THE CONTEST CORNER

No. 21

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The problems featured in this section have appeared in, or have been inspired by, a mathematics contest question at either the high school or the undergraduate level. Readers are invited to submit solutions, comments and generalizations to any problem. Please email your submissions to `crux-contest@cms.math.ca` *or mail them to the address inside the back cover. Electronic submissions are preferable.*

Submissions of solutions. *Each solution should be contained in a separate file named using the convention* LastName_FirstName_CCProblemNumber *(example Doe_Jane_OC1234.tex). It is preferred that readers submit a LaTeX file and a pdf file for each solution, although other formats are also accepted. Submissions by regular mail are also accepted. Each solution should start on a separate page and name(s) of solver(s) with affiliation, city and country should appear at the start of each solution.*

To facilitate their consideration, solutions should be received by the editor by **1 June 2015**, *although late solutions will also be considered until a solution is published.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, 7, and 9, English will precede French, and in issues 2, 4, 6, 8, and 10, French will precede English. In the solutions' section, the problem will be stated in the language of the primary featured solution.

CC101. Find all pairs of whole numbers a and b such that their product ab is divisible by 175 and their sum $a+b$ is equal to 175.

CC102. In pentagon $ABCDE$, angles B and D are right. Prove that the perimeter of triangle ACE is at least $2BD$.

CC103. Let a and b be two rational numbers such that $\sqrt{a}+\sqrt{b}+\sqrt{ab}$ is also rational. Prove that $\sqrt{a}$ and $\sqrt{b}$ must also be rationals.

CC104. Compare the area of an incircle of a square to the area of its circumcircle.

CC105. Knowing that $3.3025 < \log_{10} 2007 < 3.3026$, determine the left-most digit of the decimal expansion of 2007^{1000}.

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CC101. Déterminer toutes les paires d'entiers non négatifs a et b dont le produit ab est divisible par 175 et la somme $a+b$ est égale à 175.

CC102. Dans le pentagone $ABCDE$, les angles B et D sont droits. Démontrer que le périmètre du triangle ACE est supérieur ou égal à $2BD$.

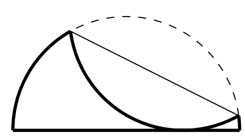
CC103. Soit deux nombres rationnels a et b tels que $\sqrt{a} + \sqrt{b} + \sqrt{ab}$ est aussi rationnel. Montrez que $\sqrt{a}$ and $\sqrt{b}$ sont aussi des rationnels.

CC104. Comparer l'aire du cercle inscrit dans un carré à l'aire du cercle circonscrit au carré.

CC105. En partant du fait que $3.3025 < \log_{10} 2007 < 3.3026$, déterminer le premier chiffre à gauche dans l'écriture décimale de 2007^{1000}.

CONTEST CORNER SOLUTIONS

CC51. A semicircular piece of paper with radius 2 is creased and folded along a chord so that the arc is tangent to the diameter as shown in the diagram. If the contact point of the arc divides the diameter in the ratio $3 : 1$, determine the length of the crease.



Originally problem 5 of 1997 Invitational Mathematics Challenge, Grade 11.

Solved by L. Bobo ; R. Hess ; S. Muralidharan ; A. Plaza ; N. Stanciu ; and T. Zvonaru. We present the solution by S. Muralidharan.

Let $O(0, 0)$ be the centre of the semi-circle with radius 2. Let C be the centre of the circle formed by extending the folded portion. Let R be the point of tangency. Let DE be the length of the crease. Let S be the point where the line joining the centres intersects the crease.

The folded circular portion also has radius 2 and since it touches the diameter at the point $(1, 0)$, its centre is at $C(1, 2)$. Since these are equal circles, the line joining their centres and the common chord (the crease) bisect each other. Thus, $OS = \frac{OC}{2} = \frac{\sqrt{5}}{2}$ and hence

$$DE = 2DS = 2\sqrt{OD^2 - OS^2} = 2\sqrt{4 - \frac{5}{4}} = \sqrt{11}.$$