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SYNOPSIS

417 Skoliad No. 128 *Lily Yen and Mogens Hansen*

- Mathematics Association of Quebec Contest, 2010, Secondary level
- Concours de l'Association mathématique du Québec, 2010, Ordre secondaire
- Solutions to questions of the 27th New Brunswick Mathematics Competition, 2009, Grade 9, Part C

423 Mathematical Mayhem *Ian VanderBurgh*

423 Mayhem Problems: M457–M462

425 Mayhem Solutions: M420–M425

430 Problem of the Month *Ian VanderBurgh*

432 Square Triangles *Peter Hurthig*

435 The Olympiad Corner: No. 289 *R.E. Woodrow*

Featuring:

the Croatian National Mathematical Competition, 2007, Grades 3 & 4; the 51st National Mathematical Olympiad in Slovenia, 2007, 1st and 2nd Selection Exams; the Correspondence Mathematical Competition in Slovakia, 2006/7, Round 1, First Set; the 57th Latvian Mathematical Olympiad, 2007, 3rd Round, Grades 11 and 12; the Finnish National High School Mathematics Competition, 2007, Final Round; the IX Olimpiada Matemática de Centroamérica y El Caribe; and solutions from readers to some problems from

- Croatian Mathematical Olympiad, 2006, National Competition, 4th Grade;
- Balkan Mathematical Olympiad, 2006;
- Finnish Mathematical Olympiad 2006, Final Round;
- Estonian Mathematical Olympiad 2005/6, Final Round;

450 Book Reviews *Amar Sodhi*

450 *Who Gave You the Epsilon? & Other Tales of Mathematical History*
Edited by Marlow Anderson, Victor Katz, and Robin Wilson
Reviewed by Jeff Hooper

451 *The Princeton Companion to Mathematics*

Edited by Timothy Gowers, June Barrow-Green, and Imre Leader
 Reviewed by R.W. Richards

453 When do the Curves $xy \equiv 1 \pmod{n}$ and $x^2 + y^2 \equiv 1 \pmod{n}$ Intersect?

by Sara Hanrahan and Mizan R. Khan

It is a fact that the hyperbola $xy = 1$ does not intersect the unit circle $x^2 + y^2 = 1$ in \mathbb{R}^2 . In the course of some investigations in modular arithmetic, the authors asked themselves whether the unit modular circle

$$\mathcal{C}_n = \{(x, y) : x, y \in \mathbb{Z}_n \text{ and } x^2 + y^2 \equiv 1 \pmod{n}\}$$

could intersect the modular hyperbola

$$\mathcal{H}_n = \{(x, y) : x, y \in \mathbb{Z}_n \text{ and } xy \equiv 1 \pmod{n}\}.$$

For example, $\mathcal{C}_{17} \cap \mathcal{H}_{17} = \emptyset$, but $\mathcal{C}_{37} \cap \mathcal{H}_{37} \neq \emptyset$.

Eventually they discovered

Theorem 1 It is the case that $\mathcal{C}_n \cap \mathcal{H}_n \neq \emptyset$ if and only if every prime in the canonical factorization of n is congruent to 1 modulo 12.

Enjoy!!

459 Problems: 3576–3587

This month's "free sample" is:

3586. *Proposed by Shai Covo, Kiryat-Ono, Israel.*

For each positive integer n , a_n is the number of positive divisors of n of the form $4m + 1$ minus the number of positive divisors of n of the form $4m + 3$ (so $a_4 = 1$, $a_5 = 2$, and $a_6 = 0$). Evaluate the sum

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_n}{n}.$$

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3586. *Proposé par Shai Covo, Kiryat-Ono, Israël.*

Pour tout entier positif n , soit a_n le nombre des diviseurs positifs de n de la forme $4m + 1$ moins le nombre de ceux ayant la forme $4m + 3$

(p.ex. $a_4 = 1$, $a_5 = 2$ et $a_6 = 0$). Evaluer la somme $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_n}{n}$.

464 Solutions: 3475–3487