MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as a Mathematical Journal for and by High School and University Students. It continues, with the same emphasis, as an integral part of Crux Mathematicorum with Mathematical Mayhem.

The Mayhem Editor is Ian VanderBurgh (University of Waterloo). The other staff members are Monika Khbeis (Ascension of Our Lord Secondary School, Mississauga), Eric Robert (Leo Hayes High School, Fredericton), Larry Rice (University of Waterloo), and Ron Lancaster (University of Toronto).

Mayhem Problems

Please send your solutions to the problems in this edition by 15 June 2008. Solutions received after this date will only be considered if there is time before publication of the solutions.

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier of the University of Montreal for translations of the problems.

M338. Proposed by the Mayhem Staff.

Two students miscopy the quadratic equation $x^2 + bx + c = 0$ that their teacher writes on the board. Jim copies $b$ correctly but miscopies $c$; his equation has roots 5 and 4. Vazz copies $c$ correctly, but miscopies $b$; his equation has roots 2 and 4. What are the roots of the original equation?

M339. Proposed by the Mayhem Staff.

(a) Determine the number of integers between 100 and 199, inclusive, which contain exactly two equal digits.

(b) An integer between 1 and 999 is chosen at random, with each integer being equally likely to be chosen. What is the probability that the integer has exactly two equal digits?

M340. Proposed by the Mayhem Staff.

Let $ABC$ be an isosceles triangle with $AB = AC$, and let $M$ be the mid-point of $BC$. Let $P$ be any point on $BM$. A perpendicular is drawn to $BC$ at $P$, meeting $BA$ at $K$ and $CA$ extended at $T$. Prove that $PK + PT$ is independent of the position of $P$ (that is, the value of $PK + PT$ is always the same, no matter where $P$ is placed).
M341. **Proposed by the Mayhem Staff.**

Let $ABC$ be a right triangle with right angle at $B$. Sides $BA$ and $BC$ are in the ratio $3:2$. Altitude $BD$ divides $CA$ into two parts that differ in length by 10. What is the length of $CA$?

M342. **Proposed by the Mayhem Staff.**

Quincy and Celine have to move 10 small boxes and 10 large boxes. The chart below indicates the time that each person takes to move each type of box.

<table>
<thead>
<tr>
<th></th>
<th>Celine</th>
<th>Quincy</th>
</tr>
</thead>
<tbody>
<tr>
<td>small box</td>
<td>1 min.</td>
<td>3 min.</td>
</tr>
<tr>
<td>large box</td>
<td>6 min.</td>
<td>5 min.</td>
</tr>
</tbody>
</table>

They start moving the boxes at 9:00 am. What is the earliest time at which they can be finished moving all of the boxes?

M343. **Proposed by the Mayhem Staff.**

The Fibonacci numbers are defined by $f_1 = f_2 = 1$ and, for $n \geq 2$, by $f_{n+1} = f_n + f_{n-1}$. The first few Fibonacci numbers are $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$. Find the sum of the first 100 even Fibonacci numbers.

M338. **Proposé par l’Équipe de Mayhem.**

Deux étudiants font une erreur en recopiant l’équation quadratique $x^2 + bx + c = 0$ que leur professeur écrit au tableau. Jean copie $b$ correctement, mais pas $c$; son équation possède alors les racines 5 et 4. Victor copie $c$ correctement, mais pas $b$; son équation possède les racines 2 et 4. Quelles sont les racines de l’équation originale?

M339. **Proposé par l’Équipe de Mayhem.**

(a) Déterminer le nombre d’entiers entre 100 et 199, bornes comprises, contenant exactement deux chiffres égaux.

(b) Un entier entre 1 et 999 est choisi au hasard, chaque entier ayant la même chance d’être choisi. Quelle est la probabilité pour que cet entier ait exactement deux chiffres égaux?

M340. **Proposé par l’Équipe de Mayhem.**

Soit $ABC$ un triangle isocèle avec $AB = AC$, et soit $M$ le point milieu de $BC$. Soit $P$ un point quelconque sur $BM$. Par $P$, on dessine une perpendiculaire à $BC$, coupant $BA$ en $K$ et la droite $CA$ en $T$. Montrer que $PK + PT$ est indépendant de la position de $P$ (c’est-à-dire, la valeur de $PK + PT$ est toujours la même, peu importe la position de $P$).
M341. Proposé par l'Équipe de Mayhem.

Soit ABC un triangle rectangle, d'angle droit en B. Ses côtés BA et BC sont dans le rapport 3 : 2. La hauteur BD divise CA en deux parties dont la différence des longueurs est 10. Quelle est la longueur de CA?

M342. Proposé par l'Équipe de Mayhem.

Sophie et Céline doivent déplacer des boîtes, 10 grandes et 10 petites. Le tableau ci-dessous indique les temps requis pour ce faire, dans chaque cas et pour chaque personne.

<table>
<thead>
<tr>
<th></th>
<th>Céline</th>
<th>Sophie</th>
</tr>
</thead>
<tbody>
<tr>
<td>petite boîte</td>
<td>1 min.</td>
<td>3 min.</td>
</tr>
<tr>
<td>grande boîte</td>
<td>6 min.</td>
<td>5 min.</td>
</tr>
</tbody>
</table>

Leur travail commence à 9 heures du matin. Trouver à quelle heure, au plus tôt, elles pourraient finir leur déménagement?

M343. Proposé par l'Équipe de Mayhem.

Les nombres de Fibonacci sont définis par $f_1 = f_2 = 1$ et, pour $n \geq 2$, par $f_{n+1} = f_n + f_{n-1}$. Voici donc le début de la liste des nombres de Fibonacci : 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... Trouver la somme des 100 premiers nombres pairs de cette liste.

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**Mayhem Solutions**

M288. Proposed by Bruce Shayer, Memorial University of Newfoundland, St. John's, NL.

The following figure can be cut into two pieces and reassembled into a square, by simply cutting off the 'tab' and placing it in the cutaway at the top, as shown in the second image.

![Image](image1.png)

Determine a method to cut the given figure into three pieces which can be reassembled to form a square. (Find a method which is essentially different from cutting it into two pieces; for example, cutting the tab into two pieces would not be considered different from the two-piece dissection.)