A Mathematical Performance
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Two of us girls were members of the S.M.A.R.T. Circle in Edmonton. The acronym stands for Saturday Mathematical Activities, Recreations & Tutorials. In 2010, the Circle sent two teams of four to the Junior High Division of the International Mathematics Competition (IMC). Five Circle members, reinforced by three students elsewhere from Alberta, went to the host city Incheon for the event. Hee-Joo, the third girl, came from Calgary. She was particularly excited since she was originally from South Korea. We had the highest percentage of female representation in the history of the competition, except that Iran, not fully understanding the rules in their first year of participation, sent nine all-girl teams!

The individual and team contest papers are given in the second part of this article, which will appear in Volume 41, issue 10 of Crux. For further details, see their website http://www.imc-official.org/en_US/; for solutions, see the book [1].

Apart from academic matters, the I.M.C. emphasized international friendship and understanding. The most wonderful feature was the Cultural Evening, when teams from various countries gave short performances. These are usually in the form of songs and dances, but there had been innovative presentations as well.

The activity we chose was Platonic Metamorphosis. We used six coloured strings to construct the skeleton of each of the five Platonic solids, in six steps in a continuous transformation. The clever idea came from Karl Schaffer. See his paper [2]. However, we made our own adaptation. Ten students were required, preferably in a six-to-four gender mix. Fortunately, Giavanna’s younger brother and sister came along on a family holiday, and we had the perfect combination.

The official languages for the I.M.C. were English and Chinese, along with Korean for 2010. During our performance, narration was done by Giavanna in English, Ling-Feng in Chinese and Hee-Joo in Korean. This was extremely well-received by the audience.

Step 1. Construction of the Tetrahedron

Start off with the four girls identified as N(orth), S(outh), E(ast) and W(est). Each designates one hand as the U(pper) hand and the other hand as the L(ower) hand. N and S hold out their U hands while E and W hold out their L hands. String 1 is held between UN and LW, string 2 between UN and LE, string 3 between LW and LE, string 4 between LW and US, string 5 between LE and US, and string 6 between UN and US. The completed tetrahedron is shown in Figure 1, with string 6 drawn in such a way to facilitate the description of the next step.
Step 2. Transformation into the Cube

Each of the four girls holds out the other hand and places it at the center of one of the four faces of the tetrahedron, as shown on the left side of Figure 2. Each of these hands will grab the three sides of the triangular face. The end result is a cube, as shown on the right side of Figure 2. Each string forms a face of the cube.

Step 3. Transformation into the Dodecahedron

First, the cube is redrawn as shown in Figure 3.
Now the six boys enter the picture. They are identified as T(op face), B(ottom face), H (northwest face), I (southwest face), J (southeast face) and K (northeast face). Each of them holds out both hands and places them symmetrically about the center of the assigned face of the cube. The line segment joining the two hands of each student is parallel to a side of the cube, and the segments on adjacent faces are perpendicular to each other.

Each pair of these hands will grab the two sides of the square face parallel to the segment they form. Each hand will also grab the nearer one of the remaining two sides of the square face. **It should be emphasized that while each face of the cube is formed of one string, no part of this string is to be grabbed by the hands assigned to this face.** Instead, the other four strings joining adjacent pairs of vertices of the face are grabbed, as illustrated in Figure 4.

![Figure 4](image)

The end result is a dodecahedron, as shown in Figure 5. Failure to observe the caution in the preceding paragraph will still produce a dodecahedron, but the whole structure will then fall apart in Step 4.

![Figure 5](image)
Step 4. Transformation into the Icosahedron

The four girls let go of their strings. The end result is an icosahedron, as shown in Figure 6.

![Figure 6](image1.png)

Step 5. Transformation into the Octahedron

Each of the six boys slides both hands together. The end result is an octahedron, as shown in Figure 7. Two strings which are opposite sides of the original tetrahedron now form the same square cross-section of the octahedron.

![Figure 7](image2.png)
Step 6. Return to the Tetrahedron

The four girls N, S, E and W re-enter the picture. N puts the U hand in triangle HKT (north and top), S puts the U hand in triangle IJT (south and top), E puts the L hand in triangle JKB (bottom and east) and W puts the L hand in triangle HIB (bottom and west). This is shown in Figure 8.

![Figure 8](image)

Each hands grabs the three strings it originally holds, and then the six boys let go of theirs. The end result is once again a tetrahedron, as shown in Figure 9.

![Figure 9](image)

Bibliography:
