MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as **a Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of ***Crux Mathematicorum with Mathematical Mayhem***.

The Mayhem Editor is Ian VanderBurgh (University of Waterloo). The other staff member is Monika Khbeis (Our Lady of Mt. Carmel Secondary School, Mississauga, ON). The editor thanks Rachael Verbruggen, University of Waterloo, for her assistance with this month's solutions.

Mayhem Problems

Please send your solutions to the problems in this edition by ***15 February 2011****. Solutions received after this date will only be considered if there is time before publication of the solutions.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier of the University of Montreal for translations of the problems.

M457. *Proposed by the Mayhem Staff.*

Suppose that A is a digit between 0 and 9, inclusive, and that the tens digit of the product of $2A7$ and 39 is 9. Determine the digit A.

M458. *Proposed by the Mayhem Staff.*

Convex quadrilateral $ABCD$ has $AB = AD = 10$ and $BC = CD$. Also, AC is perpendicular to BD, with AC and BD intersecting at P. If $BP = 8$ and $CD = CP + 2$, determine the area of quadrilateral $ABCD$.

M459. *Proposed by Neven Jurič, Zagreb, Croatia.*

Determine whether or not it is possible to create a collection of ten distinct subsets of $S = \{1, 2, 3, 4, 5, 6\}$ so that each subset contains three elements, each element of S appears in five subsets, and each pair of elements from S appears in two subsets.

M460. *Proposed by Dragoljub Milošević, Gornji Milanovac, Serbia.*

Let a and b be positive real numbers. Define $A = \frac{a+b}{2}$, $G = \sqrt{ab}$, and $K = \sqrt{\frac{a^2+b^2}{2}}$. Prove that (a) $G^2 + K^2 = 2A^2$, (b) $A^2 \geq KG$, (c) $G + K \leq 2A$, and (d) $G^4 + K^4 \geq 2A^4$.

M461. *Proposed by Landelino Arboniés, Colegio Marcelino Champagnat, Santo Domingo, Dominican Republic.*

A *Champagnat* number is equal to the sum of all the digits in a set of consecutive positive integers, one of which is the number itself. Thus, 42 is a Champagnat number, since 42 is the sum of all of the digits of 39, 40, 41, 42, 43, 44. Prove that there exist infinitely many Champagnat numbers.

M462. *Proposed by Alex Song, Detroit Country Day School, Detroit, MI, USA and Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON.*

Let $\lfloor x \rfloor$ denote the greatest integer not exceeding x and let $\lceil x \rceil$ denote the smallest integer greater than or equal to x. For example, $\lfloor 3.1 \rfloor = 3$, $\lceil 3.1 \rceil = 4$, $\lfloor -1.4 \rfloor = -2$, and $\lceil -1.4 \rceil = -1$. Determine all real numbers x for which $\lfloor x \rfloor \lceil x \rceil = x^2$.

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M457. *Proposé par l'Équipe de Mayhem.*

On suppose que A est un chiffre entre 0 et 9 inclusivement, et que le chiffre des dizaines du produit $2A7$ et 39 est 9. Trouver A.

M458. *Proposé par l'Équipe de Mayhem.*

Soit $ABCD$ un quadrilatère convexe avec $AB = AD = 10$ et $BC = CD$. De plus, soit AC perpendiculaire à BD et P leur point d'intersection. Si $BP = 8$ et $CD = CP + 2$, trouver l'aire du quadrilatère $ABCD$.

M459. *Proposé par Neven Jurič, Zagreb, Croatie.*

Déterminer si oui ou non, il est possible de créer une collection de dix sous-ensembles distincts de $S = \{1, 2, 3, 4, 5, 6\}$ de sorte que chaque sous-ensemble contienne trois éléments, que chaque élément de S apparaisse dans cinq sous-ensembles, et que chaque paire d'éléments de S apparaisse dans deux sous-ensembles.

M460. *Proposé par Dragoljub Milošević, Gornji Milanovac, Serbie.*

Soit a et b deux nombres réels positifs. On définit $A = \dfrac{a+b}{2}$, $G = \sqrt{ab}$ et $K = \sqrt{\dfrac{a^2+b^2}{2}}$. Montrer que (a) $G^2 + K^2 = 2A^2$, (b) $A^2 \geq KG$, (c) $G + K \leq 2A$, et (d) $G^4 + K^4 \geq 2A^4$.

M461. *Proposé par Landelino Arboniés, Colegio Marcelino Champagnat, Santo Domingo, République dominicaine.*

Un nombre de Champagnat n est défini comme la somme de tous les chiffres d'une suite d'entiers consécutifs comprenant n. Ainsi, 42 est un nombre de Champagnat puisqu'il est dans la suite 39, 40, 41, 42, 43, 44. Montrer qu'il existe une infinité de nombres de Champagnat.

M462. *Proposé par Alex Song, Detroit Country Day School, Detroit, MI, É-U et Edward T.H. Wang, Université Wilfrid Laurier, Waterloo, ON.*

On désigne par $\lfloor x \rfloor$ le plus grand entier n'excédant pas x et par $\lceil x \rceil$ le plus petit entier plus grand ou égal à x. Par exemple, $\lfloor 3.1 \rfloor = 3$, $\lceil 3.1 \rceil = 4$, $\lfloor -1.4 \rfloor = -2$, et $\lceil -1.4 \rceil = -1$. Déterminer tous les nombres réels x pour lesquels $\lfloor x \rfloor \lceil x \rceil = x^2$.

Mayhem Solutions

M420. *Proposed by the Mayhem Staff.*

Riley is a poor starving university student, but is mathematically astute. He notices that five suppers in residence cost the same as seven lunches. After one week of skipping supper most nights, he notices that five lunches and one supper cost $\$48$ in total. How much do 16 suppers cost?

Solution by Oscar Xia, student, St. George's School, Vancouver, BC.

Let one lunch cost x dollars and one supper cost y dollars.

Since five suppers cost the same as seven lunches, then $5y = 7x$. Since five suppers and one lunch cost $\$48$, then $5x + y = 48$.

From the first equation, $x = \frac{5}{7}y$ and so we obtain $5\left(\frac{5}{7}y\right) + y = 48$, or $\frac{32}{7}y = 48$, or $y = \frac{21}{2}$.

Therefore, 16 suppers cost $16\left(\frac{21}{2}\right) = 168$ dollars.

Also solved by MATTHEW BABBITT, home-schooled student, Fort Edward, NY, USA; JACLYN CHANG, student, Western Canada High School, Calgary, AB; NATALIA DESY, student, SMA Xaverius 1, Palembang, Indonesia; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; LEWIS HUGHES, Auburn University, Montgomery, AL, USA; HUGO LUYO SÁNCHEZ, Pontificia Universidad Católica del Peru, Lima, Peru; RICARD PEIRÓ, IES "Abastos", Valencia, Spain; GILI RUSAK, student, Shaker Junior High School, Loudanville, NY, USA; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; and JIXUAN WANG, student, Don Mills Collegiate Institute, Toronto, ON.

M421. *Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.*

Let $\lfloor x \rfloor$ be the greatest integer less than or equal to the real number x. Determine all real numbers x such that

$$\left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{3}{x} \right\rfloor = 4\,.$$

Solution by Matthew Babbitt, home-schooled student, Fort Edward, NY, USA.

Since x cannot be 0, then we let $y = \frac{1}{x}$. Therefore, we are looking for all real nonzero solutions to $\lfloor y \rfloor + \lfloor 3y \rfloor = 4$. Note that y cannot be