Mayhem Problems

Please send your solutions to the problems in this edition by 1 March 2005. Solutions received after this date will only be considered if there is time before publication of the solutions.

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5, and 7, English will precede French, and in issues 2, 4, 6, and 8, French will precede English.

The editor thanks Jean-Marc Terrier and Martin Goldstein of the University of Montreal for translations of the problems.

M151. Proposed by Babis Stergiou, Chalkida, Greece.

Let $a, b, c$ be real numbers with $abc = 1$. Prove that

$$a^3 + b^3 + c^3 + (ab)^3 + (bc)^3 + (ca)^3 \geq 2(a^2b + b^2c + c^2a).$$

M152. Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John’s, NL.

A right-angled triangle has the property that, when a square is drawn externally on each side of the triangle, the six vertices of the squares that are not vertices of the triangle are concyclic. Characterize such triangles.

M153. Proposed by Yufei Zhao, student, Don Mills Collegiate Institute, Toronto, ON.

Two similar triangles $APB$ and $BQC$ are erected externally on a triangle $ABC$. If $R$ is a point such that $PBQR$ is a parallelogram, show that triangles $ARC$ and $APB$ are congruent.

M154. Proposed by the Mayhem Staff.

Eight rooks are placed randomly on different squares of a chessboard. What is the probability that none of the rooks is under attack by another rook?

M155. Proposed by K.R.S. Sastry, Bangalore, India.

Determine the cubic $x^3 + px + q = 0$, given that

(i) it has one repeated root, and

(ii) $p$ and $q$ are integers such that $q$ is the smallest permissible positive integral multiple of $p$. 
M156. Proposed by the Mayhem Staff.

Solve for $x$ where $0 \leq x < 2\pi$:

$$2^{1+3 \cos x} - 10(2)^{-1+2 \cos x} + 2^{2+\cos x} - 1 = 0.$$