BOOK REVIEWS

Amar Sodhi

Heavenly Mathematics: The Forgotten Art of Spherical Trigonometry
by Glen Van Brummelen
Princeton University Press, 2012
ISBN: 978-0-69114-892-2, Hardcover, 192 + xvi pages, US$35
Reviewed by ***J. Chris Fisher****, University of Regina, Regina, SK*

The amusing title, playing on the two meanings of the word *heavenly*, sets the tone of the book — the author with his relaxed and playful style has produced a work that is as enjoyable as it is engrossing. Yet the book is quite serious in the goal suggested by its subtitle, *The Forgotten Art of Spherical Trigonometry.* With applications to navigation, astronomy, and geography, spherical trigonometry had been an essential part of every mathematician's education until the mid 1950s when the subject suddenly disappeared from North American schools. The thoroughness of its disappearance is evident in the pages of *Crux*, where geometry problems rarely deal with more than two dimensions, and when they do, they fail to attract much attention.

The book is not intended to be a thorough treatment of spherical trigonometry (among other things, the theorems are not presented in their full generality), nor is it a scholarly history of the subject. Instead it is a pleasant blend of history and mathematics. Each topic is introduced in a historical context. The problems that arise are then solved with the help of the basic theorems of spherical geometry, which are proved, for the most part, by adapting a historical proof. Van Brummelen declares early on that he appreciates the subject for its beauty; he finds the theorems elegant and often surprising; the proofs he provides have a visual impact that makes them informative and convincing.

Chapter 1 addresses the question of how to measure the earth's radius, which is accomplished by ascending a mountain in the fashion of al-Bīrūnī (around AD 1000), as opposed to the more familiar method of Eratosthenes (3rd century BC), which appears as an exercise. To complete the calculation one needs a sine table, which is constructed using ideas from Ptolemy (2nd century AD). The chapter ends with a calculation of the distance to the moon (also following Ptolemy). Chapter 2 provides an introduction to spherical geometry motivated by discussing the celestial sphere with its great circles representing the equator, ecliptic, and horizon. This leads to the question of how to describe the position of heavenly objects, the sun in particular, with respect to the celestial equator. Three answers are provided in the subsequent four chapters along with solutions to several other important problems: the ancient approach based on the spherical version of Menelaus's theorem, the medieval approach based on the rule of four quantities, and the modern approach based on Napier's discovery of how the parts of a spherical triangle are related.

These first six chapters form the basis for the courses and workshops that the author has taught. The text seems to be suitable for a university course for math and science majors, or for liberal arts students who are not afraid of mathematical formulas and proofs. The final three chapters evolved from student projects; they deal with areas, angles, polyhedra, stereographic projection, and navigating by the stars. Each chapter ends with numerous problems that illustrate and extend the material; many are taken from historical textbooks but, unfortunately, the author does not provide the solutions. The book can be read with profit from cover to cover by the casual reader armed only with the basics of plane trigonometry — very little is required beyond the geometric meaning of sine, cosine and tangent, and perhaps the sine and cosine laws. For the very casual reader whose interest is mainly in the pictures and anecdotes, the author has indicated the beginning and end of detailed arguments that, he says, can be omitted without losing the general flow of ideas. On the other hand, I found the mathematics fascinating; although I already knew much of the story I learned something, historical and mathematical, from every chapter.

Mathematical Excursions to the World's Great Buildings by Alexander J. Hahn
Princeton University Press, 2012
ISBN: 978-0-69114-520-4, Hardcover, 318 + ix pages, US$49.50
Reviewed by ***David Butt****, architect*
and ***J. Chris Fisher****, mathematician, Regina, SK*

By 1296 the Italian city of Florence was already becoming prosperous and important, and so it required a cathedral commensurate with its new status. Of course the town fathers wanted the latest Gothic vaulting, but to make their structure exceptional the plans called for the largest dome imaginable. As was customary in those days, construction of the nave and transept went on for over a century with no idea of how, or even if it might be possible to build such a dome — the octagonal drum on which the dome was to rest was 145 feet across, and it had to support a weight that was perhaps twenty times greater than that of any dome that had ever been built, yet obtrusive exterior buttresses were forbidden by the commission that oversaw the construction. Moreover, the drum itself reached 180 feet above the floor, a height which made infeasible the timber bracing built up from the ground that traditionally supported a dome as it was being constructed. An innovative solution was needed, and Filippo Brunelleschi (1377-1446), a brilliant mathematician, artist, craftsman, inventor, and architect had the skill to devise that solution.

Alexander Hahn lays a readable foundation for the breakthrough building projects we witness today. His analysis of historical structures should make every student of architecture proud. What better record do we have of man's creative evolution than our architecture? Architects and "master builders" rely heavily on the creative engineering skills of their design partners. The story of the building of the Florence Cathedral is one of a half-dozen compelling excursions to the world's great buildings. Perhaps even more exciting was the construction of the Sydney Opera House (1957-1973), which also required its designers to devise innovative solutions to numerous unprecedented problems; then, at the last minute, they had to bring in a bit of elementary mathematics to avert inundation by cost overruns.

The author has organized his book around two historical narratives: one explores the architectural form and structure of a sampling of great buildings; the other develops mathematics from a historical perspective. In addition to six stories that are recounted in some detail, there are numerous brief discussions, some quite interesting, but many only superficial. All, however, are accompanied by well-chosen pictures and diagrams. As for the mathematics in the book, the most successful deal with analysis of thrusts, loads, tensions, and compressions. Hahn uses modern mathematics (vectors, trigonometry, and calculus) to analyze structures whose builders had no access to such tools; the builders relied instead on experience, ingenuity, and faith. For example, the author is able to explain why cracks developed in Brunelleschi's magnificent dome, and to assure us that corrective measures are in place to secure the stability of the structure for centuries to come.

On the other hand, we suspect that readers of this journal would find most of the mathematics in the book to be boring; much of it falls far short of providing insight into the architecture. Large chunks of the book are devoted to elementary mathematics that has only a tenuous relationship to the main topic. It is not clear to us who the intended audience for those passages might be — perhaps the author used the book for a mathematics course that he taught, so he felt obliged to include more mathematics. He does not say. Architectural students would likely skip the mathematical digressions because they are distracting; readers with little background in mathematics would probably find the content too concise and not particularly well explained. In the chapter on Renaissance buildings, for example, the fifteen pages devoted to perspective drawings consist of calculations involving Cartesian coordinates that not only explain very little, but they are most certainly inappropriate for the job. After a messy three-page calculation to demonstrate that a circle becomes an ellipse in a perspective drawing, in place of the steps that determine the endpoint of the major axis he declares that, "This is an unpleasant computation that we will omit." After all that effort, we fail to learn how a Renaissance architect might have used drawings to design his building projects. It might be unusual for a review appearing in a mathematics journal to advise the reader to skip the mathematics, but following that advice, you will likely find Hahn's book fascinating reading.