MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as **a Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of ***Crux Mathematicorum with Mathematical Mayhem***.

The Mayhem Editor is Ian VanderBurgh (University of Waterloo). The other staff members are Monika Khbeis (Ascension of Our Lord Secondary School, Mississauga) and Eric Robert (Leo Hayes High School, Fredericton).

Mayhem Problems

*Please send your solutions to the problems in this edition by **1 February 2010**. Solutions received after this date will only be considered if there is time before publication of the solutions. The Mayhem Staff ask that each solution be submitted on a separate page and that the solver's name and contact information be included with each solution.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier of the University of Montreal for translations of the problems.

M407. *Proposed by Neven Jurič, Zagreb, Croatia.*

Determine whether or not the square at right can be completed to form a 4×4 magic square using the integers from 1 to 16. (In a magic square, the sums of the numbers in each row, in each column, and in each of the two main diagonals are all equal.)

			12
	16	1	10
	2	15	8

M408. *Proposed by Ovidiu Furdui, Campia Turzii, Cluj, Romania.*

Determine all three-digit positive integers $\underline{abc}$ that satisfy the equation $\underline{abc} = \underline{ab} + \underline{bc} + \underline{ca}$. (Here $\underline{abc}$ denotes the three-digit positive integer with hundreds digit a, tens digit b, and units digit c.)

M409. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL and the Mayhem Staff.*

The three altitudes of a triangle lie along the lines $y = x$, $y = -2x + 3$, and $x = 1$. If one of the vertices of the triangle is at $(5, 5)$, determine the coordinates of the other two vertices.

M410. *Proposed by Matthew Babbitt, student, Albany Area Math Circle, Fort Edward, NY, USA.*

A cube with edge length a, a regular tetrahedron with edge length b, and a regular octahedron with edge length c all have the same surface area. Determine the value of $\frac{\sqrt{bc}}{a}$.

M411. *Proposed by Neculai Stanciu, George Emil Palade Secondary School, Buzău, Romania.*

Triangle ABC has side lengths a, b, and c. If

$$2a + 3b + 4c = 4\sqrt{2a-2} + 6\sqrt{3b-3} + 8\sqrt{4c-4} - 20\,,$$

prove that triangle ABC is right-angled.

M412. *Proposed by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON.*

For a real number x, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x, and let $\{x\} = x - \lfloor x \rfloor$ denote the fractional part of x. Determine all real numbers x for which $\lfloor x \rfloor \cdot \{x\} = x$.

. . . .

M407. *Proposé par Neven Jurič, Zagreb, Croatie.*

Décider si oui ou non le carré ci-contre peut être complété en un carré magique de 4×4 en utilisant les entiers de 1 à 16. (Dans un carré magique, les sommes des nombres dans chaque ligne, chaque colonne et chaque diagonale principale sont toutes égales.)

			12
	16	1	10
	2	15	8

M408. *Proposé par Ovidiu Furdui, Campia Turzii, Cluj, Roumanie.*

Trouver tous les entiers positifs de trois chiffres $\underline{abc}$ satisfaisant l'équation $\underline{abc} = \underline{ab} + \underline{bc} + \underline{ca}$. (On désigne ici par $\underline{abc}$ l'entier de trois chiffres dont le chiffre des centaines est a, celui des dizaines b et celui des unités c.)

M409. *Proposé par Bruce Shawyer, Université Memorial de Terre-Neuve, St. John's, NL et l'Equipe de Mayhem.*

Les trois hauteurs d'un triangle sont sur les droites d'équation $y = x$, $y = -2x+3$, et $x = 1$. Si l'un des sommets du triangle a comme coordonnées $(5, 5)$, trouver celles des deux autres sommets.

M410. *Proposé par Matthew Babbitt, étudiant, Albany Area Math Circle, Fort Edward, NY, É-U.*

Un cube d'arête a, un tétrahèdre régulier d'arête b et un octahèdre régulier d'arête c ont tous la même surface. Trouver la valeur de $\frac{\sqrt{bc}}{a}$.

M411. *Proposé par Neculai Stanciu, École secondaire George Emil Palade, Buzău, Roumanie.*

Soit a, b et c les côtés du triangle ABC. Si

$$2a + 3b + 4c \;=\; 4\sqrt{2a-2} \;+\; 6\sqrt{3b-3} \;+\; 8\sqrt{4c-4} \;-\; 20\,,$$

montrer que ABC est un triangle rectangle.

M412. *Proposé par Edward T.H. Wang, Université Wilfrid Laurier, Waterloo, ON.*

Si x est un nombre réel, on désigne par $\lfloor x \rfloor$ le plus grand entier plus petit ou égal à x, et par $\{x\} = x - \lfloor x \rfloor$ la partie fractionnaire de x. Trouver tous les nombres réels x pour lesquels $\lfloor x \rfloor \cdot \{x\} = x$.

Mayhem Solutions

M376. *Proposed by the Mayhem Staff.*

Determine the value of x if $\left(10^{2009}+25\right)^2 - \left(10^{2009}-25\right)^2 = 10^x$.

Solution by Edin Ajanovic, student, First Bosniak High School, Sarajevo, Bosnia and Herzegovina.

Expanding, we have

$$\begin{aligned}
&\left(10^{2009}+25\right)^2 - \left(10^{2009}-25\right)^2 \\
&\quad = \left[\left(10^{2009}\right)^2 + 2\cdot 25\cdot 10^{2009} + 25^2\right] \\
&\qquad - \left[\left(10^{2009}\right)^2 - 2\cdot 25\cdot 10^{2009} + 25^2\right] \\
&\quad = 4\cdot 25\cdot 10^{2009} \\
&\quad = 100\cdot 10^{2009} \\
&\quad = 10^{2011}\,,
\end{aligned}$$

and so $x = 2011$.