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SYNOPSIS

417 Skoliad No. 128  Lily Yen and Mogens Hansen
- Mathematics Association of Quebec Contest, 2010, Secondary level
- Concours de l'Association mathématique du Québec, 2010, Ordre secondaire
- Solutions to questions of the 27th New Brunswick Mathematics Competition, 2009, Grade 9, Part C

423 Mathematical Mayhem  Ian VanderBurgh
423 Mayhem Problems:  M457-M462
425 Mayhem Solutions:  M420-M425
430 Problem of the Month  Ian VanderBurgh
432 Square Triangles  Peter Hurthig

435 The Olympiad Corner: No. 289  R.E. Woodrow
Featuring:
the Croatian National Mathematical Competition, 2007, Grades 3 & 4;
the 51st National Mathematical Olympiad in Slovenia, 2007, 1st and 2nd Selection Exams; the Correspondence Mathematical Competition in Slovakia, 2006/7, Round 1, First Set; the 57th Latvian Mathematical Olympiad, 2007, 3rd Round, Grades 11 and 12; the Finnish National High School Mathematics Competition, 2007, Final Round; the IX Olimpiada Matemáticos de Centroamérica y El Caribe; and solutions from readers to some problems from

- Croatian Mathematical Olympiad, 2006, National Competition, 4th Grade;
- Balkan Mathematical Olympiad, 2006;
- Finnish Mathematical Olympiad 2006, Final Round;
- Estonian Mathematical Olympiad 2005/6, Final Round;

450 Book Reviews  Amar Sodhi

450 Who Gave You the Epsilon? & Other Tales of Mathematical History
Edited by Marlow Anderson, Victor Katz, and Robin Wilson
Reviewed by Jeff Hooper
451 The Princeton Companion to Mathematics
Edited by Timothy Gowers, June Barrow-Green, and Imre Leader
Reviewed by R.W. Richards

453 When do the Curves \( xy \equiv 1 \pmod{n} \) and \( x^2 + y^2 \equiv 1 \pmod{n} \) intersect?

by Sara Hanrahan and Mizan R. Khan

It is a fact that the hyperbola \( xy = 1 \) does not intersect the unit circle \( x^2 + y^2 = 1 \) in \( \mathbb{R}^2 \). In the course of some investigations in modular arithmetic, the authors asked themselves whether the unit modular circle

\[
C_n = \{(x, y) : x, y \in \mathbb{Z}_n \text{ and } x^2 + y^2 \equiv 1 \pmod{n}\}
\]

could intersect the modular hyperbola

\[
\mathcal{H}_n = \{(x, y) : x, y \in \mathbb{Z}_n \text{ and } xy \equiv 1 \pmod{n}\}.
\]

For example, \( C_{17} \cap \mathcal{H}_{17} = \emptyset \), but \( C_{37} \cap \mathcal{H}_{37} \neq \emptyset \).

Eventually they discovered

**Theorem 1** It is the case that \( C_n \cap \mathcal{H}_n \neq \emptyset \) if and only if every prime in the canonical factorization of \( n \) is congruent to 1 modulo 12.

Enjoy!!

459 Problems: 3576–3587

This month’s “free sample” is:

3586. Proposed by Shai Covo, Kiryat-Ono, Israel.

For each positive integer \( n \), \( a_n \) is the number of positive divisors of \( n \) of the form \( 4m + 1 \) minus the number of positive divisors of \( n \) of the form \( 4m + 3 \) (so \( a_4 = 1 \), \( a_5 = 2 \), and \( a_6 = 0 \)). Evaluate the sum

\[
\sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_n}{n}.
\]

3586. Proposé par Shai Covo, Kiryat-Ono, Israël.

Pour tout entier positif \( n \), soit \( a_n \) le nombre des diviseurs positifs de \( n \) de la forme \( 4m + 1 \) moins le nombre de ceux ayant la forme \( 4m + 3 \) (p.ex. \( a_4 = 1 \), \( a_5 = 2 \) et \( a_6 = 0 \)). Evaluer la somme \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_n}{n} \).

464 Solutions: 3475–3487