

## BOOK REVIEWS

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### *Putnam and Beyond*

By Răzvan Gelca and Titu Andreescu, Springer Science + Business Media LLC, New York, 2007

ISBN-13: 978-0-387-25765-5, softcover, 798 + xvi pages, US\$69.95

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Reviewed by **Jeff Hooper**, Acadia University, Wolfville, NS

One of my favourite problem books is the one by Andreescu and Gelca, *Mathematical Olympiad Challenges* (Birkhäuser, 2000), in which the authors collect numerous problems centred mainly around past Olympiads, and group them together by similar topic. In *Putnam and Beyond*, the authors again combine to deliver a similar problem book, based this time around on the Putnam Competition.

Structurally, the book consists of six major chapters: Methods of Proof, Algebra, Real Analysis, Geometry and Trigonometry, Number Theory, and Combinatorics and Probability. Each chapter is divided into numerous sections and subsections, each of which focuses on an important problem idea. For instance, the initial section of the Algebra chapter is *Identities and Inequalities*, and here we find the topic divided into a number of important ideas: Algebraic Identities, the equation  $x^2 \geq 0$ , the Cauchy-Schwartz Inequality, the Triangle Inequality, the AM-GM Inequality, and so on. Each section contains explanations of the key ideas and several worked problems, along with a number of problems to solve. Full solutions are provided at the back of the book, along with sources and/or helpful references where necessary. The initial chapter is a particularly nice introduction for students.

There is some overlap, of course, with the topics covered in the authors' previous book, and I was expecting there to be much in common. To their credit, the authors have avoided that sort of mild cheating: for the topics in common with their first book, the examples and problems they offer are new. This new book is also a much larger and far more extensive effort. In addition to all of the examples provided, the book contains more than 900 problems. And these are Putnam-level problems, so they are mainly a level up from the earlier book. In fact the topics in the book go very deep, and cover most of the major ideas found in the undergraduate mathematics curriculum. So there's lots here.

The book is not error-free, however, and a teacher or coach who uses this book should be a little careful. I certainly haven't worked through every problem in the book, but did dip into them in a number of sections. Just to give an example, Problem 33 asks the solver, "Given 50 distinct positive integers strictly less than 100, prove that some two of them sum to 99." A little thought shows that one can take the numbers 50, 51, . . . , 99 and the statement fails.

But these problems are of the minor variety. This wonderful book is an excellent problems resource and should become a part of any serious library for problem solving. By collecting together problems by topic, the authors provide readers the chance to study each of these important problem-solving techniques and ideas in isolation, and help them begin to see the inherent patterns. This should be one of the first books considered as a resource by anyone coaching groups of problem solvers.

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*Mathematical Connections: A Companion for Teachers and Others*

By Al Cuoco, Mathematical Association of America, 2005

ISBN 978-0-88385-739-7, hardcover, 239+xix pages, US\$54.95

Reviewed by **Peter S. Brouwer**, State University of New York, Potsdam, NY, USA

Al Cuoco is the Director of the Center for Mathematics Education at the Education Development Center in Newton, MA, where he works in the areas of curriculum development and professional development of teachers. This book joins a number of recent others in addressing secondary school mathematics content topics from an advanced (or deeper) perspective. The primary audience is high school mathematics teachers, and the book is based on the assumption that providing a more advanced treatment of some of the mathematical topics taught at that level is a valuable form of professional development. The author's emphasis is on making connections between topics and developing mathematical habits of mind.

The choice of topics is somewhat idiosyncratic, and reflects the inter-related topics that Cuoco is interested in exploring. The chapter titles are: 1. Difference Tables and Polynomial Fits, 2. Form and Function: The Algebra of Polynomials, 3. Complex Numbers, Complex Maps, and Trigonometry, 4. Combinations and Locks, and 5. Sums of Powers.

The strength of this book is that it is essentially a problems book (on the above topics). There are many problems, including 90 in the first chapter alone, and the reader is asked to work them sequentially while reading through the text. These are grouped by themes, which aids coherence. In addition, there are many problems given as exercises. The author includes helpful notes on selected problems at the end of each chapter.

Many of the problems in this book are quite challenging, but its incremental and thematic approach helps. As polynomial algebra (and patterns in polynomial coefficients) appear in every chapter, the reader must be comfortable manipulating rather complicated algebraic expressions. I would recommend this book for serious, mathematics-based professional development programs as well as for experienced independent readers who would enjoy pursuing a fruitful intellectual journey through selected advanced secondary mathematics topics.