and Goldbach's Conjecture

by Apostolos Doxiadis  
Faber and Faber 2000  
pp. 210



This review is reprinted with permission from the April 2001 issue of the *AustMS Gazette*

Don't be put off by the unfortunate title. This short novel is an excellent piece of work from any point of view.

Firstly, in the popular parlance, it's a "good read" – one wants to know how it progresses and ends, so the pages turn, and as a narrative, it is very well crafted, so there are no slow patches, and no fruitless by-ways. Secondly, the whole story is believable, the psychological underpinnings are convincing, and the background history seems accurate. Thirdly, the two main characters are well enough realised to fully engage the reader's sympathy and interest, and the "minor" characters given just enough attention to elucidate their influence on the action. They are not just decorative cameos. Incidentally these minor characters include Carathéodory, Hardy, Littlewood, Ramanujan, and Gödel, as well as some less familiar to a mathematician. The mathematically educated will also recognize some familiar technical charac-

ters as well. Just as war novels include strategy, politics and machinery as de facto springs of action, so here the mathematician will be aware of methods, collaborations and conjectures.

The theme of the book is *obsession*. The narrator is Petros's nephew, who observes obsession in his uncle, experiences it himself, and reacts to, and survives both partial success and failure, all in the context of obsession in mathematical research.

But this is not a dry consideration of such matters; in fact, it starts with a very human family puzzle. Why do Petros's brothers consider him a failure, and both a danger to the young and a salutary lesson to them? Does our developing picture of Petros accord with this view? What will be the devastating effects (on Uncle and Nephew) of the nephew's probing? And, as things progress, larger matters loom. I am sure that this book will be enjoyed by *any* reader of novels, especially since any necessary technical stuff is brilliantly and succinctly explained, at least as well as in any top-grade spy thriller or political romance. There is no clumsy didacticism here; it all flows naturally with the narrative, as in Le Carré or Deighton or Price.

(Why do I mention these "espionage" writers? Well, because of the reader-catching opening pages, a teasing move they often use. Here, the Nephew asserts that Petros's life story is bounded by a letter written in 1724, and the content of pages 183 to 198 of issue 38 of *Monatshefte für Mathematik und Physik*. That move, which sets up what needs explanation, is a favourite tactic of those writers. Of course, in the event, it turns out that quite a few other things will need explanation first, and that is the case in this novel, too.)

However, to the mathematician, there must be a deeper appreciation and enjoyment than to the layman. It is

clear *from the writing* that the author has himself engaged in mathematical research, and has subjected his experience to much more perceptive introspection than is usual. I would say that he succinctly conveys the flavour of that experience to the general public through this novel, perhaps better than professional mathematicians such as Hardy or Halmos, and that just as part of a *wider* aspect of human behaviour! Also, for the mathematician, there are quite a few sly quips and enjoyable comments on the profession and its icons, and cogent observations about proofs, claims, publication priority, appointments. There is even a whiff of gunpowder and *realpolitik* about the reasons the British invite Petros to Cambridge.

Although the human story of the protagonists comes to a reasonable conclusion by the last page, for the mathematician the reverberations continue. In the context of elementary number theory, should we read Gödel's theorem as a timely warning, or as just another challenge to our ingenuity? Political novels often end with just this sort of sting in the tail, and in this case, it is this little bit of spice that makes this novel so satisfying

Incidentally, the wrapper delivers the information that the author was born in Australia, raised in Athens, entered Columbia at age 15 on the grounds of his original research, did postgraduate work in Paris, and still has had a life as a prize-winning film maker, theatre director, translator of plays, and novelist, (This is his fourth novel). As a matter of fact, now I think of it, this would make a gripping movie. I can imagine some of the scenes: the German-educated intense and passionate Petros against the foil of the essentially English Hardy ("The Queen of Number Theory" Petros calls him), or the awful revelations in the Petros-Turing interview in Petros's rooms in

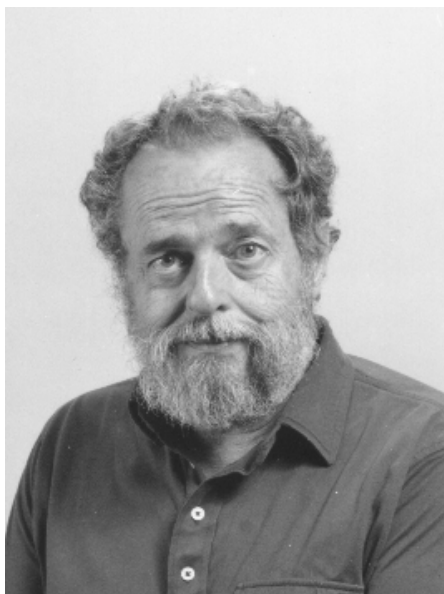
Cambridge, the symbolism of the beans on the floor (You'll have to read it to see that one), not least, the figure of Gödel drifting into insanity at the I.A.S. The

author has set them up in just a few succinct words, but they are visually bright.

You can gather that I really enjoyed

this novel. Thank you, Apostolos Doxiadis!

## CRM-Fields Prize 2002



*John B. Friedlander*

The Centre de recherches mathématiques and the Fields Institute are pleased to announce that Professor John B. Friedlander of the University of Toronto is the 2002 recipient of the annual CRM-Fields Prize.

Professor Friedlander is one of the world's foremost analytic number theorists, and is a recognized leader in the theory of prime numbers and L-functions. He received his B.Sc. from the University of Toronto in 1965, an M.A. from the University of Waterloo in 1966, and a Ph.D. from Penn State in 1972. He was a lecturer at M.I.T. in 1974-76, and has been on the faculty of the University of Toronto since 1980, where he served as Chair during 1987-91. He has also spent several years at the Institute for Advanced Study where he has collaborated with E. Bombieri and many others.

Friedlander is a Fellow of the Royal Society of Canada (1988), was an invited lecturer at the 1994 ICM in Zürich and delivered the CMS Jeffery-Williams Lecture in 1999. He has contributed significantly to mathematics in other ways, especially in Canada, through his role at NSERC (Mathematics GSC, 1991-94), as Mathematics Convenor of the Royal Society of Canada (1990-93), and as a Council member (1989-95) and Scientific Advisory Panel member (1996-2000) of the Fields Institute. He has served on the Editorial Board of the Canadian Journal of Mathematics and the Canadian Mathematical Bulletin for the past 4 years.

Friedlander's publications are vast in number, and many are published in journals such as the *Annals of Mathematics* and *Inventiones Mathematicae*. Perhaps his first truly striking result was published in *Acta Arithmetica* in 1976, "On the class numbers of certain quadratic extensions", in which he tackled the classical and difficult problem of computing class numbers of number fields. In this case he provides a computable lower bound for the class numbers of totally imaginary quadratic extensions of certain totally real number fields.

Another theme of his research has been the distribution of primes in arithmetic progressions, and some of his most significant work appears in three papers with Bombieri and Iwaniec (*Acta Math.* 1986, *Math. Ann.* 1987, and *J. Amer. Math. Soc.* 1989). Let  $q$  be a positive integer and  $a$  an integer relatively prime to  $q$ . For any real number  $x$ , let  $\pi(x, q, a)$  be the number of primes of the form  $a + nq$  which are  $\leq x$ . In the 19th century, Lejeune-Dirichlet proved that *all* primes  $\leq x$  are roughly equally distributed among the possibilities for  $a$  (of which there are essentially  $\phi(q)$  where  $\phi$  is the Euler totient function). Equivalently  $\pi(x, q, a)$  is asymptotic to  $\text{li}(x)/\phi(q)$  where  $\text{li}(x)$  is the logarithmic integral of  $x$ . Friedlander, Bombieri and Iwaniec investigated this more deeply by studying the sum of  $|\pi(x, q, a) - \text{li}(x)/\phi(q)|$  over the integers  $q$  below a given bound  $Q$  and over all  $a$ . They proved that the sum is small for values of  $Q$  which were larger than previously considered (successfully). Friedlander and Granville (*Annals of Math* 1989) subsequently proved that the primes are not uniformly distributed for different values of  $a$  if  $q$  is large enough – a shocking result to many number theorists. This also enabled them to disprove conjectures of Montgomery and of Elliot and Halberstam which had stood for more than 15 years.

Many number theorists consider the "crown jewel" of Friedlander's work to be the proof (with Iwaniec, *Annals of Math.* 1998) that there are infinitely many primes of the form  $x^2 + y^4$ . This result has been characterized as a "monumental breakthrough" in a featured review last year in *Math Reviews* because of the fact that the sequence  $x^2 + y^4$  is much "thinner" (less dense) than any sequence treated previously – the number of integers in this sequence up to  $N$  grows like  $N^{3/4}$ . In doing this they introduce radically new ideas into "sieve theory". These ideas were subsequently adapted by Heath-Brown and Moroz in finding the asymptotic distribution of primes represented by a general binary cubic form.

Friedlander's expertise in sieve methods and exponential sums has recently been applied to cryptography. In a paper with R. Canetti and I. Shparlinski (J. London Math. Soc. 1999), he proved that, if  $g$  is a primitive root of the prime  $p$  and  $x, y = 1, \dots, p-1$ , the triples  $(g^x, g^y, g^{xy})$  are uniformly distributed. This problem is of interest in cryptography where it is often assumed that such triples cannot be distinguished from random triples in feasible computation time. Their theorem implies that this is at least true for a constant fraction of the most significant bits, and for a constant fraction of the least significant bits. Two subsequent papers (with Pomerance and Shparlinski) were featured in a "popular" article by B. Cipra in the SIAM News last year.

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Le Centre de recherches mathématiques et le Fields Institute sont heureux d'annoncer que le Professeur John B. Friedlander de l'Université de Toronto reçoit le Prix annuel CRM-Fields pour l'année 2002.

Le Professeur Friedlander est l'un des meilleurs spécialistes de la théorie analytique des nombres et il est reconnu comme un chef de file dans la théorie des nombres premiers et des fonctions L. Il reçut son B.Sc. de l'Université de Toronto en 1965, un M.A. de l'Université de Waterloo en 1966 et un Ph.D. de Penn State en 1972. Après avoir enseigné au M.I.T. de 1974 à 1976, il s'est joint au Département de Mathématiques de l'Université de Toronto en 1980, et il en fut le directeur de 1987 à 1991. Il a aussi passé plusieurs années à l'Institute for Advanced Study, où il a collaboré avec E. Bombieri et plusieurs autres.

Friedlander est membre de la Société Royale du Canada. Il fut conférencier invité à l'ICM de Zürich en 1994 et prononça la Conférence Jeffery-Williams de la SMC en 1999. Il a également contribué de façon significative à la vie des mathématiques, tout particulièrement au Canada, par sa participation au Comité de sélection des subventions du CRSNG (1990-1994) et au Comité de candidature des nouveaux membres en mathématiques à la Société Royale du Canada (1990-1993). Il fut membre du Bureau de direction (1989-1995) ainsi que du Comité consultatif scientifique (1996-2000) du Fields Institute. Depuis 4 ans, il fait partie du Comité de rédaction du Journal canadien de mathématiques et du Bulletin canadien de mathématiques.

Friedlander a un vaste nombre de publications à son actif, dont plusieurs dans des journaux tels que *Annals of Mathematics* et *Inventiones Mathematicae*. Son premier résultat frappant, peut-être, fut publié dans *Acta Arithmetica* en 1976, "On the class numbers of certain quadratic extensions", où il s'attaquait au problème classique et difficile de calculer les nombres de classes des corps de nombres. Dans ce cas, il obtient une borne inférieure calculable pour les nombres de classes d'extensions quadratiques totalement imaginaires de corps de nombres totalement réels.

Ses recherches ont aussi porté sur la distribution des nom-

bres premiers dans des progressions arithmétiques, et certains de ses résultats les plus significatifs se trouvent dans trois articles conjoints avec Bombieri et Iwaniec (*Acta Math.* 1986, *Math. Ann.* 1987, and *J. Amer. Math. Soc.* 1989). Soient  $q$  et  $a$ , deux entiers premiers entre eux,  $q$  étant positif. Pour tout nombre réel  $x$ , soit  $\pi(x, q, a)$  le nombre de nombres premiers de la forme  $a + nq$  qui soient  $\leq x$ . Au 19<sup>ième</sup> siècle, Lejeune-Dirichlet prouva que *tous* les nombres premiers  $\leq x$  sont, en gros, distribués également parmi les valeurs possibles de  $a$  (dont le nombre, essentiellement, est donné par la fonction eulérienne  $\phi(q)$ ). De façon équivalente,  $\pi(x, q, a)$  est asymptotique à  $\text{li}(x)/\phi(q)$ , où  $\text{li}(x)$  est le logarithme intégral de  $x$ . Friedlander, Bombieri et Iwaniec approfondirent la question en étudiant la somme de  $|\pi(x, q, a) - \text{li}(x)/\phi(q)|$  sur les entiers  $q$  inférieurs à une borne  $Q$  donnée et sur tous les  $a$ . Ils prouvèrent que la somme est petite pour des valeurs de  $Q$  supérieures à celles qui avaient été considérées avec succès jusque-là. Friedlander et Granville (*Annals of Math.* 1989) prouvèrent par la suite que les nombres premiers ne sont pas distribués uniformément pour diverses valeurs de  $a$  si  $q$  est suffisamment grand – un résultat choquant pour plusieurs théoriciens des nombres. Ils purent aussi réfuter des conjectures de Montgomery ainsi que de Elliot et Halberstam, qui étaient ouvertes depuis plus de quinze ans.

Pour plusieurs théoriciens des nombres, le plus beau fleuron de la couronne de Friedlander est la preuve (avec Iwaniec, *Annals of Math.* 1998) qu'il y a un nombre infini de nombres premiers de la forme  $x^2 + y^4$ . Tenant la vedette dans une recension de *Math. Reviews* l'année dernière, ce résultat a été décrit comme une "percée monumentale" par suite du fait que la suite  $x^2 + y^4$  est beaucoup "plus mince" (moins dense) que toute suite considérée précédemment – le nombre d'entiers dans cette suite jusqu'à  $N$  croît comme  $N^{3/4}$ . Pour en arriver là, ils introduisirent des idées radicalement nouvelles dans la "théorie du crible". Ces idées furent adoptées subséquemment par Heath-Brown et Moroz pour trouver la distribution asymptotique des nombres premiers représentés par une forme binaire cubique générale.

L'expertise de Friedlander dans les méthodes de cribles et dans les sommes exponentielles a récemment été appliquée à la cryptographie. Dans un article avec R. Canetti et I. Shparlinski (*J. London Math. Soc.* 1999), il prouva que si  $g$  est une racine primitive du nombre premier  $p$  et  $x, y = 1, \dots, p-1$ , les triplets  $(g^x, g^y, g^{xy})$  sont distribués uniformément. L'intérêt de ce problème pour la cryptographie vient de ce que l'on suppose fréquemment qu'il est impossible de distinguer de tels triplets de triplets aléatoire en un temps de calcul réalisable. Leur théorème implique que c'est au moins vrai pour une fraction constante des bits les plus significatifs, et pour une fraction constante des bits les moins significatifs. Deux articles ultérieurs (avec Pomerance et Shparlinski) ont fait la une des *SIAM News* dans un article de vulgarisation de B. Cipra l'année dernière.

## RESEARCH NOTES

Noriko Yui, Column Editor

At the CMS Winter Meeting held in Toronto in December 2001, the Coxeter-James Lecture was delivered by Kai Behrend (UBC) and the Doctoral Prize Lecture by Nathan Ng (University of Georgia). Here are the citations that accompanied the awards.

**The 2001 CMS Doctoral Prize**  
**Le 2001 prix de doctorat du SMC**



*Nathan Ng*

Nathan Ng's Ph.D. thesis is entitled "Limiting distributions and zeros of Artin L-functions". This thesis is in the field of analytic number theory. In his thesis, Nathan extends and develops in a far-reaching fashion the analysis of the fine structure of the distribution of prime counting functions and of the Möbius functions.

The first part of his thesis extends to a non-abelian setting the work of Rubinstein and Sarnak on "prime number races" between congruence classes of primes. The second part of his thesis concerns the important function  $M(x)$ , the summatory function of the Möbius function, whose behaviour reflects properties of the distribution of the zeros of the Riemann zeta function. His results have been called "remarkable, surprising and unexpected".

Nathan Ng has mastered large areas of both analytic and algebraic number theory, and is very good at difficult computational problems. He is also a talented teacher, and won a departmental prize as a graduate teaching assistant.

La thèse de doctorat de Nathan Ng, intitulée «Limiting distributions and zeros of Artin L-functions», relève du domaine de la théorie analytique des nombres. Dans sa thèse, Nathan Ng fait une analyse poussée de la structure de la distribution des fonctions qui comptent les nombres premiers et de la fonction de Möbius.

Dans la première partie de sa thèse, il étudie l'environnement non-abélien des travaux de Rubinstein et de Sarnak sur les «races» de nombres premiers parmi les classes de congruence de nombres premiers. Dans la seconde partie, il aborde l'importante fonction  $M(x)$ , fonction sommatoire de la fonction Möbius, dont le comportement illustre les propriétés de la distribution des zéros de la fonction Riemann zeta. Ses résultats se sont valus les qualificatifs de «remarquables, surprenants et inattendus».

M. Ng maîtrise de grandes parties à la fois de la théorie analytique et de la théorie algébrique des nombres, et il excelle à la résolution de problèmes computationnels complexes. Il est de plus un professeur de talent et a remporté, à ce titre, un prix de son département quand il était assistant d'enseignement diplômé.

**The 2001 CMS Coxeter–James Lecturer**  
**La 2001 conférence Coxeter–James du SMC**



*Kai Behrend*

Kai Behrend is one of the world's leading experts in the theory of algebraic stacks and the geometry of moduli spaces



of stable maps, which has become one of the most important areas in algebraic geometry because of the unexpected predictions in enumerative algebraic geometry made by physicists based on string theory. In two Inventiones papers, one joint with B. Fantecchi, Kai used the theory of algebraic stacks to define the virtual fundamental class of a moduli space, which enables one to evaluate the Gromov-Witten invariants of the moduli space, and hence obtain enumerative results. According to a referee, "this realizes a program proposed by Deligne and Kontsevich and provides the foundation for a general counting procedure generalizing the usual technique of counting points in varieties over finite fields in the definition of zeta functions. One of the great challenges is now to combine counting procedures from GW theory and from finite fields to get a generating function with rich structural properties. Its arithmetic analogue will probably open the door to understanding the zeta functions controlling the numbers of integer solutions of systems of algebraic equations." These papers are among the most widely cited works in algebraic geometry over the last five years. Kai's recent work on differential graded stacks is an extremely important contribution to the construction of extended moduli spaces which are crucial to theory of mirror symmetry. Kai is also an author of a much anticipated joint research monograph on the cohomology of stacks, to be published by Springer-Verlag.

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Kai Behrend est un des experts mondiaux dans la théorie des champs algébriques et la géométrie du moduli des applications stables. Cette théorie est devenue l'un des domaines les plus importants de la géométrie algébrique en raison des prédictions inattendues en géométrie algébrique énumérative réalisées par des physiciens dans la théorie des cordes. Dans deux articles originaux, dont un écrit avec B. Fantecchi, Kai a utilisé la théorie des champs algébriques pour définir la classe virtuelle fondamentale d'un espace de moduli et par la suite obtenir des résultats énumératifs. Selon un examinateur, «ceci réalise un programme proposé par Deligne et Kontsevich et sert de fondement à une procédure générale de compte généralisant la technique usuelle de compter les points d'une variété sur des corps finis dans la définition des fonctions zeta. Un des grands défis est maintenant de combiner les techniques de compte de la théorie de GW et des corps finis pour obtenir une fonction génératrice avec de riches propriétés structurales. L'analogie arithmétique de cette fonction permettra probablement de comprendre, à la longue, les fonctions zeta qui contrôlent le nombre de solutions entières des systèmes d'équations algébriques.» Ces deux articles sont considérés parmi les travaux les plus cités dans le domaine de la géométrie algébrique depuis les cinq dernières années. Le travail récent de Kai sur les champs différentiels gradués est

une contribution très importante à la construction des espaces de moduli généralisés qui sont cruciaux pour la théorie de la symétrie du Miroir. Kai est aussi l'auteur de la monographie d'une recherche conjointe sur la cohomologie des champs, un travail qui sera publié par Springer-Verlag.

### CRM Update

The Centre de recherches mathématiques 2002-2003 thematic program will be in the Mathematics of Computer Science. It covers a whole range of areas in which mathematics and computer science interact: complexity, quantum computing, combinatorics and algorithmics, logic, cryptography, random numbers and machine learning. The three Aisenstadt chairs for the year are Manuel Blum (Carnegie Mellon), Laszlo Lovasz (Microsoft Research), Endre Szemerédi (Rutgers University).

In the meantime, the first session of this year's program in Groups and Geometry was in topology and geometry in low dimensions, a true success. A similar session on the Langlands program for function fields is planned for next April and May as well as many other workshops. Consult the CRM website for more details: [www.crm.umontreal.ca](http://www.crm.umontreal.ca)

**News:** This fall a new laboratory of the Network for computing and mathematical modelling was created: the Laboratoire Universitaire sur le Temps Extrême (LUTE). This laboratory is the fruit of an accord concluded with Environment Canada, which includes not only some very important credits for research as well as an important contribution of computing time, but also a major commitment of research personnel.

A special activity was held on January 18, 2002, at the CRM, in honor of Dr. André Aisenstadt who died last October. Three speakers took part in the event. Niky Kamran (McGill), first recipient of the André-Aisenstadt Prize in 1991, delivered a lecture entitled «L'opérateur de Dirac en géométrie de Kerr». Francis Clarke (Institut Universitaire de France et Université de Lyon), director of the CRM from 1984 to 1993 followed with his lecture entitled: «La conception de retours d'état (feedbacks) en théorie du contrôle: une introduction». Finally, Jingyi Chen (University of British Columbia), recipient of the André-Aisenstadt Prize 2001 gave the lecture: «Quaternionic mappings between hyperkähler manifolds».

**CRM Prizes:** The CRM-FIELDS Prize 2001 winner, William T. Tutte (University of Waterloo), gave a lecture on November 9 entitled "Some adventures in Graph Theory". Tutte works in graph theory and related areas of discrete math, and has done so for the last 60 years or so. The CRM-SSC Prize in Statistics went to Ms. Colleen Cutler (University of Waterloo). Dr. Cutler's work has had a very substantial impact in probability and statistics, but also in other areas of science.

## Whither Mathematics?

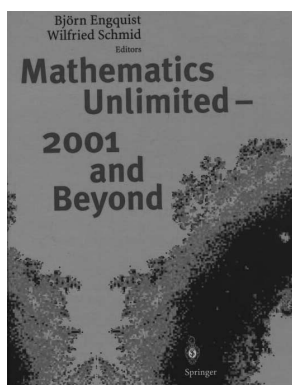
Book Review by Michael Barr, McGill University

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### Mathematics Unlimited—2001 and Beyond

by Björn Engquist and Wilfried Schmid, Editors  
Springer-Verlag New York 2001  
1237 pp

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It always irritates me to read a review in which the reviewer wishes the authors (or, in this case the editors) had produced a different book. Nonetheless, I will argue exactly that since they do not seem to have produced the book they claimed or even the book they intended. The preface is extremely brief (not over a half page) and not very informative. It essentially reduces to the statement that this anthology attempts to shed light on the questions of what are the important developments in present mathematics and where is it heading. In fact, only the first is addressed in any serious way, but that is fair since, as some sage said, prediction is hard, especially of the future.

What is the audience for the book? The editors bring up Hilbert's 23 problems as having set the agenda for mathematics of the 20th century and claim that their book is intended to do the same for the 21st. All I can say to that is that if that is what 21st century mathematics is going to be, I am glad my career was in the 20th. By my (necessarily arbitrary) count, there were 22

(of 62) articles in physics, engineering and applied math, seven each in computer science and number theory, six in geometry, five in probability and statistics, three in economics and finance, and two each in algebra and analysis. The remaining ones were either interviews, a catch-all that, for want of a better term, I call philosophy, or not mathematics at all. Imagine that, more articles on economics than on either algebra or analysis!

I do not know what instructions were given to the authors nor what editorial control was exercised, but the level of the articles range from highly technical and unreadable except by an expert to one horror on mathematics in the entertainment industry that was written in a celebratory mode but was, from a mathematical viewpoint, content-free. I learned from that article that rotations of space can be described by unit quaternions, but not how. (It is by conjugation, thinking of 3-dimensional vectors as purely imaginary quaternions and there is a kernel, the subgroup  $\{1, -1\}$ . There is now more mathematics in this review than in the article in question.)

The articles are not numbered and they are not classified, but rather sorted alphabetically by author's (first) name. I imagine I could find a more useless way of sorting them if I tried (by size, maybe). Since no one will want to (or be able to) read more than a fraction of the articles, a classification would have been very helpful. Notice how this contrasts with Hilbert's list of problems, whose statements, at least, were generally accessible to all the mathematicians of his day.

I could not read every article, nor report on each one if I had, but I attempted a few and I will discuss those below.

The first article I looked at was called, "Mathematics of Financial Markets" by Mark Davis. The article dis-

cusses some models of stock prices. I had always thought that a mathematical model of a real world situation should be constructed in something like the following way. Study the situation in sufficient detail to understand how it works. Choose relevant variables and write down, as a result of the studies, equations that describe the interactions. Solve them, exactly if possible, numerically if necessary, and compare them with reality. If they fit, then you suppose that you have captured the situation in sufficient detail and you can say that you have an explanatory theory and use it to make further predictions. If they don't fit, then go back to step one and refine your analysis. Judging from this paper, what you do is conjecture a conclusion, abandon it without discussing its accuracy and conjecture another conclusion until you find one that no one objects to. The first theory of stock price variations was that stock prices move by a process akin to Brownian motion. This led to certain predictions, although there is no discussion of how well it mirrors reality. The original model led to a Gaussian distribution of stock price movements. This model was eventually abandoned, not because it didn't work (that question is not discussed), but because the tail of the Gaussian distribution would allow negative stock prices and stock prices cannot be negative! Is this a serious objection? To me it seems fatuous, since while stock prices cannot be negative, that is a figment of the laws that created limited liability corporations and the value underlying a stock can certainly be negative (cf. the Enron Corp.). At any rate, this "model" was replaced by a new model that led to an exponential distribution of prices that could not go negative. Does it have any actually predictive value? That does not seem to be discussed. The point I am trying to make is that these models are *a priori* rather than based on an analysis of

the stock markets. In fact my reaction to the whole subject is summarized in, “Garbage in, garbage out”.

The next paper I looked at happened to be the immediate preceding paper, “Some open problems and research directions in the mathematical study of fluid dynamics”. Now this is genuine mathematics and, although I know nothing of the subject, I thought I might like to at least get an idea of how it worked. It started well enough with Navier-Stokes partial differential equation which is derived by identifying the relevant variables as pressure, density (assumed constant, at least as a first approximation) and velocity and applying Newton’s law. Another constant, viscosity, is used (although this depends on temperature, but I am not objecting to simplifying assumptions). The questions that concern him are the existence, uniqueness, and regularity of the solutions of these equations and the rest of the article is concerned with these questions. I would have liked to have seen a discussion of what failure of these questions actually means. Does lack of

regularity imply turbulence or is something else involved? Since real systems lead to real behaviour, what could non-existence of solutions mean? If the solutions turn out to exist but be non-unique, does that mean that the physical system is indeterminate or that the equations have been oversimplified? Instead what we get are technical details that are likely to interest only experts for whom this survey is unnecessary.

The next article I attempted was “From finite sets to Feynman diagrams” by John Baez and James Dolan. Baez, a cousin of the famous folk singer, is well known for his regular internet posting, “This week in physics”, although his degrees and current affiliation are in mathematics. I was expecting something in physics. To my surprise what he wrote was an introduction to category theory. It begins with a clear and convincing—to me—explanation of why category theory ought to be interesting to mathematicians. He then develops a bit of the theory, including some ideas that

will be new to most people. The whole article seems both clear and accessible, without omitting anything essential. It may not be the only such essay in this volume, but it is the only one I have read so far.

The last article I will mention explicitly is that by Marie-Françoise Roy, “Three problems in real algebraic geometry and their descendants.” If you are going to have a collection of this kind, this is pretty much a model article. It takes three problems (the first being the Hilbert problem of showing that every real polynomial in several variables that is never negative is a sum of squares of rational functions) and states them, discusses the motivation, gives a sketch of the mathematics involved in solving them and discusses further questions along the same lines. The article has mathematical content to be sure, but is not overly technical and explains why the questions are interesting and even how the future might develop. If only the other articles had been like this.

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**UNIVERSITY OF SASKATCHEWAN – SASKATOON, SASKATCHEWAN**  
**DEPARTMENT OF MATHEMATICS AND STATISTICS**  
**Position in Mathematics or Statistics**

The Department of Mathematics Statistics at the UNIVERSITY OF SASKATCHEWAN invites applications for two term positions in all areas of Mathematics and Statistics at the Assistant Professor level starting July 1, 2002. We are searching for individuals who have demonstrated exceptional promise in establishing vigorous independent research programs and who will have a commitment to undergraduate and graduate teaching. The department’s web page can be found at <http://math.usask.ca/>. Applications, including a curriculum vitae and descriptive statements of research plans and teaching activities, should be sent by **March 31, 2002** to:

**Department of Mathematics and Statistics,  
 University of Saskatchewan,  
 Room 142 McLean Hall,  
 106 Wiggins Road,  
 Saskatoon, SK, S7N 5E6  
 Email: [math@sask.usask.ca](mailto:math@sask.usask.ca).**

Please arrange for letters of reference to be sent, in confidence, from three referees. The positions are subject to final budgetary approval.

The University of Saskatchewan is committed to Employment Equity. Members of Designated Groups (women, aboriginal people, people with disabilities and visible minorities) are encouraged to self-identify on their application. In accordance with Canadian immigration requirements, this advertisement is directed to Canadian citizens and permanent residents in the first instance. However, this position has been cleared for advertising at the two-tier level. Applications are invited from qualified individuals, regardless of their immigration status.

## EDUCATION NOTES

Ed Barbeau and Harry White, Column Editors

### Outreach from the Institut des sciences mathématiques

The *Institut des sciences mathématiques* is a Québec consortium of several universities, Concordia, Laval, McGill, Montréal, UQÀM and Sherbrooke. Its activities, described on the website [www.math.uqam.ca/ISM/](http://www.math.uqam.ca/ISM/), includes educational as well as research-oriented activities. It sponsors not only a weekly seminar at which graduate students can present the fruits of their research “dans une ambiance détendue et amicale”, but also a series over a dozen lectures given by members of constituent universities each year to enable teachers in colleges to gain an appreciation of recent developments in mathematics.

### A report on the mathematical education of teachers

*The mathematical education of teachers*, Conference Board of the Mathematical Sciences, Issues in Mathematical Education, Volume 11. AMS & MAA, 2001. ISBN 0-8218-2899-1.

<http://www.maa.org/cbms>

This report, produced under the auspices of a nine member steering committee chaired by James Lewis and written by Alan Tucker, James Fey, Deborah Schifter and Judith Sowder, was prepared for the US Conference Board of the Mathematical Sciences. It sets out recommendations for the formation of K-12 mathematics teachers, emphasizing that there is intellectual structure in school mathematics and that the mathematical knowledge required for teaching has special characteristics. Consistent with the NCTM *Principles and Standards for School Mathematics*, it is intended to be read by leaders and members of mathematics departments and faculties of education, by educational policy makers at the board and provincial levels, mathematics organizations, funding bodies and accreditation and certification organizations.

After a brief introductory chapter that questions the assumption the elementary teachers learn all the mathematics they need in their own schooling and indicates that issues of day-to-day instruction require deeper levels of understanding, the authors lay out their 10 core recommendations:

1. *Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.*

2. *Although the quality of mathematical preparation is more important than the quantity, the following amount of mathematics coursework for prospective teachers is recommended:*

(i) *Prospective elementary grade teachers should be required*

*to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.*

(ii) *Prospective middle grades teachers of mathematics should be required to take at least 21 semester-hours of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.*

(iii) *Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics, that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics.*

3. *Courses on fundamental ideas of school mathematics should focus on a thorough development of basic mathematical ideas. All courses designed for prospective teachers should develop careful reasoning and mathematical “common sense” in analyzing conceptual relationships and solving problems.*

4. *Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching.*

5. *Teacher education must be recognized as an important part of mathematics departments’ mission at institutions that educate teachers. More mathematicians should consider becoming deeply involved in K-12 mathematics education.*

6. *The mathematical education of teachers should be seen as a partnership between mathematics faculty and mathematics education faculty.*

7. *There needs to be greater cooperation between two-year and four-year colleges in the mathematical education of teachers.*

8. *There needs to be more collaboration between mathematics faculty and school mathematics teachers.*

9. *Efforts to improve standards for school mathematics instruction, as well as for teacher preparation accreditation and teacher certification, will be strengthened by the full-fledged participation of the academic mathematics community.*

10. *Teachers need the opportunity to develop their understanding of mathematics and its teaching throughout their careers, through both self-directed and collegial study, and through formal coursework.*

11. *Mathematics in middle grades (grades 5-8) should be taught by mathematics specialists.*

The remainder of the report provides specific recommendations for the preparation of K-4, 5-8 and 9-12 teachers in

separate pairs of chapters, one summary and the other detailed. These are accompanied by vignettes mostly from classrooms to bring into focus the role of the university instruction.

In the early K-4 years, investigations and problems should be extensively employed so that mathematics makes sense to the pupils. Accordingly, their teachers need to “reconnect with their own capacities for mathematical thought”. They must be able to deconstruct their mathematical knowledge into elemental components both accessible and visible. Courses for intending teachers cover numbers and operations, algebra and functions, geometry and measurement, as well as data analysis, statistics and probability. I will let the reader check the website for details and just make a few observations. A central idea in the elementary curriculum is ratio and proportion, and already by Grade 4, pupils may have been exposed to rates, prices, units, scale and linear formulae. So the concept needs to be explored thoroughly and the notion of linearity highlighted with prospective teachers, more than the recommendations seem to allow. Quite a lot is said about fractions in the report, and it is surprising that the number line does not make an appearance. A tape measure is within the experience of many children, and it seems that placing fractions along a number line and interpreting addition as a concatenation of lengths might be a productive way of securing the concept of fractions. Indeed, it might be less fraught with peril than using the standard pizza model or sets of individual objects, where confusion may arise about the unit. While the vignettes indicate that children are involved in reasoning, mathematical reasoning seems to be but a small byproduct of the suggested regime in university. A program for prospective teachers at any level must focus more on reasoning and analysis, even if it is not done formally. One vehicle to achieve this would be to include recreational mathematics and some traditional tricks and puzzles, a step with the useful byproduct of inducting teachers into mathematical lore and history.

Pupils in the middle school should be taught by mathematics specialists with a firm base of knowledge and sense of the connections between elementary and advanced topics. The formation of such teachers should emphasize making sense of mathematics, reasoning, proportionality and variation. They should study four main areas: (1) numbers and operations, including the number line, fields axioms, prime factorization and common divisor; (2) algebra and functions, including characteristics of linear, quadratic and exponential functions, and qualitative graphing; (3) measurement and geometry, including the use of software for exploration; (4) data analysis, statistics and probability. These can be augmented by courses in calculus focussing on concepts and applications, discrete mathematics, history and linear algebra. In the report, the discussion on reasoning seems to emphasize specific types, such as proportional, spatial, statistical, and

not give enough attention to the broader issue of pursuing an argument and drawing conclusions. This could be remedied by having a more open-ended course in problem-solving that might include not only recreational mathematics, but introduce combinatorics and graph theory. It would also be useful for teachers at this level to have some exposure to physics and chemistry, which would provide a good context for dealing with variability and proportionality.

The report urges that the education of high school teachers “should develop: (1) deep understanding of the fundamental ideas in grades 9-12 curricula and strong technical skills for application of those ideas; (2) knowledge of the mathematical understandings and skills that students acquire in their elementary and middle school experiences, and how they affect learning in high school; (3) knowledge of the mathematics that students are likely to encounter when they leave high school for collegiate study, vocational training or employment; (4) mathematical maturity and attitudes that will enable and encourage continued growth of knowledge in the subject and its teaching ... Core mathematics major courses can be re-designed to help future teachers make insightful connections between the advanced mathematics they are learning and the high school mathematics they will be teaching. Mathematics departments can support the design, development and offering of a capstone course sequence for teachers in which conceptual difficulties, fundamental ideas and techniques of high school mathematics are examined from an advanced standpoint.” These should involve a collaboration of education and mathematics faculty. The capstone courses would treat algebra and number theory, geometry and trigonometry, functions and analysis, data analysis, statistics and probability, as well as discrete mathematics and computer science. The report warns that didactic research indicates that increasing the mathematical intensity of the undergraduate programme could be counterproductive, so that emphasis on subject matter should be accompanied by similar emphasis on pedagogy, fostering of deep understanding, and sense of the historical and scientific roots of mathematics.

There is much worthwhile discussion in the long chapter on the preparation of secondary teachers. The increased emphasis on geometry and history is welcome, and there is a brief section on the tertiary learning environment and recent pedagogical experiments there. But it is probably worth explicitly discussing strategies for learning. Retention of mathematical ideas depends on the learner summarizing the knowledge in her own way and developing a worldview; this is not an ability that many students naturally have, and we need to explicitly address how this can be fostered. Finally, there is a largely missing dimension, exemplified by the writings of Martin Gardner, and that is the occasion in the formation of teachers for them to get the joy and experience of looking at nice mathematical problems that should inform the culture of every teacher of the subject.

This section of the *Notes* is open to reactions from colleagues who have studied the report, and wish to comment on the views expressed in it or in this review. It provides a solid basis upon which we can hone our own ideas and develop our policies.

*Ed Barbeau*

### More articles from IJMEST

In December and February, I presented summaries of some articles from Volume 31, Number 1 (January-February, 2000) of *International Journal of Mathematical Education in Science and Technology*. Here are three more.

*The student experience of mathematical proof at university level*, Keith Jones, England. "The aim of the paper is to review what is known of the student experience of mathematical proof at this level and to provide some evidence of the conceptions of mathematical proof that recent graduates bring to the postgraduate course to teach high school mathematics." Researchers in mathematics agree that many undergraduates have trouble constructing proofs and appreciating their import, even though they might be able to reproduce a formally taught proof. The analytic tool used by the author is the *concept map*, pioneered by J.D. Novak and B. Gowin, where the student subjects (75 secondary teacher trainees) treat as nodes on a graph concepts they identify together as pertinent to proof and join by edges those that are connected. Examples of concept maps are provided and discussed. While there was a rough correlation between the grade standing of the students and the complexity of their maps, there was evidence that some of the less qualified had a richer appreciation of the connections and the potential to be better teachers. The author identifies possible causes in university courses that might not engage students in questions of significance and context, and of the failure of most students to "grasp mathematics as a field of intricately related structures".

*Asking mathematical questions mathematically*, John Mason, England. Lecturers and tutors are urged to carefully attend to the style and format of questions they ask students, as they may in fact frustrate their intended purposes. Questions should be orchestrated so that students become sensitized to the mathematical structure and context, and begin to formulate questions on their own. Core mathematical themes, such as invariance amidst change, doing and undoing, freedom and constraint, should be explicitly engaged. As examples, the author discusses how the following two problems can be used to generate deeper mathematical questions: (1) *Find  $\int x \cos ax dx$  using integration by parts*; (2) *Find the extreme values of  $f(x) = x^5 + x - \sqrt{x+1}$  on the interval  $[0, 1]$ .*

*Assessment of learning in university mathematics*, by Geoff Smith & Leigh Wood, Australia. The teaching that leads to qualitative changes in the demeanour of learners and encourages them to understand reality in a new way requires support from our methods of assessment. Students are capable of changing their approach to learning from superficial to profound. The authors have developed a taxonomy, *Mathematical Assessment Task Hierarchy*, (MATH), that classifies tasks by the activity required to complete them successfully rather than by difficulty. Sample tasks are: (1) *to evaluate the arguments of two students as to whether the matrix equation  $MAX = MB$  has the same solutions as  $AX = B$* ; (2) *to compare the method of undetermined coefficients with the method of variation of parameters in solving second order differential equations*; (3) *to study the effect on the solution graph of changing the parameters in a boundary value problem with a second order differential equation*; (4) *analyzing and correcting typical errors of first year students*. A brief discussion of the effects of changes in assessment, of the effects of external factors such as technology, and of barriers to change, is given.

*Ed Barbeau*

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## Online Mathematics Textbooks

George Cain, Georgia Institute of Technology, maintains a website [www.math.gatech.edu/cain/textbooks/onlinebooks.html](http://www.math.gatech.edu/cain/textbooks/onlinebooks.html) which he introduces as follows:

The writing of textbooks and making them freely available on the web is an idea whose time has arrived. Most college mathematics textbooks attempt to be all things to all people

and, as a result, are much too big and expensive. This perhaps made some sense when these books were rather expensive to produce and distribute—but this time has passed. Here follow links to the online mathematics texts of which I am aware. I urge everyone to let me know of others in order that I may add them to the list.

*- noted in the AustMS Gazette, Oct 2001*

## UPCOMING CONFERENCES

### ICM 2002 Update

Zhiming Ma, President of the Chinese Mathematical Society and Chairman of the Local Organizing Committee for ICM 2002, has announced that

- the Second Announcement is available and has been mailed to all pre-registrants,
- registration is now open, and
- short communications and posters may now be submitted.

For details please look under "General Information" at the ICM 2002 website [www.icm2002.org.cn](http://www.icm2002.org.cn). The lists of the 20 plenary and 169 sectional speakers are also available there. The plenary speakers are: Noga Alon, Tel Aviv University, Israel

Douglas Norman Arnold, Institute for Mathematics and its Applications, University of Minnesota, USA

Alberto Bressan, S.I.S.S.A. Italy

Luis Angel Caffarelli, University of Texas at Austin, USA

Sun-Yung Alice Chang, Princeton University, USA

David Leigh Donoho, Stanford University, USA

Ludwig Dmitrievich Faddeev, Steklov Mathematical Institute, Russia

Shafi Goldwasser, MIT Laboratory for Computer Science, USA ; Weizmann Institute of Science, Israel

Uffe Haagerup, University of Southern Denmark, Denmark

Michael Jerome Hopkins, MIT, USA

Victor Kac, Massachusetts Institute of Technology, USA

Harry Kesten, Cornell University, USA

Frances Clare Kirwan, Mathematical Institute, University of Oxford, United Kingdom

Laurent Lafforgue, Institut Des Hautes Etudes Scientifiques (IHES), France

David B. Mumford, Brown University, USA

Hiraku Nakajima, Kyoto University, Japan

Yum-Tong Siu, Harvard University, USA

Richard Lawrence Taylor, Harvard University, USA

Gang Tian, MIT, USA; Peking University, China

Edward Witten, Institute for Advanced Study, School of Natural Sciences, USA

### Banach Algebras and their Applications – Banach Algebras 2003

Edmonton, Alberta - July 27 to August 9, 2003

Organizers: Anthony To-Ming Lau and Volker Runde

This conference is the sixteenth in a series of conferences on Banach algebras that started 1974 in Los Angeles. We expect that most specialists in Banach algebras as well as leading mathematicians from related areas will attend this conference. In the past, these conferences have always led to fruitful inter-

action between the participants, and we expect this tradition to continue.

In addition to the regular conference program consisting of one hour and half hour talks by the participants, we also plan to hold five workshops on the following topics, each of which will be chaired by an internationally recognized specialist in the respective area:

\* Banach algebras in harmonic analysis (to be held in the honor of Eberhard Kaniuth on the occasion of his retirement)  
Chair: Anthony To-Ming Lau (Edmonton).

\* Banach algebras in operator theory  
Chair: Michael M. Neumann (Starkville; USA).

\* Banach algebras and operator spaces  
Chair: Zhong-Jin Ruan (Urbana-Champaign; USA).

\* K-theory of Banach algebras  
Chair: Joachim Cuntz (Muenster; Germany)

\* Topological homology.  
Chair: Alexander Ya. Helemskii (Moscow; Russia).

Each workshop will occupy two afternoons. The chairs are completely free to decide on the format of their workshops.

For more detailed information - including a list of invited speakers -, see the conference website at <http://www.math.ualberta.ca/ba03/>.

MAKING MATHEMATICAL CONNECTIONS:  
2002 & BEYOND

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NCTM Canadian Regional Conference

17 – 19 October 2002  
Regina, Saskatchewan

Join us for the upcoming NCTM conference. This is an opportunity for professional development for all levels of mathematics educators. Latest research, resources, ideas, networking and more to help us educate our students for the 21<sup>st</sup> Century. Come and experience what the National Council of Teachers of Mathematics' Canadian Regional Conference can bring to your teaching.



Hosted by the Saskatchewan Mathematics Teachers' Society. Further information will be available in mid April on our web sites: <http://www.nctm.org/> and <http://mathcentral.uregina.ca/SMTS/>

### Invited speakers for ICIAM 2003

The names of invited speakers for the 5th International Congress on Industrial and Applied Mathematics, to be held 7-11 July 2003 in Sydney, Australia, have been announced, as follows:

Brian Anderson, Australia

Yann Brenier, France

Jennifer Chayes, USA

James Demmel, USA  
 David Donoho, USA  
 Alice Guionnet, France  
 Jonathan Keating, UK  
 Nancy Kopell, USA  
 Peter Markowich, Austria  
 Harald Neiderreiter, Singapore  
 George Papanicolaou, USA  
 Phillipe Toint, Belgium  
 Henk van der Vorst, Netherlands  
 Vladimir Zakharov, Russia/USA  
 Marsha Berger, USA  
 Franko Brezzi, Italy

Mark Davis, UK  
 Peter Deuffhard, Germany  
 Yoshikazu Giga, Japan  
 Tom Hou, USA  
 Rupert Klein, Germany  
 Tom Leighton, USA  
 Alexander Mielke, Germany  
 Michael Ortiz, USA  
 Neil Sloane, USA  
 rnie Tuck, Australia  
 Ying Lung-an, China  
 Further details are available at the congress website  
[www.iciam.org](http://www.iciam.org)

## Endowment Grants Committee

Reports on projects supported in 2001  
 Chair, James Timourian, University of Alberta

We received 12 applications for the 2001 Endowment Grants Competition. A total of 8 were received using our online HTML form. The remaining applications were received as e-mail attachments in either Word or LaTeX.

The total amount applied for was \$130,800 and the Endowment Grants Committee (EGC) was allotted \$60,000 to grant. Unlike in past years (1999 and 2000) all of the money allocated was awarded.

The EGC made an unusual decision in the case of one application: we decided to make an award for the second year of the program, subject to additional financing being obtained for the first year.

Applicants have been unofficially notified of the results by e-mail, and official letters will be sent from the Executive Office in Ottawa. Successful applications can be viewed on the web at <http://www.cms.math.ca/Grants/EGC/>.

The Endowment Grants Competition is now three years old and the enabling resolutions called for a review at this time. The new Chair of the Endowment Grants Committee is Kathryn Hare; the ex-Chair will briefly report early in the New Year on the Committee's experience of the past three years, as part of the review process.

Once a proposal is funded the applicants are obligated to report on how things turned out. Not all of the reports that were due in 2001 have been received, but once received they will appear on the web as well as extracts published in the CMS Notes.

Nous avons reçu 12 demandes dans le cadre du Concours de bourses du fonds de dotation 2001. Huit nous sont parvenues par le formulaire Web et les quatre autres, en annexe à des messages de courrier électronique, en format Word ou LaTeX.

Le total des demandes s'élevait à 130 800 \$, et le CABFD avait 60 000 \$ à distribuer. Contrairement aux années passées (1999 et 2000), la totalité de la somme a été attribuée.

Le CABFD a pris une décision inhabituelle dans un des cas. Les membres du Comité ont en effet décidé d'accorder une bourse pour la seconde année du programme à la condition que le candidat en question obtienne du financement supplémentaire pour la première année.

Les candidats ont été informés officieusement des résultats (par courriel), et le bureau administratif d'Ottawa se chargera d'envoyer les lettres officielles. Les demandes acceptées sont publiées sur le Web au <http://www.smc.math.ca/Grants/EGC/2001res.f>

Le programme de bourses du fonds de dotation en est maintenant à sa troisième année. Conformément à sa résolution de formation, le comité doit procéder à un examen à ce moment-ci. La nouvelle présidente du Comité est Kathryn Hare; l'ancien président présentera, au début de l'année, un bref rapport des activités du Comité au cours des trois dernières années, dans le cadre du processus d'examen.

Mentionnons qu'une fois un projet subventionné, les candidats doivent produire des rapports d'étape. Les rapports de 2001 ne sont pas tous arrivés comme prévu, mais dès qu'ils arriveront, nous les publierons sur le Web et en ferons paraître des extraits dans les Notes de la SMC.



## Best Approximation in Inner Product Spaces

Book Review by Thomas Ransford, Université Laval

### Best Approximation in Inner Product Spaces

by Frank Deutsch  
Springer-Verlag New York 2001



I can still remember being surprised and delighted to discover, back in high school, that drawing the ‘best’ straight line to fit a set of data was not a matter of artistic skill, but a mathematically precise problem with a simple, elegant solution. This problem, widely known as least-squares regression, is an elementary example in the theory of best approximation in inner product spaces.

The general set-up is as follows. Let  $X$  be an inner product space, with the norm defined as usual in terms of the inner product as  $\|x\| = \langle x, x \rangle^{1/2}$ . Given a subset  $K$  of  $X$  and a point  $x \in X$ , we seek a point  $y_0 \in K$  such that  $\|y_0 - x\| = \min_{y \in K} \|y - x\|$ . Such a point  $y_0$ , if it exists, is called a *best approximation* to  $x$  from  $K$ . It turns out that many approximation problems can be reformulated in these terms.

Here are some of the questions that arise. Given  $x$  and  $K$ , does a best approximation  $y_0$  exist? If so, then is it unique? How does one recognize it? Are there useful algorithms for finding it? How does it vary as  $x$  varies? Can

one readily compute the error of approximation  $\|y_0 - x\|$ , or, if not, then at least estimate it?

To answer these questions, it is necessary to develop a certain number of functional analytic tools. For example, the problem of existence and uniqueness of best approximation leads one rapidly into the theory of convex sets, linear functionals and duality. Further considerations require the open mapping and closed range theorems. Although much of this background material is standard, the approach adopted ensures that it is particularly well motivated.

The book divides roughly into two parts. The first eight chapters treat the easier questions, including best approximation from subspaces, half-spaces and cones. Here there are often explicit formulas available with interesting applications. Two of the highlights are Müntz’s beautiful extension of the Weierstrass approximation theorem for polynomials, and a nicely motivated exposition of the theory of generalized inverses of matrices.

The last four chapters are more advanced, and contain mostly material which has not previously appeared in book form. The highlight here is surely Dykstra’s algorithm, an ingenious and far-reaching generalization of von Neumann’s method of alternating projections. Suppose that  $K_1, \dots, K_r$  are closed, convex subsets of a Hilbert space (i.e. a complete inner product space) and that one already has a method of computing the best approximation from each  $K_i$ . Dykstra’s algorithm is an iterative method which always converges towards the best approximation from  $\bigcap_1^r K_i$ . For example, it allows us to compute the best approximation from a polyhedron, because a polyhedron is just a finite intersection of half-spaces and, from earlier in the book, we already have a formula for the best approximation from a half-space.

The book concludes with an interesting open problem which dates back to 1961. Let  $K$  be a subset of Hilbert space  $H$ . It is called a *Chebyshev set* if each point of  $H$  has a unique best approximation from  $K$ . We would like to characterize such sets. It is proved early on that every closed, convex subset of  $H$  is Chebyshev. In the other direction, it is easy to see that if  $K$  is Chebyshev then it must be closed in  $H$ . Question: must it also be convex? It is shown that the answer is yes if  $H$  is finite-dimensional, and, more generally, if  $H$  is arbitrary and  $K$  is *boundedly compact* (i.e. the intersection of  $K$  with each closed ball is compact). The general case, however, remains a subject for current research.

So much for what is in the book: now for a few words about what is not. The author restricts himself to inner product spaces, as opposed to arbitrary normed spaces. This means that the geometry is easier, but one misses out on topics such as best uniform approximation and best  $L^1$ -approximation, both of practical interest and both with nice theories attached to them. This is deliberate: the book is intended to be an introduction, and references are given for more advanced topics.

I myself would have been tempted to go even further, and consider only *complete* inner product spaces. Indeed, the author himself suggests as much as a means of shortening the time needed to cover the material. In fact, if I have a complaint about the book, it is that too much prominence is given to incomplete spaces, and that in any case many of the results would be more transparent if one worked in a completion. To give one (not very important) example: the ‘key lemma’ 6.6, apparently an inequality about linear functionals, can also be viewed as an inequality about inner products in the completion. As such it is much easier to prove, and one sees easily that the best constant is  $\sqrt{2}$

and not 8.

This quibble aside, I found the book informative and easy to follow. The author says in the preface that he has tried to err on the side of including too much detail, so that the book might prove

valuable for self-study. Each chapter concludes with about twenty exercises, as well as some interesting historical notes (did you know that Schwarz established his inequality in a paper where he studied the problem of a vi-

brating membrane?). At over 100\$, the book is perhaps too pricey to buy for a casual read, but every university library should have a copy.

(*JW-LECTURE continued from page 4*)

$\{A(L, M) = 0\}$  one will have a unique solution  $(z_1, \dots, z_n)$  of (7 $\dagger$ ) and hence a well-defined function  $V(L, M) = \sum_{j=1}^n D(z_j)$ . (The extension of  $V(L, M)$  to the singular points of the variety  $\{A(L, M) = 0\}$  is one of the matters that is best omitted in this sketch).

Then, using a classical formula of Schläfli concerning the volume of infinitesimally deformed polyhedra, one shows that

$$dV = -2 \left( \log |L| d(\arg M) - \log |M| d(\arg L) \right). \tag{9}$$

This important result is due to Hodgson [Ho] with improvements by Dunfield [Du]. Given (9), one can begin to believe that this has something to do with Mahler’s measure since if one restricts  $M$  to the unit circle  $M = e^{i\theta}$  then  $dV = -2 \log |L| d\theta$ .

**The Mahler measure of A-polynomials**

Using (9) and Jensen’s formula, one finds for any irreducible factor  $A_0(L, M)$  of  $A(L, M)$  that one has

$$2\pi m(A_0(L, M)) = \sum_{\rho} V(\rho), \tag{10}$$

where  $\rho$  runs over a finite set of solutions of (7 $\dagger$ ). So each of the terms  $V(\rho)$  is a sum of dilogarithms  $\sum_{j=1}^n D(\xi_j)$ , where the  $\xi_j$  are algebraic numbers. The  $\xi_j$  may be real, in which case  $D(\xi_j) = 0$ , or have  $Im \xi_j < 0$  in which case the tetrahedron  $\Delta(\xi_j)$  is negatively oriented. So  $\rho$  describes a *pseudo-triangulation* of  $X$ .

The formula (10) thus expresses  $m(A_0)$  as an explicit sum of dilogarithms of algebraic numbers. This is in itself remarkable – for general polynomials, one would not expect such a representation. For example, the families of polynomials studied in [Bo2,RV] have  $m(A)$  a rational multiple of  $L(E_A, 2)/\pi^2$ , where  $E_A$  is an elliptic curve. The key here is that  $dV$  is an exact form.

The use of the symbol  $\rho$  in (10) reminds us of the fact that these solutions of (7 $\dagger$ ) correspond to certain representations of the fundamental group  $\pi_1(X)$  into  $PSL(2, \mathbb{C})$ . This is the point of view adopted in [CCGLS] where the A-polynomial is first defined.

In particular, there will be a factor  $A_0$  of  $A$  corresponding to the geometric solution  $\rho_0$  of (7), (i.e. with  $L^2 = M^2 = 1$ ), and one or more of the terms in (10) will be  $V(\rho_0) = vol(X)$ .

If we are so lucky that (10) contains only a single term, then we will have our desired formula

$$2\pi m(A_0) = vol(X). \tag{11}$$

For knot complements and many other manifolds one finds that  $A_0(L, M) = B(L, M^2)$  and then there are two equal terms in (10) corresponding to  $M = \pm 1$ . If there are no other terms then, instead of (11), one obtains

$$\pi m(A_0) = \pi m(B) = vol(X). \tag{12}$$

**Example 1 - the figure eight knot complement** For example, the figure eight knot  $4_1$  has a complement that was shown by Riley to be a hyperbolic manifold  $X = \mathbb{H}^3/\Gamma$  with  $\Gamma$  an index 12 subgroup of  $PSL(2, O_3)$  [Mi]. Thurston [Th1,Th2] showed that  $X$  can in fact be triangulated by 2 regular ideal tetrahedra, (i.e. with shapes  $\zeta_6 = (1 + \sqrt{-3})/2$ ), so  $vol(X) = 2D(\zeta_6) = 2\pi d_3 = 12\pi d_3/6$ .

Here  $A(L, M) = B(L, M^2)$  where

$$B(x, y) = -y^4 + x(1 - y - 2y^2 - y^3 + y^4) - x^2.$$

In this case (10) has two equal terms so (12) applies and we have

$$\pi m(B(x, y)) = vol(X) = 2\pi d_3, \tag{13}$$

so  $m(B(x, y)) = 2d_3$ . Of course  $B(x, y)$  is not nearly as simple as Smyth’s example  $1 + x + y$  but it does show that the appearance of  $d_3$  in both (2) and (5) is not a coincidence. The formula (13) had in fact been conjectured on the basis of numerical evidence in [Bo2]. It was the coincidental appearance of  $B(x, y)$  in [CCGLS] that led to this entire line of investigation.

**Example 2** In the Hildebrand–Weeks census [HW] of hyperbolic manifolds triangulated by at most 7 tetrahedra, there is a manifold m009 made up of three tetrahedra with shapes in the field  $\mathbb{Q}(\sqrt{-7})$ . As in Example 1, the A-polynomial is  $A(L, M) = B(L, M^2)$ , where

$$B(x, y) = x^2y + x(-y^3 + 2y^2 + 2y - 1) + y^2.$$

The group  $\Gamma$  is a subgroup of  $PSL(2, O_7)$  of index 3 and there are two equal terms in (10) so we have  $m(A) = m(B) =$

$d_7/2$ . This is quite a bit simpler than Ray’s example in [Ra] but not as simple as some of the examples in [BRV].

**Example 3** Another example from the Hildebrand–Weeks census, there is a two-cusped manifold called m412 that can be triangulated with 5 regular ideal tetrahedra. Hence  $vol(m412) = 5D(\zeta_6) = 5\pi d_3$ . The computation of  $A(L, M)$  is not particularly simple, but nevertheless the final result is very simple:  $A_0(L, M) = B(L, M^2)$ , where

$$B(x, y) = (y + 1)x^2 + (-y^2 + 6y - 1)x + y(y + 1). \tag{14}$$

Again (12) applies and we obtain  $m(B) = 5d_3$ , a result also conjectured in [Bo2]. By a remarkable coincidence, at about the same time as (14) was computed and  $m(B) = 5d_3$  established by this method, Rodriguez-Villegas gave a direct proof by directly writing  $dV$  explicitly as a sum of 10 terms of the form  $dD(f_j)$  and Bertin [Be] derived the result by using the methods of [RV]. So now we have three distinct proofs of this result.

**Example 4** However, we are not always so fortunate. For example, the manifold v3551 is the largest volume manifold in the census. It can be triangulated with 7 regular ideal tetrahedra so  $vol(v3551) = 7D(\zeta_6) = 7\pi d_3$ . Here  $A_0(L, M)$  is of degree  $12 \times 12$  in  $(L, M)$  and is not even in  $M$ . But there are 4 terms in (10) so that  $2\pi m(A_0) = \sum_{k=0}^3 V(\rho_k)$ . Here  $V(\rho_0) = 7d_3$ , but  $V(\rho_k) = \pi Z_F/2$ , for  $k = 1, 2, 3$ , where  $F$  is the quartic field of discriminant  $-1192$ . So we have

$$m(A_0) = 72d_3 + 34Z_F \tag{15}$$

In fact, while we know that the first term in (15) is exactly  $7d_3/2$ , we only know that the second term is of the form  $rZ_F$  for some rational  $r$  with  $|r - 34| < 10^{-50}$ . This curious state of affairs is due to the use of a theorem of Borel as in [BRV] to treat the terms  $V(\rho_k)$  with  $k = 1, 2, 3$ .

**Some new cases of Chinburg’s conjecture**

Darren Long and Alan Reid kindly provided me with A-polynomials for the full Bianchi manifold  $\mathbb{H}^3/PSL(2, O_f)$  for the cases  $f = 11, 19$  and  $23$ . For  $f = 23$ , (12) applies so

$$m((x-x^3+x^5)(y^2+1)+(1-6x+12x^3-6x^5+x^6)y) = 16d_{23},$$

producing the first proof of Chinburg’s conjecture for  $f = 23$ . On the other hand, for  $f = 19$ , one finds that  $m(A) = 16d_{19} + 16d_3$ .

However, by a different method, Rodriguez Villegas and I found the example

$$m((x+x^2+x^3+x^4+x^5)(y^2+1)+(1+6x+2x^2-8x^3+2x^4+6x^5+x^6)y) \stackrel{?}{=} 25d_{19}. \tag{16}$$

This is the only example currently known of (3) for  $f = 19$ . Here the  $\stackrel{?}{=}$  means that (16) has been verified to

50 decimal places but not yet proved. We expect to be able to prove by computations in the Bloch group as in [BRV] that the ratio of the two sides of (16) is a rational number. But to show this rational number is exactly 1 is another matter. The denominator 5 in the RHS of (16) shows that this does not come from a subgroup of  $PSL(2, O_{19})$  but perhaps it still has a “geometric” origin. There is much left to be explained!

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(PRÉSIDENTE—continu de page 1)

des cours de mathématiques dans les programmes techniques et plusieurs programmes pré-universitaires. Notre discipline continue à avoir mauvaise presse et à être perçue comme favorisant l'élitisme et cause de décrochage scolaire.

C'est dans cette perspective que la Société mathématique du Canada a décidé de se lancer dans une série de forums nationaux en enseignement mathématique. Le premier forum se tiendra à l'UQAM (Montréal) les 16-18 mai 2003. Il sera suivi d'un deuxième forum en Ontario en 2004. Ces forums continueront la tradition initiée en 1995 lors du premier forum national en éducation mathématique qui s'est tenu à Québec en mai 1995 sous la présidence de Katherine Heinrich.

Ces forums rassembleront environ 200 participants de toutes les provinces et territoires du Canada, représentant les différents groupes ayant un intérêt ou un impact au niveau de la formation mathématique au niveau secondaire ou collégial comme: les professeurs d'écoles et de collèges, les enseignants universitaires en mathématiques ou didactique mathématique, les administrateurs d'école, les ministères de l'éducation provinciaux, les conseillers pédagogiques, les maisons d'édition, des participants non canadiens, les associations provinciales d'enseignants.

La SMC espère, au travers de ces forums, permettre des contacts fructueux permettant de faire progresser l'enseignement des mathématiques et la reconnaissance de la discipline et de son importance partout au pays. C'est pourquoi elle travaillera à ce projet en partenariat le plus large possible en invitant à devenir partenaires les instituts mathématiques, les associations mathématiques provinciales, les associations œuvrant en sciences mathématiques ou en didactique mathématique, les organismes subventionnaires, les ministères de l'éducation au pays, etc.

Le premier forum sera une occasion d'échanger sur les objectifs et les meilleures réalisations à l'échelle du pays. On y identifiera des directions de travail sur lesquelles des sous-groupes feront des recherches en préparation du deuxième forum. Ces recherches seront publiées électroniquement et sur papier et diffusées à grande échelle. Avec George Bluman du département de mathématiques de UBC j'assumerai la présidence scientifique de l'événement.

Nous sommes très reconnaissants à l'UQAM d'avoir accepté de s'occuper de l'organisation locale. Le comité local

d'organisation sera présidé par Louis Charbonneau. Il sera aidé de Pierre Bouchard et Manzoor Ahmad.

Voici quelques-uns des grands thèmes que nous espérons voir traités pendant le Forum:

- Les mathématiques à l'école: comment les rendre intéressantes?
- Les mathématiques, formation de l'esprit.
- La formation des professeurs de mathématiques à l'élémentaire et au secondaire.

Voici quelques-uns de nos prochains rendez-vous. La réunion d'été 2002 de la Société mathématique du Canada se tiendra dans la belle ville de Québec les 15-17 juin 2002. Il y a déjà 158 conférenciers invités mais les organisateurs lancent un appel pour des communications courtes qui ne seront pas en compétition avec les conférences des sessions spéciales! Nous vous encourageons donc à envoyer vos étudiants de maîtrise ou de doctorat à cette réunion et à leur faire présenter une communication: ce sera pour eux une occasion de faire des contacts et d'enrichir leur CV par leur participation à une rencontre scientifique d'envergure.

Et le prochain congrès canadien des étudiants en mathématiques se tiendra à l'Université Calgary les 2-7 juillet 2002. C'est déjà le moment d'en aviser vos étudiants de premier cycle et d'encourager certains à préparer une communication. Vous pourriez même les encourager à présenter la candidature de votre département pour accueillir les prochains congrès. Vous serez fiers d'eux!

Enfin, pour terminer, nous vous annonçons déjà la tenue d'une rencontre conjointe entre la Société mathématique du Canada, la Société canadienne de mathématiques appliquées et industrielles, la Société mathématique de France et la Société française de mathématiques appliquées à Toulouse en juillet 2004. Nous espérons greffer à cette rencontre une rencontre satellite francophone avec les sociétés mathématiques de plusieurs pays d'Afrique du Nord (Maroc, Tunisie, etc.) Ce sera pour plusieurs d'entre nous l'occasion de renouer avec nos anciens étudiants de doctorat!

Je saisis cette occasion que j'ai de vous adresser la parole pour vous parler de l'Université Indépendante de Moscou. Petite université fondée en 1991, elle s'est donnée comme mission de former des futurs chercheurs en mathématiques et de maintenir les meilleures traditions de la grande

école mathématique russe. L'Université Indépendante de Moscou offre maintenant un programme "Math in Moscow" à l'intention des étudiants étrangers. Ceux-ci sont invités à passer un trimestre à Moscou pendant lequel ils suivront des cours en anglais. Les frais de scolarité sont au montant de US\$3500 par trimestre. Des détails sur le programme peuvent être obtenus à:

[www.mccme.ru/mathinmoscow](http://www.mccme.ru/mathinmoscow)

L'Université Indépendante de Moscou a maintenant de nombreuses accords de coopération avec plusieurs institutions, dont l'École Normale Supérieure. En 1998 l'AMS a publié une collection d'articles de recherche par les professeurs et les étudiants des cycles supérieurs de l'Université

Indépendante de Moscou ("Mathematics at the Independent University of Moscow", AMS translations, Series 2, vol. **185**, 1998). L'Université Indépendante de Moscou vient de lancer un journal, le "Moscow Mathematical Journal", publiant des articles de recherche ou d'exposition de très haut calibre. La version électronique se trouve à:

[www.ams.org/distribution/mmj](http://www.ams.org/distribution/mmj)

Le coût de l'abonnement à ce journal est très modeste: US\$ 150 pour les institutions et US\$ 75 pour les individus. Je vous encourage donc à ajouter ce journal à la collection des périodiques de votre bibliothèque et à publiciser le programme "Math in Moscow" auprès de vos étudiants des cycles supérieurs.

## CALL FOR SESSIONS / APPEL AUX COMMUNICATIONS

Additional self-supported sessions play an important role in the success of the Society's semi-annual meetings. The SMC welcomes and invites proposals for self-supported sessions for **Summer 2003 (University of Alberta, Edmonton, Alberta)**.

Proposals should include a brief description of the focus and purpose of the session, the number and expected length of the talks, as well as the organizer's name, complete address, telephone number, e-mail address, etc.

These additional sessions will be incorporated with the other sessions, time blocks allocated by the Meeting Director and advertised in the *CMS Notes*, on *Camel* and, if possible, in the *Notices of the AMS* and in publications of other societies. Speakers in these additional sessions will be requested to submit abstracts which will be published in the meeting programme.

The following provides information on the sessions confirmed to date.

**Those wishing to organize a session should send a proposal to the Meeting Director by the deadline below.**

**Deadline: April 15, 2002 / Date limite : le 15 avril 2002**

**Conformal Field Theory / Théorie des champs conformes**  
Terry Gannon (Alberta) and / et Mark Walton (Lethbridge)

**Brouwer Groups / Groupes de Brouwer**

Mazi Shirvani (Alberta)

**Computational and Analytical Techniques in Modern Applications / Techniques numériques et analytiques dans les applications modernes**

Peter Mineev (Alberta) and / et Tony Ware (Calgary)

**Infinite Dimensional Dynamical Systems / Systèmes dynamiques en dimensions infinies**

XiaoQiang Zhao (Memorial) and / et Thomas Hillen (Alberta)

**Applied Harmonic Analysis / Analyse harmonique ap-**

Les sessions autofinancées contribuent de plus en plus au succès des réunions semi-annuelles de la Société. La SMC encourage ces initiatives et invitent les organisateurs(trices) potentiel(les) à soumettre leurs projets pour ce type de sessions à l'occasion de **la réunion d'été 2003 (Université d'Alberta, Edmonton, Alberta)**.

Les projets doivent inclure une brève description du thème et de la motivation de la session, le nombre et la durée des communications prévues, ainsi que le nom et les coordonnées physiques et électroniques de l'organisateur(trice).

Ces sessions additionnelles feront partie du programme, leur horaire sera établi par le directeur de la réunion, et elles seront publicisées dans les *Notes de la SMC*, sur *Camel* et, si possible, dans les *Notices de l'AMS* et les publications d'autres sociétés. Les conférenciers devront soumettre un résumé de leur communication, qui paraîtra dans le programme de la réunion.

**Toute personne désireuse d'organiser une session doit faire parvenir un projet au directeur de réunion avant la date ci-dessous.**

**pliée** RongQing Jia (Alberta) and / et Bin Han (Alberta)

**New and Successful Courses and Programmes in Mathematics / Nouveaux programmes de mathématiques et programmes a succes**

Ted Lewis (Alberta)

**YanPing Lin, Meeting Director / Directeur de la réunion**

Department of Mathematical and Statistical Sciences

CAB 632

University of Alberta

Edmonton, Alberta, Canada T6G 2G1

Tel: (780) 492-7880 Fax: (780) 492-6826

e-mail: [ylin@math.ualberta.ca](mailto:ylin@math.ualberta.ca)

## OBITUARIES / AVIS DE DÉCÈS



**Ivan Rival  
(1948–2002)**

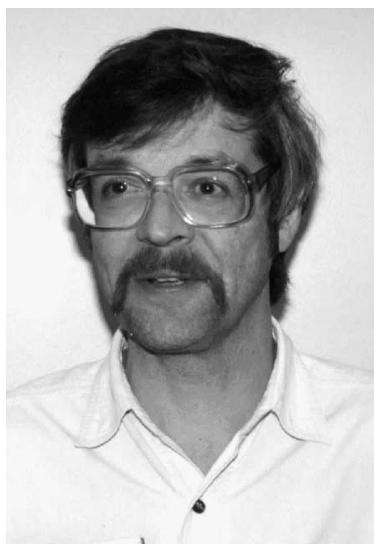
Ivan Rival, professor in the University of Ottawa School of Information Technology and Engineering, passed away suddenly on January 22nd, 2002. Dr. Rival died of heart failure. He was 54.

Dr. Rival joined the University in 1986 as a professor of computer science and mathematics, and was chair of the Department of Computer Science between 1988 and 1991. Gifted with an inventive mind and an instinct for practical applications, Dr. Rival created and developed “Degree navigator,” an innovative software product combining a sense of aesthetics and science that is widely used in universities as a student curriculum planning and management tool. He will be sadly missed by family, colleagues and friends.

Dr. Rival is survived by his wife Hetje, children Robert, David and

Katja, daughter-in-law Chantal, and his parents, Edith and Zoltan.

A memorial service took place at 11 a.m. Monday, January 28 in the Chapel of Tabaret Hall, University of Ottawa. Interment followed in Beechwood Cemetery, Ottawa.



**Murray Bell  
(1950–2001)**

Murray Bell, professor in the Department of Mathematics of the University of Manitoba, passed away suddenly on December 9th, 2001.

Murray was born in Montreal on November 17, 1950. He attended the University of Manitoba, receiving his B.Sc. and M.Sc. there. He obtained his doctorate at the University of Alberta in 1977 under the supervision of Steve Willard. His doctoral thesis was

devoted to the study of supercompact spaces.

Murray joined the Department of Mathematics at the University of Manitoba in 1981 and remained there (apart from sabbatical leaves) for the rest of his career.

Murray’s mathematical research was renowned internationally. His primary interests lay in set-theoretic topology with a combinatorial flavour. He made major contributions to the study of compact Hausdorff spaces, especially supercompactness, chain conditions, dyadic spaces and the interaction between graph theory and set-theoretic topology. A more detailed discussion of his research will appear elsewhere. He collaborated with many in the topological community and had a major influence on it.

Murray will be remembered around the world as a brilliant lecturer, a deeply imaginative researcher and a mathematician who created many compelling ideas of startling clarity. He was passionate about ideas and contributed greatly to our understanding of compact Hausdorff spaces.

Murray was a frequent participant at topological conferences. He is remembered as a friendly and intense man who preferred dialogue to conversation in larger groups. He was always ready to discuss mathematics and his colleagues were always happy to see him again.

He is survived by his wife Marianne. The funeral announcement is at <http://at.yorku.ca/i/a/a/i/42.htm>

## CALL FOR NOMINATIONS / APPEL DE CANDIDATURES

### Associate Editors - CJM and CMB / Rédacteurs associés - JCM et BCM

The Publications Committee of the CMS solicits nominations for one Associate Editor for the Canadian Journal of Mathematics (CJM) and the Canadian Mathematical Bulletin (CMB). The appointment will be for five years beginning January 1, 2003. The continuing members (with their end of term) are below.

#### CJM Editors-in-Chief / Rédacteurs-en-chef du CJM :

Henri Darmon and/et Niky Kamran, McGill (2006)

#### Rédacteurs-en-chef du BCM / CMB Editors-in-Chief:

James Lewis, Arturo Pianzola; Alberta and/et Noriko Yui; Queen's (2005)

#### Associate Editors / Rédacteurs associés :

The deadline for the submission of nominations is **April 15, 2002**. Nominations, containing a curriculum vitae and the candidate's agreement to serve should be sent to the address below.

Le comité des publications de la SMC sollicite des mises en candidatures pour une poste de rédacteur associé du Journal canadien de mathématiques (JCM) et Bulletin canadien de mathématiques (BCM). Le mandat sera de cinq ans et débutera le 1 janvier 2003. Les membres qui continuent suivent.

F. Shahidi, Purdue (2005)	M. Barlow, UBC (2004)
F. Lalonde, Montréal (2003)	P. Borwein, SFU (2004)
J. Millson, Maryland (2003)	N. Pippenger, UBC (2004)
C. Sulem, Toronto (2003)	G. Elliott, Toronto (2005)
A. Geramita, Queen's (2006)	V. Kac, MIT (2006)
M. Zworski, California (2006)	J. Bland, Toronto (2002)
R. Murty, Queen's (2006)	

L'échéance pour proposer des candidats est le **15 avril 2002**. Les mises en candidature, accompagnés d'un curriculum vitae ainsi que du consentement du candidat(e), devrait être envoyées à l'adresse ci-dessous.

**Keith Taylor, Chair / Président**  
**CMS Publications Committee / Comité des publications de la SMC**  
**Department of Mathematics and Statistics**  
**University of Saskatchewan, McLean Hall, 106 Wiggins Road**  
**Saskatoon, Saskatchewan S7N 5E6**  
**chair-pub@cms.math.ca**

### Coxeter-James / Jeffery-Williams / Krieger-Nelson Prize Lectureships Prix de conférence Coxeter-James / Jeffery-Williams / Krieger-Nelson

The CMS Research Committee is inviting nominations for three prize lectureships.

The Coxeter-James Prize Lectureship recognizes outstanding young research mathematicians in Canada. The selected candidate will deliver the prize lecture at the Winter 2002 Meeting in Ottawa, Ontario. Nomination letters should include at least three names of suggested referees.

The Jeffery-Williams Prize Lectureship recognizes outstanding leaders in mathematics in a Canadian context. The prize lecture will be delivered at the Summer 2003 Meeting in Edmonton, Alberta. Nomination letters should include three names of suggested referees.

The Krieger-Nelson Prize Lectureship recognizes outstanding female mathematicians. The prize lecture will be delivered at the Summer 2003 Meeting in Edmonton, Alberta. Nomination letters should include three names of suggested referees.

The deadline for nominations is **September 1, 2002**. Let-

ters of nomination should be sent to the address below.

Le Comité de recherche de la SMC invite les mises en candidatures pour les trois prix de conférence de la Société, la Conférence Coxeter-James, la Conférence Jeffery-Williams et la Conférence Krieger-Nelson.

Le prix Coxeter-James rend hommage à l'apport exceptionnel des jeunes mathématiciens au Canada. Le candidat choisi présentera sa conférence lors de la réunion d'hiver 2002 à Ottawa (Ontario). Les lettres de mises en candidatures devraient inclure les noms d'au moins trois répondants possibles.

Le prix Jeffery-Williams rend hommage à l'apport exceptionnel des mathématiciens d'expérience au Canada. La Conférence sera présentée lors de la réunion d'été 2003 à Edmonton (Alberta). Les lettres de mises en candidature devraient inclure les noms d'au moins trois répondants possibles.

Le prix Krieger-Nelson rend hommage à l'apport exceptionnel des mathématiciennes au Canada. La Conférence sera présentée lors de la réunion d'été 2003 à Edmonton (Alberta). Les lettres de mises en candidatures devraient inclure les noms

d'au moins trois répondants possibles.

La date limite pour les mises en candidatures est le **1er septembre 2002**. Les lettres de mises en candidatures devraient être envoyées à :

**Douglas Stinson, CMS Research Committee / Comité de recherche de la SMC**  
**Department of Pure Mathematics,**  
**University of Waterloo**  
**200 University Ave West,**  
**Waterloo, ON Canada**  
**N2L 3G1**

### 2002 Adrien Pouliot Award / Prix Adrien-Pouliot 2002

Nominations of individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada are solicited. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programmes, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

Nominations must be submitted on the "Nomination Form" available from the CMS Office. To assure uniformity in the selection process, please follow the instructions precisely. Documentation exceeding the prescribed limits will not be considered by the Selection Committee. Individuals who made a nomination in 2001 can renew this nomination by simply indicating their wish to do so by the deadline date. Only materials updating the 2001 Nomination need be provided as the original has been retained.

Nominations must be received by the CMS Office no later **April 30, 2002**. Please send six copies of each nomination to the following address:

**The Adrien Pouliot Award / Le Prix Adrien-Pouliot**  
**Canadian Mathematical Society / Société mathématique du Canada**  
**577 King Edward, Suite 109,**  
**P.O. Box 450, Station A / C.P. 450, Succ. A**  
**Ottawa, Ontario**  
**K1N 6N5**

Nous sollicitons la candidature de personnes ou de groupe de personnes ayant contribué de façon importante et soutenue à des activités mathématiques éducatives au Canada. Le terme "contributions" s'emploie ici au sens large; les candidats pourront être associés à une activité de sensibilisation, un nouveau programme adapté au milieu scolaire ou à l'industrie, des activités promotionnelles de vulgarisation des mathématiques, des initiatives, spéciales, des conférences ou des concours à l'intention des étudiants, etc.

Les candidatures doivent nous être transmises via le "Formulaire de mise en candidature" disponible du bureau de la direction de la SMC. Pour garantir l'uniformité du processus de sélection, veuillez suivre les instructions à la lettre. Toute documentation excédant les limites prescrites ne sera pas considérée par le comité de sélection. Il est possible de renouveler une mise en candidature présentée l'an dernier, pourvu que l'on en manifeste le désir avant la date limite. Dans ce cas, le présentateur n'a qu'à soumettre des documents de mise à jour puisque le dossier original a été conservé.

Les mises en candidature doivent parvenir au bureau de la SMC avant le **30 avril 2002**. Veuillez faire parvenir vos mises en candidature en six exemplaires à l'adresse suivante:

### CMS Distinguished Service Award / Prix de la SMC pour service méritoire

In 1995, the Society established this award to recognize individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society.

The first awards were presented at the 1995 Winter Meeting in Vancouver to Donald Coxeter, Nathan Mendelsohn,

John Coleman, Maurice L'Abbé and George Duff. Awards were presented at the 1996 Winter Meeting in London, Ontario to David Borwein and P.G. (Tim) Rooney, at the 1999 Summer Meeting in St. John's, Newfoundland to Michael Doob and S. Swaminathan, and at the 2000 the Winter Meeting in Vancouver, British Columbia to Arthur Sherker. The



2001 award was presented to James Timourian at the Winter Meeting in Toronto, Ontario.

Nominations should include a reasonably detailed rationale and be submitted by **March 31, 2002**, to the address below.

\*\*\*\*\*

En 1995, la Société mathématique du Canada a créé un nouveau prix pour récompenser les personnes qui contribuent de façon importante et soutenue à la communauté mathématique canadienne et, notamment, à la SMC.

Les premiers lauréats, Donald Coxeter, Nathan Mendelsohn, John Coleman, Maurice L'Abbé et George Duff, furent

honorés lors de la réunion d'hiver 1995 à Vancouver. Les lauréats, David Borwein et P.G. (Tim) Rooney, furent honorés lors de la réunion d'hiver 1996 à London, Ontario. Les lauréats, Michael Doob et S. Swaminathan, furent honorés lors de la réunion d'été 1999 à St. John's, Terre-Neuve. Arthur Sherk fut honoré à la réunion d'hiver 2000 à Vancouver, Colombie-britannique et James Timourian fut honoré à la réunion d'hiver 2001 à Toronto, Ontario.

Pour les mises en candidature prière de présenter des dossiers suffisamment détaillés et de les faire parvenir, le **31 mars 2002** au plus tard, à l'adresse ci-dessous.

**Selection Committee / Comité de sélection**  
**Distinguished Service Award / Prix pour service méritoire**  
**577 King Edward, Suite 109**  
**C.P./P.O. 450, Succursale / Station A**  
**Ottawa, Ontario K1N 6N5 Canada**

## CALENDAR OF EVENTS / CALENDRIER DES ÉVÉNEMENTS

### MARCH 2002

**18–23** Third Annual Colloquiumfest (University of Saskatchewan, Saskatoon, SK)  
<http://math.usask.ca/fvk/Mb3.htm>

**26–April 4** Instructional Conference on Combinatorial Aspects of Mathematical Analysis (ICMS, Edinburgh, UK)  
<http://www.ma.hw.ac.uk/icma/current/>

### APRIL 2002

**5–6** The 28th Annual New York State Regional Graduate Mathematics Conference (Syracuse University, Syracuse, New York)  
<http://math.syr.edu/mgo/conference/conf.html>

**8–19** Invariant Theory (Queen's University, Kingston, ON)  
[activites@crm.umontreal.ca](mailto:activites@crm.umontreal.ca),  
<http://www.CRM.UMontreal.CA/geometry/>

**30–May 17** Concentration Period on the Langlands Programme for Function Fields (CRM, Université de Montréal, Montréal)  
[activites@crm.umontreal.ca](mailto:activites@crm.umontreal.ca),  
<http://www.CRM.UMontreal.CA/geometry/>

### MAY 2002

**3–5** AMS Eastern Section Meeting (CRM, Université de Montréal)  
<http://www.ams.math.org/meetings/>

**17–31** 6th PIMS Industrial Problem Solving Workshop (UBC, Vancouver, BC)  
[frigaard@math.ubc.ca](mailto:frigaard@math.ubc.ca)

### MARS 2002

**19–25** Canadian Number Theory Association Conference (CRM, Université de Montréal, Montréal)  
<http://www.math.mcgill.ca/cnta7>

**24–26** Annual Meeting, Canadian Society for History and Philosophy of Mathematics / Société canadienne d'histoire et de philosophie des mathématiques (University of Toronto)  
<http://www.cshpm.org>

**24–28** 25th Anniversary Meeting of the Canadian Mathematics Education Study Group (CMESG), (Queen's University, Kingston, ON)  
[david.reid@acadiau.ca](mailto:david.reid@acadiau.ca)

**27–June 10** Computational Lie Theory (CRM, Université de Montréal, Montréal)  
[activites@crm.umontreal.ca](mailto:activites@crm.umontreal.ca),  
<http://www.CRM.UMontreal.CA/geometry/>

### JUNE 2002

**3–8** Abel Bicentennial Conference 2002 (University of Oslo, Oslo, Norway)  
<http://www.math.uio.no/abel/conference/index.html>

**4–13** Linear Algebra Workshop (Bled, Slovenia)  
[luzius@mathstat.dal.ca](mailto:luzius@mathstat.dal.ca), <http://www.ijp.si/ftp/pub/stop/law/>

**6–8** CAIMS 2002 (University of Calgary)  
[Samuel Shen: shen@maildrop.srv.ualberta.ca](mailto:Samuel.Shen@maildrop.srv.ualberta.ca)

**6–8** Conference on zero-dimensional schemes and related topics, in honor of Tony Geramita's 60th birthday (Acireale, Italy (Sicily))  
<http://cocoa.dima.unige.it/conference/acireale/first.html>

**10–15** Algebraic Transformation Groups (CRM, Université de Montréal, Montréal)

### JUN 2002

*activites@crm.umontreal.ca,*  
<http://www.CRM.UMontreal.CA/geometry/>

**15– 17 CMS Summer Meeting / Réunion d'été de la SMC (Université Laval, Québec, Québec)**

<http://www.cms.math.ca/Events/summer02/>

**17–21** Seventh International Conference on p-adic Functional Analysis, (University of Nijmegen, The Netherlands)

<http://www.sci.kun.nl/math/p-adic2002/>

**17–21** Householder Symposium on Numerical Linear Algebra (Peebles Hydro Hotel, near Edinburgh, Scotland)

*p.a.knight@strath.ac.uk*

<http://www.maths.strath.ac.uk/matrix/>

**24–28** Special Activity in Analytic Number Theory (Max Planck Institute, Bonn) *moroz@mpim-bonn.mpg.de*

**25–28**, 8th International Conference on Applications of Computer Algebra (Volos, Greece)

<http://www.uth.gr>, <http://www.volos-m.gr>

**JULY 2002**

**JUILLET 2002**

**1–5** Congrès à la memoire de Jacques-Louis Lions (College de France, Paris)

<http://acm.emath.fr/congres-jllions/>

**7–12** The 5th Americas Conference in Differential Equations and Nonlinear Dynamics (University of Alberta, Edmonton)

<http://www.math.ualberta.ca/mli/americas.htm>

*email: mli@math.ualberta.ca*

**8–19** SMS-NATO ASI: Normal Forms, Bifurcations, and Finiteness Problems in Differential Equations (Université de Montréal, Montréal)

<http://www.dms.umontreal.ca/sms>

**15– Aug.10** Conference on Representation Theory of Algebras and Related Topics (ICRA X) (The Fields Institute for Research in Mathematical Sciences, Toronto)

*icrax@fields.utoronto.ca*

**22–30** 44th International Mathematical Olympiad (University of Strathclyde, Glasgow, UK)

**22–Aug 16** Atlantic Association for Research in the Mathematical Sciences Summer School (Memorial University of Newfoundland, St. John's)

<http://www.math.mun.ca/aarms/SS2002> or *email Edgar Goodaire (edgar@math.mun.ca)*

**31–Aug. 3** Novel Kananaskis Symposia on Pressure Distribution

[www.wcb2002.com](http://www.wcb2002.com), *info@wcb2002.com*

**AUGUST 2002**

**AOÛT 2002**

**2–3** Banff Symposia on Skeletal Muscle

[www.wcb2002.com](http://www.wcb2002.com), *info@wcb2002.com*

**3–10** Logic Colloquium 2002, ASL European Summer Meeting (WestfWilhelms-Universität, München, Germany)

<http://www.math.uni-muester.de/LC2002>

**4–9** World Congress of Biomechanics

[www.wcb2002.com](http://www.wcb2002.com), *info@wcb2002.com*

**7–12** Marsden Workshop on Geometry, Mechanics and Dynamics (The Fields Institute for Research in Mathematical Sciences, Toronto)

*marsden60@fields.utoronto.ca*

**15–18** The International Conference on Mathematical Biology (a satellite meeting of ICM-2002)(Guangxi Normal University, Guilin, Guangxi Province, PR China)

*gxnu@public.glptt.gx.cn*

**20–28** International Congress of Mathematicians (Beijing, China) <http://icm2002.org.cn/>

**25–Sept 1** 40th International Symposium on Functional Equations (Gronow, Poland)

*isfe40@uz.zgora.pl* <http://www.isfe40.uz.zgora.pl>

**SEPTEMBER 2002**

**SEPTEMBRE 2002**

**Sept.– Dec. 2002** Set Theory and Analysis Program, (The Fields Institute for Research in Mathematical Sciences, Toronto)

<http://www.fields.utoronto.ca/maillist/>

**23–28** Workshop on Categorical Structures for Descent and Galois Theory, Hopf Algebras and Semiabelian Categories, (The Fields Institute for Research in Mathematical Sciences, Toronto)

*tholen@mathstat.yorku.ca*

**29 – Oct. 3** 18th International CODATA Conference (Hotel Delta Centreville, Montreal, Que)

*codata@dial.oleane.com*, <http://www.codata.org>

**OCTOBER 2002**

**OCTOBRE 2002**

**17 – 19** NCTM Canadian Regional Conference (Regina, Saskatchewan) Hosted by the Saskatchewan Mathematics Teachers' Society.

<http://www.nctm.org/>, <http://mathcentral.uregina.ca/SMTS/>

**DECEMBER 2002**

**DÉCEMBRE 2002**

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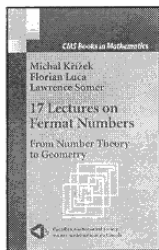
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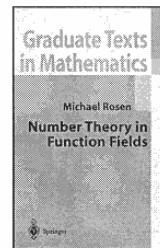


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