MATHMATICALE MAYHEM

Mathematical Mayhem began in 1988 as a Mathematical Journal for and by High School and University Students. It continues, with the same emphasis, as an integral part of Crux Mathematicorum with Mathematical Mayhem.

The Mayhem Editor is Ian VanderBurgh (University of Waterloo). The other staff members are Monika Khbeis (Our Lady of Mt. Carmel Secondary School, Mississauga, ON), and Eric Robert (Leo Hayes High School, Fredericton, NB).

Mayhem Problems

Veuillez nous transmettre vos solutions aux problèmes du présent numéro avant le 1er mars 2010. Les solutions reçues après cette date ne seront prises en compte que s'il nous reste du temps avant la publication des solutions. Chaque problème sera publié dans les deux langues officielles du Canada (anglais et français). Dans les numéros 1, 3, 5 et 7, l'anglais précédera le français, et dans les numéros 2, 4, 6 et 8, le français précédera l'anglais.

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M413. Proposé par l'Équipe de Mayhem.

Trouver le nombre d'entiers positifs formés de trois chiffres dont le produit donne 36.

M414. Proposé par l'Équipe de Mayhem.

On considère la liste des entiers positifs, rangés en ordre croissant, pouvant être exprimés comme la somme de 21 entiers (non nécessairement positifs). Déterminer le 21° entier de cette liste.

M415. Proposé par Neculai Stanciu, École secondaire George Emil Palade, Buzău, Roumanie.

Les côtés $AB$ et $CD$ d'un trapèze $ABCD$ sont parallèles. Si $AB = 15$, $CD = 30$, $AD = 9$ et $BC = 12$, trouver l'aire du trapèze $ABCD$.

M416. Proposé par Bruce Shawyer, Université Memorial de Terre-Neuve, St. John's, NL.

Montrer que 9 est un diviseur de $10^n + 3(4^n + 2) + 5$ pour tous les entiers $n$ non négatifs.

M417. Proposé par Mihály Bencze, Brasov, Roumanie.

Soit $M = \{x^2 + 4xy + y^2 : x, y \in \mathbb{Z}\}$. Montrer que le nombre 2022 appartient à $M$, mais pas le nombre 11.
M418. *Proposé par Geoffrey A. Kandall, Hamden, CT, É-U.*

Dans la figure, $F$ est sur $GE$ et $Q$ sur le prolongement de $GE$. De plus, $A$ et $H$ sont sur $PG$ de sorte que $QA$ coupe $PF$ en $B$ et $PE$ en $C$, et que $QH$ coupe $PE$ en $D$. Montrer que

$$\frac{AB}{BC} \cdot \frac{CD}{DE} \cdot \frac{EF}{FG} \cdot \frac{GH}{HA} = 1.$$  

M419. *Proposé par Joe Howard, Portales, NM, É-U.*

Soit $a$, $b$ et $c$ les longueurs des côtés d'un triangle. Montrer que

$$a(b + c) \leq \frac{a^2 + bc}{2} + \frac{b(c + a)}{2} + \frac{c(a + b)}{2} \leq 3.$$  

M413. *Proposed by the Mayhem Staff.*

Determine the number of three-digit positive integers whose digits have a product of 36.

M414. *Proposed by the Mayhem Staff.*

The positive integers that can be expressed as the sum of 21 consecutive (not necessarily positive) integers are listed in increasing order. Determine the 21st integer in this list.


In trapezoid $ABCD$, $AB$ and $CD$ are parallel. If $AB = 15$, $CD = 30$, $AD = 9$, and $BC = 12$, determine the area of the trapezoid.

M416. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

Prove that 9 divides $10^n + 3(4^n + 2) + 5$ for all nonnegative integers $n$.


Let $M = \{x^2 + 4xy + y^2 : x, y \in \mathbb{Z}\}$. Prove that the number 2022 is in $M$ but that the number 11 is not in $M$.  

2022
M418. Proposed by Geoffrey A. Kandall, Hamden, CT, USA.

In the diagram, F lies on GE and Q lies on GE extended. Also, A and H are on PG so that QA intersects PF at B, QA intersects PE at C, and QH intersects PE at D. Prove that

\[ \frac{AB}{BC} \cdot \frac{CD}{DE} \cdot \frac{EF}{FG} \cdot \frac{GH}{HA} = 1. \]

M419. Proposed by Joe Howard, Portales, NM, USA.

Let a, b, and c be the side lengths of a triangle. Prove that

\[ \frac{a(b + c)}{a^2 + bc} + \frac{b(c + a)}{b^2 + ca} + \frac{c(a + b)}{c^2 + ab} \leq 3. \]

Mayhem Solutions

M382. Proposed by the Mayhem Staff.

Determine all pairs \((x, y)\) of integers for which \(4x^2 - y^2 = 480\).

Solution by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON.

First, we note that if \((x, y)\) is a solution of the given equation with \(x\) and \(y\) integers, then so are \((x, -y)\), \((-x, y)\), and \((-x, -y)\). Hence, it suffices to find all solutions \((x, y)\) in which \(x \geq 0\) and \(y \geq 0\).

Since \(4x^2 = 480\) are both even, then \(y^2\) is even, so \(y\) is even. We thus set \(y = 2z\) for some nonnegative integer \(z\). This yields \(4x^2 = (2z)^2 = 480\), or \(x^2 - 4z^2 = 480\), or \(x^2 - 4 = 120\), or \((x - 2z)(x + 2z) = 2^4 \cdot 3 \cdot 5\).

Next we note that \((x - z) + (x + z) = 2x\), which is even, so \(x - z\) and \(x + z\) must both be even integers or both odd integers. Since their product is 120 (which is even), then each is even. Also, \(x - z \leq x + z\) since \(z \geq 0\).

We make a chart to summarize the possible values for \(x - z\) and \(x + z\), knowing that they are even positive integers whose product is 120. We obtain \(2x\) by adding \(x - z\) and \(x + z\), and we recover \(z\) by subtracting \(x\) from \(x + z\):

<table>
<thead>
<tr>
<th>(x - z)</th>
<th>(x + z)</th>
<th>(2x)</th>
<th>(x)</th>
<th>(z)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>60</td>
<td>62</td>
<td>31</td>
<td>29</td>
<td>58</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>34</td>
<td>17</td>
<td>13</td>
<td>26</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>26</td>
<td>13</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>22</td>
<td>11</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
Therefore, the nonnegative solutions are $(31, 58)$, $(17, 26)$, $(13, 14)$, and 
$(11, 2)$. Thus, the complete integer solution of $4x^2 - y^2 = 480$ consists of 
the 16 pairs $(\pm 31, \pm 58), (\pm 17, \pm 26), (\pm 13, \pm 14),$ and $(\pm 11, \pm 2)$,
where all possible combinations of signs are taken.

Also solved by EDIN AJANOVIĆ, student, First Bosniak High School, Sarajevo, Bosnia 
and Herzegovina; MATTHEW BABBITT, student, Albany Area Math Circle, Fort Edward, 
NY, USA; JACLYN CHANG, student, Western Canada High School, Calgary, AB; JOSE 
HERNÁNDEZ SANTIAGO, student, Universidad Tecnológica de la Mixteca, Oaxaca, Mexico; 
RICHARD I. HESS, Rancho Palos Verdes, CA, USA; WINDA KIRANA, student, SMPN 8, 
Yogyakarta, Indonesia; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; and KONSTANTINE 
ZELATOR, University of Pittsburgh, Pittsburgh, PA, USA. There were four incorrect and one 
incomplete solutions submitted.

M383. Proposed by the Mayhem Staff.

In rectangle $ABCD$, $P$ is on side $BC$ and $Q$ is on side $DC$ so that 
$BP = 1$, $AP = PQ = 2$ and $\angle APQ = 90^\circ$. Determine the length of $QD$.

Solution by Jadyn Chang, student, Western Canada High School, Calgary, AB.

Using the given information, we draw the diagram at right. In the diagram,

\[
\angle APQ = \angle ABP = \angle PCQ = 90^\circ, \quad AB = DC, \text{ and } AD = BC.
\]

Since $\triangle APB$ has a right angle at $B$, 
$AP = 2$, and $BP = 1$, then $\triangle APB$ is a 
$30^\circ$–$60^\circ$–$90^\circ$ triangle, so $AB = \sqrt{3}$ and 
$\angle APB = 60^\circ$.

Next, we see that

\[
\angle QPC = 180^\circ - \angle APQ - \angle APB \\
= 180^\circ - 90^\circ - 60^\circ = 30^\circ.
\]

Since $\triangle QPC$ has a $30^\circ$ angle and a 
$90^\circ$ angle, then it is also a $30^\circ$–$60^\circ$–$90^\circ$ 
triangle. Therefore, $\angle PQC = 60^\circ$ and 
$QC = \frac{1}{2} QP = 1$.

Lastly, $QD = DC - QC = AB - QC = \sqrt{3} - 1$.

Also solved by EDIN AJANOVIĆ, student, First Bosniak High School, Sarajevo, Bosnia 
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\textbf{M384. Proposed by Kunal Singh, student, Kendriya Vidyalaya School, Shillong, India.}

In the diagram at right, the point \( E \) is on \( AB \) and the point \( D \) is on \( AC \) such that \( AE = EB = DC = 1 \) and \( AD = 2 \). Determine the ratio of the area of quadrilateral \( BCDE \) to the area of triangle \( ABC \).

\textit{Solution by Scott Brown, Auburn University, Montgomery, AL, USA.}

We use the notation \( [\triangle ABC] \) for the area of \( \triangle ABC \) and \( [BCDE] \) for the area of quadrilateral \( BCDE \).

First, we note that \( [\triangle ABC] = [\triangle AED] + [BCDE] \). We will determine the ratio of \( [\triangle ABC] \) to \( [\triangle AED] \) and use this to determine the required ratio. To do this, we use the property that triangles with equal altitudes have their areas in the same ratio as the lengths of their bases.

Therefore,

\[
\frac{[\triangle ABC]}{[\triangle EAD]} = \frac{[\triangle ABC]}{[\triangle ADB]} \cdot \frac{[\triangle ADB]}{[\triangle EAD]} = \frac{AC}{AD} \cdot \frac{AB}{AE} = \frac{3}{2} \cdot \frac{2}{1} = \frac{3}{1}.
\]

Thus, \( [\triangle EAD] \) is \( \frac{1}{3} \) of \( [\triangle ABC] \). This implies that \( [BCDE] \) is \( \frac{2}{3} \) of \( [\triangle ABC] \).

In summary, \( \frac{[BCDE]}{[\triangle ABC]} = \frac{2}{3} \).

\textit{Also solved by EDIN AJANOVIĆ, student, First Bosniak High School, Sarajevo, Bosnia and Herzegovina; MATTHEW BABBITT, student, Albany Area Math Circle, Fort Edward, NY, USA; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; JACLYN CHANG, student, Western Canada High School, Calgary, AB, ZEP GYUSZI, Dimitrie Leonida Technological High School, Pitești, Romania; RICARD PEIRO, IES “Abastos”, Valenda, Spain; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; JIXUAN WANG, student, Don Mills Collegiate Institute, Toronto, ON; CHRISTOPHER WIRIAWAN, student, Surya Institute, BSD City, Indonesia; and KONSTANTINE ZELATOR, University of Pittsburgh, Pittsburgh, PA, USA.}

There was one incomplete solution submitted.

\textbf{M385. Proposed by Mihály Bencze, Brasov, Romania.}

The base 10 integer \( N = 1 \cdots 114 \cdots 44 \) starts off with 2009 consecutive digits 1 followed by 4018 consecutive digits 4. Prove that \( N \) is not a perfect square.

\textit{Solution by Jixuan Wang, student, Don Mills Collegiate Institute, Toronto, ON.}

Considering \( N \) modulo 16, we see that \( N \equiv 12 \pmod{16} \), as follows:

\[
1 \cdots 114 \cdots 44 \equiv 4444 \pmod{16} \quad \text{(since 10,000 is a multiple of 16)}
\]

\[
\equiv 44 \pmod{16} \quad \text{(since 400 is a multiple of 16)}
\]

\[
\equiv 12 \pmod{16} .
\]
Any integer $x$ is congruent modulo 16 to one of 0, ±1, ±2, ±3, ±4, ±5, ±6, ±7, or 8; so $x^2$ is congruent to one of 0, 1, 4, 9, 0, 9, 4, 1, or 0.

Therefore, every perfect square is congruent to 0, 1, 4, or 9 modulo 16, and we conclude that $N$ cannot be a perfect square.

Also solved by MATTHEW BABBITT, student, Albany Area Math Circle, Fort Edward, NY, USA; SZEP GYUSZL, Dámitie Leonida Technical High School Petrosani, Romania; JOSE HERNANDEZ SANTIAGO, student, Universidad Tecnológica de la Mixteca, Oaxaca, Mexico; RICARD PEIRO, IES "ABastos", Valencia, Spain; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; CHRISTA SOESANTO, student, Surya Institute, BSD City, Indonesia; EDWARD T.H. WANG, Wilfrid Laurier University, Waterloo, ON, and KONSTANTINE ZELATOR, University of Pittsburgh, Pittsburgh, PA, USA. There were two incomplete solutions submitted.

**M386. Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.**

Determine all real numbers $x$ for which

$$\sqrt{2 + 4x - 2x^2} + \sqrt{6 + 6x - 3x^2} = x^2 - 2x + 6.$$ 

**Solution by Bruno Salgueiro Fanego, Viveiro, Spain.**

We have

$$\sqrt{2 + 4x - 2x^2} + \sqrt{6 + 6x - 3x^2} = \sqrt{4 - (2x^2 - 4x + 2)} + \sqrt{9 - (3x^2 - 6x + 3)}$$
$$= \sqrt{4 - 2(x - 1)^2} + \sqrt{9 - 3(x - 1)^2}$$
$$\leq \sqrt{4 - 2(0)} + \sqrt{9 - 3(0)} = 2 + 3 = 5 = 0 + 5$$
$$\leq (x - 1)^2 + 5 = x^2 - 2x + 6.$$ 

For the first expression to actually equal the final expression, it must be that both inequalities are actually equalities, and so $(x - 1)^2 = 0$ or $x = 1$.

Thus, the only possible solution to the given equation is $x = 1$. We can verify by substitution that $x = 1$ is indeed a solution.

Also solved by EDIN AJANOVIĆ, student, First Bosniak High School, Sarajevo, Bosnia and Herzegovina; PAUL BRACKEN, University of Texas, Edinburg, TX, USA; CAO MINH QUANG, Nguyen Binh Khiem High School, Vinh Long, Vietnam; G.C. GREBEUL, Newport News, VA, USA; RICARD PEIRO, IES "ABastos", Valencia, Spain; FRANCISCUS SUSAN, student, Surya Institute, BSD City, Indonesia; EDWARD T.H. WANG, Wilfrid Laurier University, Waterloo, ON, and KONSTANTINE ZELATOR, University of Pittsburgh, Pittsburgh, PA, USA. There were two incorrect and one incomplete solutions submitted.

**M387. Proposed by John Grant McLoughlin, University of New Brunswick, Fredericton, NB.**

Temperature can be measured in degrees Fahrenheit ($F$) or in degrees Celsius ($C$). The two scales are related by the formula $F = 1.8C + 32$. When a two-digit integer degree temperature in Celsius is converted to Fahrenheit and rounded to the nearest integer degree, it turns out the ones and tens digits of the original Celsius temperature $C$ sometimes switch places to give the rounded Fahrenheit equivalent. Find all two-digit integer values of $C$ for which this occurs.
Solution by the Mayhem Staff.

Consider a two-digit temperature \( C = 10a + b \) in degrees Celsius, where \( a \) and \( b \) are integers with \( 1 \leq a \leq 9 \) and \( 0 \leq b \leq 9 \).

The equivalent temperature in degrees Fahrenheit is

\[
F = \frac{9}{5}C + 32 = \frac{9}{5}(10a + b) + 32 = 18a + \frac{9}{5}b + 32.
\]

We want the rounded version of this real number to equal \( 10b + a \). Therefore,

\[
\begin{aligned}
10b + a - \frac{1}{2} &< 18a + \frac{9}{5}b + 32 < 10b + a + \frac{1}{2}; \\
100b + 10a - 5 &< 180a + 18b + 320 < 100b + 10a + 5; \\
-325 &< 170a - 82b < -315; \\
315 &< 82b - 170a < 325.
\end{aligned}
\]

Since \( b \leq 9 \), then \( 82b \leq 738 \). Since \( 82b - 170a > 315 \), then \( 170a < 82b - 315 < 738 - 315 = 423 \), whence \( a < \frac{423}{170} = 2\frac{83}{170} \). Since \( a \) is an integer, then \( a \leq 2 \). Therefore, we only need to try \( a = 1 \) and \( a = 2 \).

If \( a = 1 \), the inequalities become \( 315 + 170(1) < 82b \leq 325 + 170(1) \) or \( 485 < 82b \leq 495 \) or \( 5\frac{75}{82} < b \leq 6 \frac{3}{82} \); since \( b \) is an integer, then \( b = 6 \).

Similarly, if \( a = 2 \), then \( b = 8 \).

Hence, the two possibilities are \( C = 16 \) (giving \( F = 60.8 \), which rounds to \( F \approx 61 \)) and \( C = 28 \) (giving \( F = 82.4 \), which rounds to \( F \approx 82 \)).

Also solved by MATTHEW BABBITT, student, Albany Area Math Circle, Fort Edward, NY, USA; JACLYN CHANG, student, Western Canada High School, Calgary, AB; G. C. GREUBEL, Newport News, VA, USA; RICHARD J. HESS, Rancho Palos Verdes, CA, USA; RICARD PEIRO, IES “Abastos”, València, Spain; BRUNO SALGUEIRO FANEGO, Viveiro, Spain; There was one incomplete solution submitted.

Most submitted solutions involved an explicit or implicit complete enumeration of cases from \( C = 10 \) to \( C = 39 \) after some examination of bounds.

Problem of the Month

Ian VanderBurgh

What's in a definition? Mathematics is littered with them. Often, we pay attention to them; sometimes we treat them a bit cavalierly. Here are two problems involving geometric sequences. In the second of these problems, the precision of our definition turns out to affect the answer.

Problem 1 (2009 American Invitational Mathematics Examination) Call a 3-digit number geometric if it has 3 distinct digits which, when read from left to right, form a geometric sequence. Find the difference between the largest and smallest geometric numbers.
Problem 2 (2009 Euclid Contest) If \( \log_3 x \), \( (1 + \log_4 x) \), and \( \log_8 4x \) are consecutive terms of a geometric sequence, determine the possible values of \( x \).

So what's a geometric sequence? Many of you will know this already, but by way of reminder, here is our first attempt at a definition:

**Definition 1:** A geometric sequence is a sequence of numbers in which each term after the first is obtained from the previous term by multiplying by a constant.

Often, we would call the first term in the sequence \( a \) and the multiplying factor \( r \), which gives the sequence \( a, ar, ar^2, ar^3, \ldots \). (You may notice that I've deliberately avoided the phrase “common ratio” — stay tuned!) Let's use this version of the definition to solve the first problem.

**Solution to Problem 1** The smallest 3-digit integers have hundreds digit 1. Let's see if any of these integers is geometric. Call the tens digit of our candidate number \( r \) (note that \( r \) is an integer). Since the hundreds digit is 1, the tens digit is \( r \), and the digits form a geometric sequence, then the units digit is \( r^2 \). The candidate 3-digit integer is as small as possible when \( r \) is as small as possible. Since the digits are distinct, then \( r \neq 1 \) (otherwise \( r = 1 \) would give 111) and \( r 
eq 0 \) (otherwise \( r = 0 \) would give 100). So the smallest candidate occurs when \( r = 2 \), which yields the integer 124, which must be the smallest 3-digit integer that is geometric.

The largest 3-digit integers have hundreds digit 9. Let's see if any of these integers are geometric. Consider a candidate integer and suppose that the multiplying factor between consecutive digits is \( R \). Then the tens digit is \( 9R \) and the units digit is \( 9R^2 \). Since we want this integer to be as large as possible, we try the different possibilities for \( 9R \). If \( 9R = 9 \), then \( R = 1 \), which would give the integer 999, which violates the condition of distinct digits. If \( 9R = 8 \), then \( R = \frac{8}{9} \); in this case, \( 9R^2 = \frac{64}{9} \), which is not an integer. If \( 9R = 7 \), then \( R = \frac{7}{9} \); in this case, \( 9R^2 = \frac{49}{9} \), which is not an integer. If \( 9R = 6 \), then \( R = \frac{2}{3} \), whence \( 9R^2 = (9R)R = 6R = 4 \), which yields the integer 964, which is thus the largest 3-digit integer that is geometric.

Thus, the difference between the largest and smallest 3-digit integers that are geometric is \( 964 - 124 = 840 \).

At this point, you're probably wondering about the preamble — the definition doesn't seem to be affecting anything so far. Here's another crack at the definition of a geometric sequence:

**Definition 2:** A geometric sequence is a sequence of numbers with the property that if \( a, b, c \) are consecutive terms, then \( b^2 = ac \).
And another one:

**Definition 3:** A geometric sequence is a sequence of numbers with the property that if \( a, b, c \) are consecutive terms, then \( \frac{b}{a} = \frac{c}{b} \).

Again, you may wonder what the big deal is all about. So I have a question for you: is 1, 0, 0 a geometric sequence? What do the different versions of the definition tell you?

**Solution to Problem 2** First, we express the logarithms in the three terms using a common base, namely the base 2. We obtain:

\[
1 + \log_4 x = 1 + \frac{\log_2 x}{\log_2 4} = 1 + \frac{1}{2} \log_2 x;
\]

\[
\log_6 4x = \frac{\log_2 4x}{\log_2 6} = \frac{\log_2 4 + \log_2 x}{\log_2 3} = \frac{2}{3} + \frac{1}{3} \log_2 x.
\]

Next, we make the substitution \( u = \log_2 x \) to make the next calculations less cumbersome. In terms of \( u \), our sequence is thus \( u, 1 + \frac{1}{2}u, \frac{2}{3} + \frac{1}{3}u \).

Since this sequence is geometric, then

\[
\left(1 + \frac{1}{2}u\right)^2 = u \left(\frac{2}{3} + \frac{1}{3}u\right);
\]

\[
3(2 + u)^2 = 4u(2 + u) \quad \text{(multiplying by 12)};
\]

\[
12 + 12u + 3u^2 = 4u^2 + 8u;
\]

\[
o = u^2 - 4u - 12;
\]

\[
o = (u - 6)(u + 2);
\]

and so \( u = \log_2 x = 6 \) or \( u = \log_2 x = -2 \), hence \( x = 64 \) or \( x = \frac{1}{4} \).  

So what’s the big deal? Let’s look at what the sequences are for the two possible values of \( x \).

If \( x = 64 \) (or \( u = 6 \)), the sequence is 6, 4, \( \frac{8}{3} \), which is geometric and seems pretty innocuous.

If \( x = \frac{1}{4} \) (or \( u = -2 \)), the sequence is -2, 0, 0. Oh dear! Why is this a problem? Which definition are you using? This sequence is geometric by Definition 1 and Definition 2, but according to Definition 3 it is NOT geometric. So the choice of definition (that is, one’s particular convention) changes the answer to Problem 2. Using Definition 1 or Definition 2, the answer is \( x = 64 \) or \( x = \frac{1}{4} \); but using Definition 3, the answer is \( x = 64 \) only.

There is a happy ending to this saga, though. Luckily, as the 2009 Euclid Contest was being pre-marked, the markers were alerted to this dilemma of differences of definitions and both versions, with proper justification, were accepted as correct.

So pay attention to definitions – are they completely precise? And think critically about seemingly equivalent definitions – are they really equivalent?