36: No 5 September / septembre 2010

Published by:

Canadian Mathematical Society
Société mathématique du Canada
209 - 1725 St. Laurent Blvd.
Ottawa, ON K1G 3V4, Canada
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SYNOPSIS

In the introduction the authors write "The inequalities relating angle-bisectors, medians, altitudes, and the sides of a triangle have attracted the interest of many geometers and have motivated a large number of research papers ..." They then proceed to prove five ratio-type inequalities involving these quantities associated with a triangle. The first inequality they prove is

$$\sum_{\text{cyclic}} \frac{s-a}{w_a} \;\leq\; \frac{1}{3}\sum_{\text{cyclic}} \frac{w_a}{s-a}$$

where a, b, c are the side lengths of the triangle, s is the semiperimeter, and w_a is the length of the angle bisector from vertex A.

Enjoy!!

In the introduction the author writes "One of the simplest mathematical inequalities states that the square of a real number is nonnegative. Although this inequality is straightforward, it is quite powerful." He proceeds to show how this simple observation can be used to solve a class of problems involving multivariable inequalities.

He then extends the method by utilizing polynomials that do not change sign on a given interval. Finally, he invites the reader to apply his method to solve the following problem from the 2002 Romanian National Olympiad:

Problem 9 Find all real numbers $-2 \leq a, b, c, d, e \leq 2$ that satisfy the following system:

$$\begin{aligned} a+b+c+d+e &= 0\,, \\ a^3+b^3+c^3+d^3+e^3 &= 0\,, \\ a^5+b^5+c^5+d^5+e^5 &= 10\,. \end{aligned}$$

Enjoy!!

This month's "free sample" is:

3563. *Proposed by Mikhail Kochetov and Sergey Sadov, Memorial University of Newfoundland, St. John's, NL.*

A square $n \times n$ array of lamps is controlled by an $n \times n$ switchboard. Flipping a switch in position (i, j) changes the state of all lamps in row i and in column j.

(a) Prove that for even n it is possible to turn off all the lamps no matter what the initial state of the array is. Demonstrate how to do it with the minimum number of switches.

(b) Prove that for odd n it is possible to turn off all the lamps if and only if the initial state of the array has the following property: either the number of ON lamps in every row and every column is odd, or the number of ON lamps in every row and every column is even. If this property holds, provide an algorithm to turn off all the lamps.

3563. *Proposé par Mikhail Kochetov et Sergey Sadov, Université Memorial de Terre-Neuve, St. John's, NL.*

Un tableau carré formé de $n \times n$ lampes est contrôlé par un tableau de distribution $n \times n$. Actionner un bouton en position (i, j) change l'état de toutes les lampes de la ligne i et de la colonne j.

(a) Montrer que pour n pair, il est possible d'éteindre toutes les lampes quel que soit le statut initial du tableau. Indiquer comment procéder en actionnant un minimum de boutons.

(b) Montrer que pour n impair, il est possible d'éteindre toutes les lampes si et seulement si l'état initial du tableau répond à la condition suivante : le nombre de lampes allumées dans chaque ligne et chaque colonne est soit impair, soit pair. Dans le cas où cette condition est satisfaite, proposer un algorithme pour éteindre toutes les lampes.