THE OLYMPIAD CORNER

No. 319

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The problems featured in this section have appeared in a regional or national mathematical Olympiad. Readers are invited to submit solutions, comments and generalizations to any problem. Please email your submissions to `crux-olympiad@cms.math.ca` *or mail them to the address inside the back cover. Electronic submissions are preferable.*

Submissions of solutions. *Each solution should be contained in a separate file named using the convention* LastName_FirstName_OCProblemNumber *(example Doe_Jane_OC1234.tex). It is preferred that readers submit a LaTeX file and a pdf file for each solution, although other formats are also accepted. Submissions by regular mail are also accepted. Each solution should start on a separate page and name(s) of solver(s) with affiliation, city and country should appear at the start of each solution.*

To facilitate their consideration, solutions should be received by the editor by **1 May 2015***, although late solutions will also be considered until a solution is published.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, 7, and 9, English will precede French, and in issues 2, 4, 6, 8, and 10, French will precede English. In the solutions' section, the problem will be stated in the language of the primary featured solution.

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OC161. The altitude BH dropped onto the hypotenuse AC of a right triangle ABC intersects the angle bisectors AD and CE at Q respectively P. Prove that the line passing through the midpoints of segments $[QD]$ and $[PE]$ is parallel to the line AC.

OC162. Find all functions $f : \mathbb{N} \to \mathbb{R}$ such that for all $k, m, n \in \mathbb{N}$ we have

$$f(km) + f(kn) - f(k)f(mn) \geq 1\,.$$

OC163. Let $A = \{1, 2, \ldots, 2012\}$, $B = \{1, 2, \ldots, 19\}$ and S be the set of all subsets of A. Find the number of functions $f : S \to B$ such that

$$f(A_1 \cap A_2) = \min\{f(A_1), f(A_2)\} \text{ for all } A_1, A_2 \in S\,.$$

OC164. Find all triples (m, p, q) where m is a positive integer and p, q are primes such that

$$2^m p^2 + 1 = q^5\,.$$

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OC165. Let O be the circumcenter of acute ΔABC, and let H be its orthocenter. Let $AD \perp BC$, and let EF be the perpendicular bisector of AO, with D, E on the side BC. Prove that the circumcircle of ΔADE passes through the midpoint of OH.

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OC161. L'altitude BH vers l'hypotenuse AC du triangle ABC intersecte les bissectrices de AD et CE à Q et P respectivement. Démontrer que la ligne passant par les mipoints des segments $[QD]$ et $[PE]$ est parallèle à la ligne AC.

OC162. Déterminer toutes les fonctions $f : \mathbb{N} \to \mathbb{R}$ telles que pour tout $k, m, n \in \mathbb{N}$, l'inégalité qui suit est valide

$$f(km) + f(kn) - f(k)f(mn) \geq 1\,.$$

OC163. Soit $A = \{1, 2, \ldots, 2012\}$, $B = \{1, 2, \ldots, 19\}$ et S l'ensemble de tous les sous ensembles de A. Déterminer le nombre de fonctions $f : S \to B$ telles que

$$f(A_1 \cap A_2) = \min\{f(A_1), f(A_2)\} \text{ pour tout } A_1, A_2 \in S\,.$$

OC164. Déterminer tous les triplets (m, p, q) où m est un entier positif et p, q sont des nombres premiers tels que

$$2^m p^2 + 1 = q^5\,.$$

OC165. Soit O le centre du cercle circonscrit du triangle aïgu ΔABC et soit H l'orthocentre. Soit $AD \perp BC$ et soit EF la bissectrice perpendiculaire de AO, avec D et E sur le côté BC. Démontrer que le cercle circonscrit de ΔADE passe par la mipoint de OH.