M445. Proposed by the Mayhem Staff.

The lines with equations \( y = x + 1 \), \( y = mx - 1 \), and \( y = -4x + 2m \) pass through the same point. Determine all possible values for \( m \).

M446. Proposed by J. Walter Lynch, Athens, GA, USA.

Let \( a, b, \) and \( c \) be positive digits. Suppose that \( b \) equals the product of \( a, b, \) and \( c, \) and \( ac = a + b + c. \) Determine \( a, b, \) and \( c. \) (Here \( ab \) is the two-digit positive integer with tens digit \( a \) and units digit \( b ))

M447. Proposed by Yakub N. Aliyev, Qafqaz University, Khyrdalan, Azerbaijan.

Let \( ABCD \) be a parallelogram. The sides \( AB \) and \( AD \) are extended to points \( E \) and \( F \) (respectively) so that \( E, C, \) and \( F \) all lie on a straight line. Prove that \( BE \cdot DF = AB \cdot AD. \)

M448. Proposed by the Mayhem Staff.

A polyhedron with exactly \( m + n \) faces has \( m \) faces that are quadrilaterals and \( n \) faces that are triangles. Exactly four faces meet at each vertex. Prove that \( n = 8. \)

Let \( E(x) = \frac{4^x}{4^x + 2} \).

(a) Prove that \( E(x) + E(1 - x) = 1 \).

(b) Find the value of \( E\left(\frac{1}{2010}\right) + E\left(\frac{2}{2010}\right) + \cdots + E\left(\frac{2008}{2010}\right) + E\left(\frac{2009}{2010}\right) \).

M450. Proposed by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON.

Prove that if \( n \) is an odd positive integer, then \( n^{n+2} + (n + 2)^n \) is divisible by \( 2(n + 1) \).

M445. Proposé par l’Équipe de Mayhem.

Les droites d’équations \( y = x + 1 \), \( y = mx - 1 \) et \( y = -4x + 2m \) passent toutes par le même point. Trouver toutes les valeurs possibles de \( m \).

M446. Proposé par J. Walter Lynch, Athens, GA, É-U.

On suppose que les chiffres positifs \( a \), \( b \) et \( c \) sont tels que \( b \) égale le produit de \( a \), \( b \) et \( c \) et que \( ab = a + b + c \). Déterminer \( a \), \( b \) et \( c \). (ici \( ab \) désigne l’entier positif de deux chiffres, \( a \) étant celui des dizaines et \( b \) celui des unités.)

M447. Proposé par Yakub N. Aliyev, Université de Qafqaz, Khyrdalan, Azerbaidjan.

On prolonge respectivement les côtés \( AB \) et \( AD \) du parallélogramme \( ABCD \) jusqu’aux points \( E \) et \( F \) de sorte que \( E \), \( C \) et \( F \) soient tous sur la même droite. Montrer que \( BE \cdot DF = AB \cdot AD \).

M448. Proposé par l’Équipe de Mayhem.

Un polyèdre à exactement \( m + n \) faces en possède \( m \) qui sont des quadrilatères et \( n \) qui sont des triangles. Chaque sommet est le point de rencontre d’exactement quatre faces. Montrer que \( n = 8 \).

M449. Proposé par Neculai Stanciu, École secondaire George Emil Palade, Buzău, Roumanie.

Soit \( E(x) = \frac{4^x}{4^x + 2} \).

(a) Montrer que \( E(x) + E(1 - x) = 1 \).

(b) Évaluer \( E\left(\frac{1}{2010}\right) + E\left(\frac{2}{2010}\right) + \cdots + E\left(\frac{2008}{2010}\right) + E\left(\frac{2009}{2010}\right) \).
M407. Proposed by Neven Jurič, Zagreb, Croatia.

Determine whether or not the square at right can be completed to form a $4 \times 4$ magic square using the integers from 1 to 16. (In a magic square, the sums of the numbers in each row, in each column, and in each of the two main diagonals are all equal.)

Solution by Larry Rollins, student, Auburn University Montgomery, Montgomery, Alabama, USA, modified by the editor.

Since the magic square is to contain each of the integers from 1 to 16, then the sum of the entries would be $1+2+\cdots+15+16 = \frac{1}{2}(16)(17) = 136$. Since the sum of the entries in each row is the same, then this sum should be $\frac{1}{4}(136) = 34$. Also, the sum of the entries in each column and on each diagonal should be 34.

Suppose that we can complete the square to form a magic square. In this case, the missing entry in the fourth row should be $34 - 2 - 15 - 8 = 9$, the missing entry in the third row should be $34 - 16 - 1 - 10 = 7$, and the missing entry in the fourth column should be $34 - 12 - 10 - 8 = 4$. Also, the missing entry in the “northeast” diagonal should be $34 - 9 - 16 - 4 = 5$ and the missing entry in the third column should be $34 - 5 - 1 - 15 = 13$.

At this point, we would have the square on the right. The four integers unused are 3, 6, 11, and 14. To form a magic square, the missing entries in the second column would total $34 - 16 - 2 = 16$. No pair of the unused numbers totals 16. Therefore, the square cannot be completed to form a magic square.

Also solved by JACLYN CHANG, student, Western Canada High School, Calgary, AB; BRUNO SAIGUEIRO FANEGO, Viveiro, Spain; CARL LIBIS, Cumberland University, Lebanon, TN, USA; PEDRO HENRIQUE O. PANTOJA, student, UFRN, Brazil; RICARD PEIRO, IES “Ahas- tos”, Valencia, Spain; EDWARD T.H. WANG, Wilfrid Laurier University, Waterloo, ON; JIXUAN WANG, student, Don Mills Collegiate Institute, Toronto, ON; and GUSNADI WITYOGA, student, SMPN 8, Yogyakarta, Indonesia. There were three incomplete solutions submitted.