BOOK REVIEWS

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*Explorations in Geometry*
By Bruce Shawyer
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Finally a book has appeared that is suitable for the Euclidean geometry class that I have taught at my university for many years. Such a class is directed toward education students specializing in secondary-school mathematics, although it also attracts quite a few other students who take the course as an elective. Shawyer’s students, like mine, had studied trigonometry and Euclidean geometry in high school (according to the curriculum guides), but by the time they arrived in our classrooms they seem to have long ago forgotten whatever it was they might have been exposed to. For many years there was no adequate source material for these students, hence the author’s motivation for turning his class notes into a text book. The only other text I have found that is both well written and aimed at the right level is I. Martin Isaacs’ *Geometry for College Students*, which was reviewed in *CRUX with Mayhem* [2002 : 505-506]; Isaacs’ book, whatever its merits, is overpriced by a factor of four, which makes it far less attractive than this new, reasonably priced paperback. Shawyer’s book will certainly also prove useful to students who wish to master the material on their own, perhaps to prepare themselves for a mathematics competition, or just for the enjoyment of solving interesting problems. As the title proclaims, the goal here is to explore geometry. Shawyer’s approach is exploratory; his advice for solving a geometry problem: first draw an accurate figure, add helping lines, then revise the figure retaining those parts that seem most relevant to what was given and what was required.

The book comes in three parts. The first 113 pages, arranged into six chapters, briefly describe the tools required for solving geometry problems. It starts with a very quick review of the basic Euclidean theorems; then come chapters on trigonometry, concurrency and collinearity, basic formulae (involving $R$, $r$, $s$, etc.), conic sections (with coordinates), and constructions. An unusual feature, incorporated into the section on the construction of regular polygons, is a discussion of the accuracy of the rule of thumb used by carpenters and draughtsmen for determining the centre of a regular polygon. The second part of the book, covering just 14 pages, is devoted to three fascinating problems that caught the author’s fancy. Many of you will have seen his earlier version of the third problem, “Remarkable Bisectors”, in *CRUX with Mayhem* [2006 : 434-435], which by coincidence was closely related to a 2002 Hungarian Olympiad problem [2006 : 150; 2007 : 160-161, 415-417]. The final 176 pages consist of two chapters; the first lists 124
miscellaneous problems; the second presents their solutions together with solutions to many of the other problems that appear throughout the text. The problems have been carefully selected to illustrate the many different tools that had been discussed earlier; the solutions are complete and very well explained. It is these last two chapters that make the text worthwhile.

My enthusiasm for the book, however, is dampened by what I believe to be shortcomings in Chapter 1. The author obviously wishes to keep his summary brief, but he fails to address the issue of what constitutes a proof. Nowhere is the reader told what can be used to justify a claim. The majority of students who take an introductory course in geometry have never seen a valid argument! I would expect Chapter 1 to list the theorems that are to be accepted as known; I believe, moreover, that every result discussed in that chapter should come with a proof that is complete down to the last detail. Instead, the author begins by advising his readers to see Euclid's Elements for a definitive account. I object! A sketchy "review" might be fine for the skillful self-learner, but it forces an instructor to devote valuable class time to supplying (boring!) details. The reason for writing the book in the first place was the lack of adequate source material—Euclid is not the source I would recommend to students who lack the required skill, background, and tenacity. The first theorem discussed in the book is that the sum of the angles of a triangle has measure equal to two right angles. His only explanation: "This can be deduced from Fig. 1.1", a figure that shows a line through vertex $A$ that is parallel to the base $BC$. The figure does not indicate which angles happen to be equal; nowhere in the text is it mentioned what alternate interior angles are, or when they might be equal (although this property is tacitly assumed throughout). On page two the author makes a claim followed by the question "Why?" in parentheses—where are the struggling students going to discover the answer? Is the student to learn from this that he should insert a challenge to the reader (or maybe an "it's obvious") whenever he comes across something he cannot explain? A few sentences earlier come two definitions of similar triangles—one involving corresponding angles, the other the ratios of corresponding sides; the author claims that "it is easy to see that the two definitions are equivalent." Even if that were indeed easy to see, I fail to see how such a claim belongs in a chapter that should be showing how proofs work. It is only in the valuable final chapter (where answers to most of the problems in the text have been produced in full) that the students get to see good proofs. Thus, my recommendation for the text comes with a warning to potential instructors: Be prepared to devote most of the first month of your course to filling in missing details while providing exemplary valid arguments. Until the perfect geometry text comes along, Explorations in Geometry can serve as a suitable alternative.