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SYNOPSIS

by *Sara Hanrahan and Mizan R. Khan*

It is a fact that the hyperbola $xy = 1$ does not intersect the unit circle $x^2 + y^2 = 1$ in $\mathbb{R}^2$. In the course of some investigations in modular arithmetic, the authors asked themselves whether the unit modular circle

$$\mathcal{C}_n = \{(x, y) : x, y \in \mathbb{Z}_n \text{ and } x^2 + y^2 \equiv 1 \pmod{n}\}$$

could intersect the modular hyperbola

$$\mathcal{H}_n = \{(x, y) : x, y \in \mathbb{Z}_n \text{ and } xy \equiv 1 \pmod{n}\}.$$

For example, $\mathcal{C}_{17} \cap \mathcal{H}_{17} = \emptyset$, but $\mathcal{C}_{37} \cap \mathcal{H}_{37} \neq \emptyset$.

Eventually they discovered

Theorem 1 It is the case that $\mathcal{C}_n \cap \mathcal{H}_n \neq \emptyset$ if and only if every prime in the canonical factorization of n is congruent to 1 modulo 12.

Enjoy!!

This month's "free sample" is:

3586. *Proposed by Shai Covo, Kiryat-Ono, Israel.*

For each positive integer n, a_n is the number of positive divisors of n of the form $4m + 1$ minus the number of positive divisors of n of the form $4m + 3$ (so $a_4 = 1$, $a_5 = 2$, and $a_6 = 0$). Evaluate the sum $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{a_n}{n}$.

. . . .

3586. *Proposé par Shai Covo, Kiryat-Ono, Israël.*

Pour tout entier positif n, soit a_n le nombre des diviseurs positifs de n de la forme $4m+1$ moins le nombre de ceux ayant la forme $4m+3$ (p.ex. $a_4 = 1$, $a_5 = 2$ et $a_6 = 0$). Evaluer la somme $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{a_n}{n}$.