THE CONTEST CORNER

No. 39

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The problems featured in this section have appeared in, or have been inspired by, a mathematics contest question at either the high school or the undergraduate level. Readers are invited to submit solutions, comments and generalizations to any problem. Please see submission guidelines inside the back cover or online.

To facilitate their consideration, solutions should be received by the editor by November 1, 2016, although late solutions will also be considered until a solution is published.

The editor thanks André Ladouceur, Ottawa, ON, for translations of the problems.

CC191. There are 32 competitors in a tournament. No two of them are equal in playing strength, and in a one against one match the better one always wins. Show that the gold, silver, and bronze medal winners can be found in 39 matches.

CC192. Let $M$ be a $3 \times 3$ matrix with all entries drawn randomly (and with equal probability) from $\{0, 1\}$. What is the probability that $\det M$ will be odd?

CC193. Consider the set of numbers $\{1, 2, \ldots, 10\}$. Let $\{a_1, a_2, \ldots, a_{10}\}$ be some permutation of these numbers and compute

$$|a_1 - a_2| + |a_3 - a_4| + \cdots + |a_9 - a_{10}|.$$

What is the maximum possible value of the above sum over all possible permutations and how many permutations give you this maximum value?

CC194. At a strange party, each person knew exactly 22 others. For any pair of people $X$ and $Y$ who knew one another, there was no other person at the party that they both knew. For any pair of people $X$ and $Y$ who did not know one another, there were exactly 6 other people that they both knew. How many people were at the party?

CC195. A bisecting curve is one that divides a given region into two subregions of equal area. The shortest bisecting curve of a circle is clearly a diameter. What is the shortest bisecting curve of an equilateral triangle?

CC191. Il y a 32 concurrents dans un tournoi et il n’y a pas deux concurrents de force égale. Dans n’importe quel match entre deux concurrents, le plus fort l’emporte toujours. Démontrer qu’il est possible de déterminer les récipiendaires des médailles d’or, d’argent et de bronze après 39 parties.
CC192. Soit $M$ une matrice $3 \times 3$ dont les éléments sont tous choisis de façon aléatoire dans l’ensemble \{0, 1\}. Quelle est la probabilité pour que det $M$ soit impair?

CC193. On considère l’ensemble \{1, \ldots, 10\} et une permutation \{a_1, \ldots, a_{10}\} de cet ensemble. On calcule

$$|a_1 - a_2| + |a_3 - a_4| + \cdots + |a_9 - a_{10}|.$$ 

Parmi toutes les permutations de l’ensemble, quelle est la valeur maximale de cette somme et combien de permutations donnent cette valeur maximale?

CC194. Lors d’une drôle de fête, chaque personne connaît 22 autres personnes. Pour chaque paire de personnes $X$ et $Y$ qui se connaissent l’une l’autre, il n’y a aucune autre personne à la fête que $X$ et $Y$ connaissent tous les deux. Pour chaque paire de personnes $X$ et $Y$ qui ne se connaissent pas l’une l’autre, il y a 6 personnes à la fête que $X$ et $Y$ connaissent tous les deux. Combien y a-t-il de personnes à la fête?

CC195. Une courbe bissectrice d’une surface est une courbe qui coupe la surface en deux régions de même aire. La courbe bissectrice d’un cercle est également un diamètre. Quelle est la courbe bissectrice la plus courte d’un triangle équilatéral?

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SOLUTIONS

*Statements of the problems in this section originally appear in 2014: 40(9), p. 368–369. All problems are from 42nd Ural tournament of young mathematicians, as printed in Kvant 2014(1).*

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CC141. Alice writes down 100 consecutive natural numbers. Bob multiplies 50 of them: 25 smallest ones and 25 largest ones. He then multiplies the remaining 50 numbers. Can the sum of the two products be equal to 100! = 1 \cdot 2 \cdot \ldots \cdot 100?

*Problem 2 of grade 7 level of 42nd Ural tournament of young mathematicians.*

*We received three solutions, two of which were correct. We present a composite of the two correct solutions from Konstantine Zelator and Fernando Ballesta Yagüe.*

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