SKOLIAD No. 79

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Please send your solutions to the problems in this edition by 1 March, 2005.

We will only print solutions to problems marked with an asterisk (*) if we receive them from students in grade 10 or under (or equivalent), or if we receive a unique solution or a generalization.

This month's questions are taken from the British Columbia Colleges High School Mathematics Contest 2004.

BC Colleges High School Math Contest 2004
Junior Preliminary Round
Wednesday, March 10, 2004

1. Of the 2004 students who wrote a mathematics contest, 239 wore Hip jeans and Fast runners, 252 wore Hip jeans but not Fast runners, and 1213 wore Fast runners but not Hip jeans. The number of students who wore neither is:
   (a) 298  (b) 299  (c) 300  (d) 301  (e) 302

2. Two logs found in a wood pile are identical in every respect. Using a power saw, Kate takes 9 seconds to cut the first log into four smaller logs. Assuming the time it takes her to make each cut remains constant, the time (in seconds) it takes her to cut the second log into five smaller logs is:
   (a) 12  (b) 11.5  (c) 11.25  (d) 11.75  (e) none of these

3. Using the standard $xy$-coordinate plane, the area, in square units, of a triangle whose vertices have the coordinates $(0, 0)$, $(1, 5)$, and $(7, 3)$ is:
   (a) 15  (b) 16  (c) 17.5  (d) 18  (e) none of these

4. A box contains red and blue pencils only. If the number of red pencils is two-thirds the number of blue pencils, then the proportion of pencils in the box that are red is:
   (a) $\frac{1}{3}$  (b) $\frac{2}{3}$  (c) $\frac{1}{2}$  (d) $\frac{2}{5}$  (e) $\frac{3}{5}$

5. Last year a skateboard cost $100 and a helmet cost $40. This year the cost of the skateboard increased by 12% and the cost of the helmet increased by 5%. The increase in the combined cost of the skateboard and the helmet is:
   (a) 17%  (b) 10%  (c) 9.5%  (d) 8.5%  (e) 7.5%
6. Two vertical poles, one 10 metres high and the other 15 metres high, stand 12 metres apart. The distance, in metres, between the tops of the poles is:

   (a) 16    (b) 15    (c) 14    (d) 13    (e) 12

7. The number of 4-digit numbers in which the digits sum to greater than 33 is:

   (a) 18    (b) 13    (c) 15    (d) 11    (e) none of these

8. A large square, of perimeter 20 centimetres, has double the area of a smaller square. The perimeter, in centimetres, of the smaller square is:

   (a) 10    (b) 10\sqrt{2}    (c) 20\sqrt{2}    (d) 40    (e) none of these

9. Define the operation \(*\) to mean \(A * B = \frac{A + 2B}{3}\). Then the value of 
\[[(4 \times 7) \times 8] - [4 \times (7 \times 8)]\] is:

   (a) \(-\frac{28}{9}\)  (b) \(-\frac{2}{9}\)  (c) 0  (d) \frac{8}{9}  (e) \frac{29}{9}

10. A box contains 20 yellow discs, 9 red discs, and 6 blue discs. If discs are selected at random, then the smallest number of discs that need to be selected to be assured of selecting at least two discs of each colour is:

   (a) 7  (b) 17  (c) 23  (d) 28  (e) 31

11. If \(\frac{a}{d + b + c} = \frac{4}{3}\) and \(\frac{a}{b + c} = \frac{3}{5}\), then the value of \(\frac{d}{a}\) is:

    (a) \(\frac{7}{6}\)  (b) \(\frac{6}{7}\)  (c) \(-\frac{12}{11}\)  (d) \(-\frac{11}{12}\)  (e) \(\frac{15}{11}\)

12. Lana has a collection of nickels. When she collects them in groups of three, there is one left over; when she piles them in groups of five, there are two left over; and when she puts them in piles of seven, there are three left over. The sum of the digits of the smallest number of nickels that Lana can have is:

   (a) 9  (b) 7  (c) 10  (d) 12  (e) 3

13. The value of \(A + B\) that satisfies

   \[(6^{30} + 6^{-30}) (6^{30} - 6^{-30}) = 3^A 8^B - 3^{-A} 8^{-B}\]

   is:

   (a) 30  (b) 40  (c) 60  (d) 80  (e) 120
14. Let \( x = 0.7181818\ldots \), where the digits '18' repeat. When \( x \) is expressed as a fraction in lowest terms, then its denominator exceeds its numerator by:

(a) 18 (b) 31 (c) 93 (d) 141 (e) 279

15. A student has three different Mathematics books, two different English books, and four different Science books. The number of ways that the books can be arranged on a shelf, if all books of the same subject are kept together, is:

(a) 288 (b) 864 (c) 1260 (d) 1544 (e) 1728

**BC Colleges High School Math Contest 2004**

**Senior Preliminary Round**

**Wednesday, March 10, 2004**

1. Same as question #15 on the Junior Preliminary Round above.

2. Same as question #13 on the Junior Preliminary Round above.

3. A mixing bowl is hemispherical in shape, with a radius of 12 cm. If it contains water to half its depth, then the angle through which it must be tilted before water will begin to pour out is:

(a) 15°  (b) 30°  (c) 45°  (d) 60°  (e) 75°

4. The hill behind Antonino's house is long and steep. He can walk down it at \( \frac{4\frac{1}{2}}{2} \) km/hr, but he can walk up it at only \( 1\frac{1}{2} \) km/hr. If it takes him 6 hours to make the round trip, the distance, in kilometres, from his house to the top of the hill is:

(a) 18  (b) \( \frac{27}{2} \)  (c) 9  (d) \( \frac{27}{4} \)  (e) 6

5. Same as question #12 on the Junior Preliminary Round above.

6. Suppose the line \( \ell \) is parallel to the line \( y = \frac{3}{4}x + 6 \) and four units from it. A possible equation of the line \( \ell \) is:

(a) \( y = \frac{3}{4}x \)  (b) \( y = \frac{3}{4}x + 1 \)  (c) \( y = \frac{3}{4}x + 2 \)

(d) \( y = \frac{3}{4}x + 3 \)  (e) \( y = \frac{3}{4}x + \frac{9}{2} \)
7. You have a rectangular garden twenty metres long and ten metres wide. A one-metre wide path fills up the garden. If you walk along the centre of the path from beginning to end, the number of metres that you walk is:

(a) $199\frac{1}{2}$  (b) 200  (c) $200\frac{1}{2}$  (d) $209\frac{1}{2}$  (e) 210

8. A square grid is made up of a set of parallel lines, 5 cm apart, which are intersected at right angles by another set of parallel lines, also 5 cm apart. If a circular disc of diameter 3 cm is dropped on the square grid, the probability that the disc will not touch a grid line is:

(a) 0.04  (b) 0.10  (c) 0.24  (d) 0.30  (e) 0.16

9. In the diagram, a circle is tangent to the hypotenuse of the isosceles right triangle $ABC$. Side $AB$ is extended and is tangent to the circle at $E$. Side $AC$ is extended and is tangent to the circle at $F$. If the area of triangle $ABC$ is 9, then the area of the circle is:

(a) $9\pi(3 - 2\sqrt{2})$  (b) $9\sqrt{2}\pi$  (c) $9\pi(3 + 2\sqrt{2})$
(d) $18\sqrt{2}\pi$  (e) $36\pi$

10. In the diagram at the right, segments join the vertices of a square with area 1 to midpoints of its sides. The area of the shaded quadrilateral is:

(a) $\frac{1}{2}$  (b) $\frac{2}{5}$  (c) $\frac{1}{3}$
(d) $\frac{1}{4}$  (e) $\frac{1}{5}$

11. The longer base of a trapezoid has length 15 and the line segment joining the midpoints of the two diagonals has length 3. The length of the shorter base of the trapezoid is:

(a) 6  (b) $\frac{15}{2}$  (c) 9  (d) 10  (e) 12
12. In the diagram at the right, \( \angle A = \angle B = 120^\circ \), \( EA = AB = BC = 2 \), and \( CD = DE = 4 \). The area of the pentagon \( ABCDE \) is:

(a) \( 7\sqrt{3} \)  
(b) \( 9\sqrt{3} \)  
(c) \( 3 + 6\sqrt{3} \)  
(d) 12  
(e) \( 6\sqrt{5} \)

13. Each of 600 people have at most twenty $5 bills; some may have none. Divide the 600 into groups of people with the same number of $5 bills. The smallest possible maximum size of any of these groups is:

(a) 26  
(b) 27  
(c) 28  
(d) 29  
(e) 30

14. Consider the positive integers whose first digit is 1 and which have the property that if this digit is transferred to the end of the number, the resulting number is exactly 3 times as large as the original. For example, 139 would be transformed into 391, which is not exactly 3 times as large as 139. If \( N \) is the smallest such positive integer, then the remainder when \( N \) is divided by 9 is:

(a) 0  
(b) 3  
(c) 4  
(d) 5  
(e) 8

15. The pyramid \( ABCDE \) has a square base, and all four triangular faces are equilateral. The measure of the angle \( ABD \) is:

(a) 30°  
(b) 45°  
(c) 60°  
(d) 75°  
(e) 90°