THE CONTEST CORNER

No. 19

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The problems featured in this section have appeared in, or have been inspired by, a mathematics contest question at either the high school or the undergraduate level. Readers are invited to submit solutions, comments and generalizations to any problem. Please email your submissions to `crux-contest@cms.math.ca` *or mail them to the address inside the back cover. Electronic submissions are preferable.*

Submissions of solutions. *Each solution should be contained in a separate file named using the convention* LastName_FirstName_CCProblemNumber *(example Doe_Jane_OC1234.tex). It is preferred that readers submit a LaTeX file and a pdf file for each solution, although other formats are also accepted. Submissions by regular mail are also accepted. Each solution should start on a separate page and name(s) of solver(s) with affiliation, city and country should appear at the start of each solution.*

To facilitate their consideration, solutions should be received by the editor by **1 March 2015***, although late solutions will also be considered until a solution is published.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, 7, and 9, English will precede French, and in issues 2, 4, 6, 8, and 10, French will precede English. In the solutions' section, the problem will be stated in the language of the primary featured solution.

The editor thanks André Ladouceur, Ottawa, ON, for translations of the problems.

CC87. *Correction. In issue 8, we accidentally re-printed CC33 as CC87. This is the corrected version of CC87.*
Let $ABCDE$ be a regular pentagon with each side of length 1. The length of BE is θ and the angle FEA is α, where F is the intersection of AC and BE. Find θ and $\cos\alpha$.

CC91. A line segment of constant length 1 moves with one end on the x-axis and the other end on the y-axis. The region swept out (that is, the union of all possible placements) is R. Find the equation of the boundary of R.

CC92. Each of the positive integers 2013 and 3210 has the following three properties:

1. it is an integer between 1000 and 10000,
2. its four digits are consecutive integers, and
3. it is divisible by 3.

In total, how many positive integers have these three properties?

CC93. If $x, y, z > 0$ and $xyz = 1$, find the range of all possible values of

$$\frac{x^3 + y^3 + z^3 - x^{-3} - y^{-3} - z^{-3}}{x + y + z - x^{-1} - y^{-1} - z^{-1}}.$$

CC94. If $\log_2 x, (1 + \log_4 x)$ and $\log_8 4x$ are consecutive terms of a geometric sequence, determine the possible values of x.

CC95. Positive integers x, y, z satisfy $xy + z = 160$. Determine the smallest possible value of $x + yz$.

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CC87. *Correction. Dans le numéro 8 de la revue, on a présenté le problème CC33 à la place du problème CC87. Voici le vrai problème CC87.*
Soit $ABCDE$ un pentagone régulier ayant des côtés de longueur 1. Soit θ la longueur de BE, F le point d'intersection de AC et BE et α la mesure de l'angle FEA. Déterminer θ et $\cos\alpha$.

CC91. Un segment de droite de longueur 1 se déplace de manière qu'une de ses extrémités soit toujours sur l'axe des abscisses et l'autre sur l'axe des ordonnées. Soit R la région balayée par le segment (c'est-à-dire la réunion de tous les points sur les positions du segment à mesure qu'il se déplace). Déterminer l'équation de la frontière de R.

CC92. Chacun des entiers 2013 et 3210 satisfait aux trois propriétés suivantes :

1. il est un entier entre 1000 et 10000,
2. ses quatre chiffres sont des entiers consécutifs et
3. il est divisible par 3.

Combien y a-t-il d'entiers positifs qui satisfont à ces trois propriétés ?

CC93. Sachant que $x, y, z > 0$ et $xyz = 1$, déterminer l'étendue de toutes les valeurs possibles de l'expression

$$\frac{x^3 + y^3 + z^3 - x^{-3} - y^{-3} - z^{-3}}{x + y + z - x^{-1} - y^{-1} - z^{-1}}.$$

CC94. Sachant que $\log_2 x, (1 + \log_4 x)$ et $\log_8 4x$ sont des termes consécutifs d'une suite géométrique, déterminer toutes les valeurs possibles de x.

CC95. Les entiers strictement positifs x, y, z vérifient l'équation $xy + z = 160$. Déterminer la plus petite valeur possible de $x + yz$.