MATHEMATICAL MAYHEM

Mathematical Mayhem began in 1988 as **a Mathematical Journal for and by High School and University Students**. It continues, with the same emphasis, as an integral part of ***Crux Mathematicorum with Mathematical Mayhem***.

All material intended for inclusion in this section should be sent to **~~Mathematical Mayhem, Cairine Wilson Secondary School, 975~~ Orleans Blvd., Gloucester, Ontario, Canada. K1C 2Z7**. The electronic address is

`mayhem-editors@cms.math.ca`

The Mayhem Editor is Shawn Godin (Cairine Wilson Secondary School). The Assistant Mayhem Editor is John Grant McLoughlin (University of New Brunswick). The other staff members are Paul Ottaway (Dalhousie University) and Larry Rice (University of Waterloo).

Mayhem Problems

Envoyez vos propositions et solutions à MATHEMATICAL MAYHEM, 2191 Saturn Crescent, Orleans, ON K4A 3T6, ou par courriel à

`mayhem-editors@cms.math.ca`

N'oubliez pas d'inclure à toute correspondance votre nom, votre année scolaire, le nom de votre école, ainsi que votre ville, province ou état et pays. Nous sommes surtout intéressés par les solutions d'étudiants du secondaire. Veuillez nous transmettre vos solutions aux problèmes du présent numéro avant le *premier avril 2004*. Les solutions reçues après cette date ne seront prises en compte que s'il nous reste du temps avant la publication des solutions.

Chaque problème sera publié dans les deux langues officielles du Canada (anglais et français). Dans les numéros 1, 3, 5 et 7, l'anglais précédera le français, et dans les numéros 2, 4, 6 et 8, le français précédera l'anglais.

La rédaction souhaite remercier Jean-Marc Terrier et Martin Goldstein, de l'Université de Montréal, d'avoir traduit les problèmes.

M107. *Proposé par l'Équipe de Mayhem.*

Soit a et b les longueurs des côtés de l'angle droit d'un triangle rectangle. Un cercle de rayon r touche les côtés et a son centre situé sur l'hypoténuse. Montrer que

$$\frac{1}{a}+\frac{1}{b} = \frac{1}{r}\,.$$

A right-angled triangle has legs of length a and b. A circle of radius r touches the two legs and has its centre on the hypotenuse. Show that

$$\frac{1}{a}+\frac{1}{b} = \frac{1}{r}.$$

M108. *Proposé par l'Équipe de Mayhem.*

Dans un cube dont on a coupé les huit sommets par des plans, combien de diagonales joignant les 24 nouveaux 'sommets' sont-elles comprises entièrement dans le cube ?

. . . .

Given a cube with its eight corners cut off by planes, how many diagonals joining the 24 new 'corners' lie completely inside the cube?

M109. *Proposé par l'Équipe de Mayhem.*

Si tous les plifs sont des plofs et si certains plafs sont des plifs, lesquel des énoncés X, Y, Z doivent être vrais ?

X : Tous les plifs sont des plafs.

Y : Certains plofs sont des plafs.

Z : Certains plifs ne sont pas des plafs.

. . . .

If all plinks are plonks and some plunks are plinks, which of the statements X, Y, Z must be true?

X: All plinks are plunks.

Y: Some plonks are plunks.

Z: Some plinks are not plunks.

M110. *Proposé par l'Équipe de Mayhem.*

A partir d'un nombre distinct de 1, on construit un nouveau nombre en divisant le nombre de départ augmenté d'une unité par le nombre de départ diminué d'une unité. On recommence le processus avec le nouveau nombre. Qu'arrive-t-il ? Expliquez !

. . . .

Given any starting number (other than 1), get a new number by dividing the number 1 larger than your starting number by the number 1 smaller than your starting number. Then do the same with this new number. What happens? Explain!

M111. *Proposé par l'Équipe de Mayhem.*

Un nombres-croisés est comme un mots-croisés, sauf que les réponses sont des nombres, un chiffre par case. Quelle est la somme de tous les chiffres dans la solution de ce nombres-croisés ?

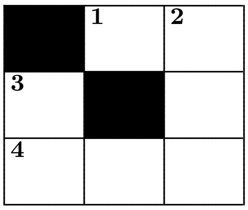
DEFINITIONS

Horizontal

1. Voir 3 Vertical
3. Cube
4. Cinq fois 3 Vertical

Vertical

2. Carré
3. Quatre fois 1 Horizontal



A crossnumber is like a crossword except that the answers are numbers with one digit in each square. What is the sum of all the digits in the solution to this crossnumber?

CLUES

Across

1. See 3 Down
3. A cube
4. Five times 3 Down

Down

2. A Square
3. Four times 1 Across

M112. *Proposé par l'Équipe de Mayhem.*

Déterminer le quotient de l'aire totale de l'hexagone régulier $ABCDEF$ et de l'aire du triangle GDE, si G est le point milieu de AB.

. . . .

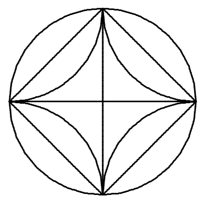
Given that $ABCDEF$ is a regular hexagon and G is the mid-point of AB, determine the ratio of the total area of hexagon $ABCDEF$ to the area of triangle GDE.

Mayhem Solutions

M57. *Proposé par J. Walter Lynch, Athens, GA, USA.*

Quatre points sont egalement espacés autour d'un cercle ayant un rayon r. Le cercle est donc divisé par 4 arcs égaux. Renversez les arcs en laissant le point du bout en place. Trouvez l'aire de la figure ainsi obtenue.

Solution de Robert Bilinski, Outremont, QC.



Puisque les quatre points sur le cercle sont également espacés, le quadrilatère formé par les quatre points est un carré. On remarque que la différence entre les aires du cercle et du carré est la même qu'entre le carré et l'étoile formée par le renversement des arcs de cercle. Le cercle a pour aire πr^2.

Le carré est formé de quatre triangles isocèles rectangles de côtés égaux r et d'hypoténuse $\sqrt{2}r$. Puisque l'hypoténuse des triangles est le côté du carré, son aire est $\left(\sqrt{2}r\right)^2$, soit $2r^2$. La différence entre l'aire du cercle et l'aire du carré est $(\pi - 2)r^2$. Donc l'aire de l'étoile est $(4 - \pi)r^2$.

M58. *Proposed by the Mayhem Staff.*

Find all positive integers x and y which satisfy the equation $x^y = y^x$.

Solution by Mihály Bencze, Brasov, Romania.

The equation is trivially true if $x = y$. We will search for solutions where $x \neq y$.

The original equation implies $\frac{\ln x}{x} = \frac{\ln y}{y}$. If we let $f(x) = \frac{\ln x}{x}$, then $f'(x) = \frac{1 - \ln x}{x^2}$. Therefore, $f(x)$ is increasing on the interval $(0, e)$ and decreasing on $(e, +\infty)$. Hence, if x, $y \in (0, e)$ and $x > y$, then $f(x) > f(y)$; similarly, if x, $y \in (e, +\infty)$ and $x > y$, then $f(x) < f(y)$. Thus, if $x > y$, we must have $y \in (0, e)$ and $x \in (e, +\infty)$. Checking $y = 1$ and $y = 2$ (the only possible values of y), we find that $(x, y) = (4, 2)$ is a solution with $x \neq y$. Therefore, all possible solutions are:

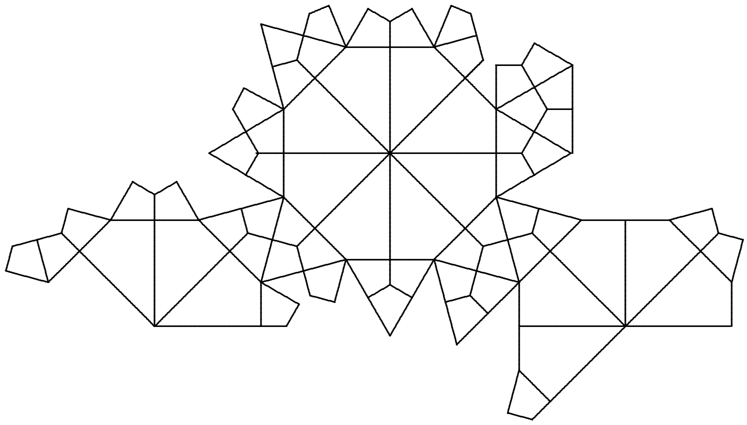
1. $x = y$;

2. $x = 2$, $y = 4$;

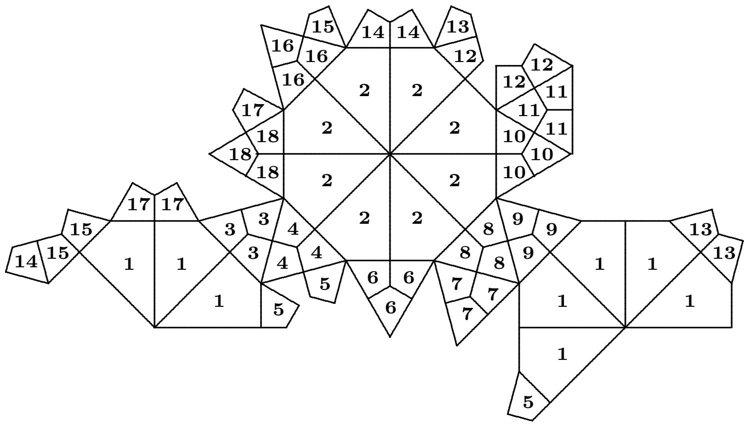
3. $x = 4$, $y = 2$.

M59. *Proposed by Izidor Hafner, Tržaška 25, Ljubljana, Slovenia.*

The diagram below represents the net of a polyhedron in which the faces of the solid are divided into smaller polygons. The task is to colour the polygons (or number them), so that each face of the original solid is a different colour.



Solution by Robert Bilinski, Outremont, QC.



M60. *Proposed by Mihály Bencze, Brasov, Romania.*

Determine all positive integers for which $\left\lfloor \sum_{k=1}^{n} \sqrt{k} \right\rfloor = n$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

Solution by the proposer.

Let $S_n = \left\lfloor \sum_{k=1}^{n} \sqrt{k} \right\rfloor$. We note that $S_1 = 1$, $S_2 = 2$, $S_3 = 4$. For $n > 3$, $S_n > S_{n-1} + 2$. Thus, $S_n = n$ only for $n = 1, 2$.

M61. *Proposed by theMayhem Staff.*

You are given 54 weights which weigh 1^2, 2^2, 3^2, ..., 54^2. Group these into three sets of equal weight.

Solution by Geneviève Lalonde, Massey, ON.

Note that summing 9 consecutive squares n^2, $(n+1)^2$, ..., $(n+8)^2$ yields $9n^2+72n+204 = 3(3n^2+24n+68)$. Among these nine squares, we cannot make 3 sets each totalling $3n^2+24n+68$ because $(n+8)^2$ cannot be grouped with two of the other squares to give the desired total (as can be easily checked). If we take

$$\text{Set 1}\left\{\begin{array}{c}(n+1)^2\\(n+3)^2\\(n+8)^2\end{array}\right., \qquad \text{Set 2}\left\{\begin{array}{c}n^2\\(n+5)^2\\(n+7)^2\end{array}\right., \qquad \text{Set 3}\left\{\begin{array}{c}(n+2)^2\\(n+4)^2\\(n+6)^2\end{array}\right.,$$

the first two sets each total $3n^2+24n+74$ and the last totals $3n^2+24n+56$.

Therefore, we can break our 54 weights into 6 groups of 9 and use our sets above within each group of 9, making sure that each of our 3 sets contains two of the subsets that total only $3n^2+24n+56$. There are many solutions, one of which is:

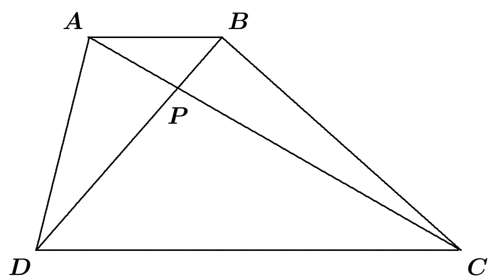
$$\begin{array}{ll}\text{Set 1:} & \{1^2, 6^2, 8^2, 11^2, 13^2, 18^2, 20^2, 22^2, 27^2, 29^2,\\ & \quad 31^2, 36^2, 39^2, 41^2, 43^2, 48^2, 50^2, 52^2\},\\ \text{Set 2:} & \{2^2, 4^2, 9^2, 10^2, 15^2, 17^2, 21^2, 23^2, 25^2, 30^2,\\ & \quad 32^2, 34^2, 38^2, 40^2, 45^2, 47^2, 49^2, 54^2\},\\ \text{Set 3:} & \{3^2, 5^2, 7^2, 12^2, 14^2, 16^2, 19^2, 24^2, 26^2, 28^2,\\ & \quad 33^2, 35^2, 37^2, 42^2, 44^2, 46^2, 51^2, 53^2\}.\end{array}$$

Each of these groups sums to 17 985.

M62. *Proposed by Richard Hoshino, Dalhousie University, Halifax, Nova Scotia.*

Let $ABCD$ be a trapezoid where sides AB and CD are parallel and the diagonals AC and BD intersect at point P. Suppose $AB = 50$, $CD = 160$, and the area of triangle PAD is 2000. Determine the area of the trapezoid.

Solution by Geneviève Lalonde, Massey, ON.



From $AB \parallel CD$, we get $\angle PAB = \angle PCD$ and $\angle PBA = \angle PDC$. Thus, $\triangle PAB$ and $\triangle PCD$ are similar. If we name the heights of P from AB and CD as h_1 and h_2, respectively, we get

$$\frac{h_1}{h_2} = \frac{AB}{CD} = \frac{5}{16}.$$

Then $h_1 = 5h$ and $h_2 = 16h$, for some real number h.

Using the notation $[ABC]$ to represent the area of the figure ABC, we have $[ADC] = \frac{1}{2}(160)(21h) = 1680h$. We also have

$$[ADC] = [ADP] + [PDC] = 2000 + \tfrac{1}{2}(160)(16h) = 2000 + 1280h.$$

Setting these two expressions equal, we get $h = 5$; whence, the height of the trapezoid is $21h = 105$. Therefore, $[ABCD] = \frac{1}{2}(50+160)(105) = 11\,025$.

Pólya's Paragon

Paul Ottaway

For this month's installment, I have decided to explore some of the curious and interesting properties of sequences and series. To begin, I would like to revisit a famous problem that is said to have been solved by the famous mathematician Gauss when he was very young. The story goes that his teacher was frustrated with how quickly he could solve the problems given out in class. Therefore, he was assigned to sum the numbers from 1 to 100, to keep him busy. Remarkably, in almost no time at all, he had solved the problem, much to the teacher's amazement.

Here is the trick he is said to have used:

$$\begin{array}{rcccccc}
S &=& 1 &+& 2 &+ \cdots +& 100\,, \\
S &=& 100 &+& 99 &+ \cdots +& 1\,, \\
2S &=& 101 &+& 101 &+ \cdots +& 101\,, \\
2S &=& 101 \cdot 100\,, &&&& \\
S &=& 5050\,. &&&&
\end{array}$$

By writing the terms forward and backward, we are able to get a very nice expression for twice the sum. The third line is the result of summing the first two lines term by term. Since we know that there are exactly 100 terms, we quickly arrive at the answer.

We would like to be able to use this trick for finding other sums as well. By generalizing, we will now call this a 'technique' which we can use for all sorts of other situations. This time, we will start with an arithmetic sequence where the first term is a, the terms increase by d, and there are n terms. Here is what happens:

$$\begin{aligned}
S &= a + (a+d) + \cdots + \big(a+(n-1)d\big)\,, \\
S &= \big(a+(n-1)d\big) + \big(a+(n-2)d\big) + \cdots + a\,, \\
2S &= \big(2a+(n-1)d\big) + \big(2a+(n-1)d\big) + \cdots + \big(2a+(n-1)d\big)\,, \\
2S &= n\big(2a+(n-1)d\big)\,, \\
S &= na + \frac{n(n-1)}{2}d\,.
\end{aligned}$$

We can use this formula to determine that the sum of the first n natural numbers is exactly $n(n+1)/2$. To see this, use $a = 1$ and $d = 1$ in the previous equation.

We might now ask ourselves what sort of sums we can achieve when the terms do not form an arithmetic progression. Here are a few more sums with interesting patterns that I will present without proof:

$$\begin{aligned}
\frac{n(n+1)}{2} &= 1+2+\cdots+n\,, \\
\frac{n(n+1)(n+2)}{3} &= 1\cdot 2 + 2\cdot 3 + \cdots + n\cdot(n+1)\,, \\
\frac{n(n+1)(n+2)(n+3)}{4} &= 1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + \cdots \\
&\qquad + n\cdot(n+1)\cdot(n+2)\,.
\end{aligned}$$

We can use these identities to discover even more sums, like the sum of squares shown here:

$$\begin{aligned}
1^2+2^2+\cdots+n^2 &= \big(1\cdot 2 + 2\cdot 3 + \cdots + n\cdot(n+1)\big) \\
&\qquad - (1+2+\cdots+n) \\
&= \frac{n(n+1)(n+2)}{3} - \frac{n(n+1)}{2} \\
&= \frac{n(n+1)(2n+1)}{6}\,.
\end{aligned}$$

Finally, I would like to look at numbers called 'triangular' numbers. The k^{th} triangular number is the sum of the first k natural numbers. The first five triangular numbers are 1, 3, 6, 10, and 15. Is there an easy way to find the sum of the first n triangular numbers? The answer is yes! Even though they do not form an arithmetic sequence, we can still find their sum.

$$\begin{aligned}
1+3+6+\cdots+\frac{n(n+1)}{2} &= \frac{1^2+1}{2}+\frac{2^2+2}{2}+\cdots+\frac{n^2+n}{2} \\
&= \frac{1}{2}\left(1^2+2^2+\cdots+n^2\right) \\
&\qquad +\frac{1}{2}\left(1+2+\cdots+n\right) \\
&= \frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6}\right)+\frac{1}{2}\left(\frac{n(n+1)}{2}\right) \\
&= \frac{n(n+1)(n+2)}{6}\,.
\end{aligned}$$

Now that you know some useful techniques and identities for finding sums, here are a couple of problems for you to try:

1. Find the sum of the first n cubes. That is, find $1^3+2^3+\cdots+n^3$.

2. Find a relationship between your result from problem 1 and one of the other identities used in this article.

3. Find the sum of the reciprocals of the triangular numbers. That is, find $\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\cdots$

   HINT: This is an infinite sum. Start by looking at half this sum, and write each term as the difference of two fractions.

Three-Pile Nim with Blocking

Arthur Holshouser and Harold Reiter

1. Introduction

Nim, also known as Bouton's Nim, is a two-player counter-pickup game that is well known in combinatorial game theory. In this paper we develop a winning strategy for a more complicated variation of Nim, in which exactly one move can be blocked at each stage of the game. Remarkably, the winning strategy for the more complicated version is much simpler than for ordinary (Bouton's) Nim.

Specifically, we explore a three-pile game with two players, a moving player and a blocking player, whose roles alternate between moves. As in ordinary Nim, a move consists of the removal of any number of counters from any single pile. Before each move, including the first move, the blocking player must eliminate exactly one of the moving player's possible moves. For example, if the moving player is confronted with piles of size 6, 10, and 10, the blocking player could forbid the removal of 7 counters from the first of the two 10-counter piles. A forbidden move is forgotten as soon as the next move is made. The winner is the last player to make an allowed move.

2. Bouton's Nim

Before developing the strategy for our game, let us review the strategy for playing Bouton's Nim. The general ideas actually apply to all "last player wins" combinatorial games. The idea is to partition the set $\mathcal{P}$ of all possible positions into two subsets $\mathcal{S}$ and $\mathcal{U}$, where the positions in $\mathcal{S}$ are "safe" to move to and the positions in $\mathcal{U}$ are "unsafe" to move to.

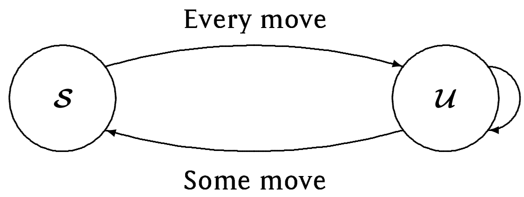
We say that a position v is *accessible* from a position u, and we write $u \longmapsto v$, if there is a move from u to v. Suppose there are two subsets $\mathcal{S}$ and $\mathcal{U}$ which partition $\mathcal{P}$ (that is, $\mathcal{S} \cup \mathcal{U} = \mathcal{P}$ and $\mathcal{S} \cap \mathcal{U} = \phi$) and which possess the following three properties:

(1) From each position v in $\mathcal{S}$, every position accessible from v belongs to $\mathcal{U}$.

(2) From each position u in $\mathcal{U}$, there is at least one position v in $\mathcal{S}$ which is accessible from u.

(3) All terminal positions belong to $\mathcal{S}$.

The sets $\mathcal{S}$ and $\mathcal{U}$ describe a winning strategy. A moving player faced with a position in $\mathcal{U}$ simply moves to a position in $\mathcal{S}$. That player can continue to move to positions in $\mathcal{S}$, ultimately winning the game. Such a strategy is depicted in the diagram below.



In three pile Nim, the members of the set $\mathcal{S}$ can be described as follows. A position in which the piles are of sizes a, b and c is denoted by (a, b, c). Associate with each such position the binary representation of the three integers a, b, c, and align these representations vertically as though we were adding them. If the number of 1's in each column is even, we say that the binary configuration is balanced, and the corresponding position belongs to $\mathcal{S}$. In other words, we take the sum in each column modulo 2. Note that $\mathcal{S}$ and its complement $\mathcal{P} - \mathcal{S}$ satisfy the three properties above.

Consider the game $(13, 15, 17)$.

$$\begin{array}{rcccccc}
13 & = & & 1 & 1 & 0 & 1 \\
15 & = & & 1 & 1 & 1 & 1 \\
17 & = & \boxed{1} & 0 & 0 & \boxed{0} & \boxed{1} \\
\hline
\text{Mod 2 Sum} & = & 1 & 0 & 0 & 1 & 1
\end{array}$$

Notice that the first, fourth, and fifth columns have 1's in the bottom row, indicating that these columns have an odd number of 1's. Also note that the three entries in the row with the 17 that are boxed need to be changed so that the columns they occupy become balanced. This can be done by replacing the pile of 17 counters with one having 2 counters. The only winning move is $(13, 15, 17) \longmapsto (13, 15, 2)$. The reason this move is unique is that the 1 in the leftmost column can be eliminated only by a move from the pile with 17 counters. The result can be depicted as:

$$\begin{array}{rccccc}
13 & = & 1 & 1 & 0 & 1 \\
15 & = & 1 & 1 & 1 & 1 \\
2 & = & & & 1 & 0 \\
\hline
\text{Mod 2 Sum} & = & 0 & 0 & 0 & 0
\end{array}$$

3. Blocking Nim

Now we consider a game where one move is blocked at each stage. Remarkably, this apparently more complicated game yields a strategy that does not require binary arithmetic. A solution is a partition $(\mathcal{S}, \mathcal{U})$ of the set $\mathcal{P}$ of positions with the following properties:

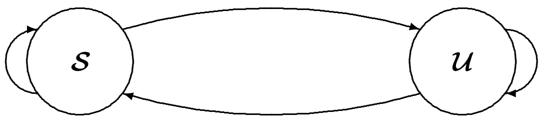
(a) Every terminal position belongs to $\mathcal{S}$.

(b) For each position u in $\mathcal{U}$, there are at least two moves from u to positions in $\mathcal{S}$.

(c) For each position v in $\mathcal{S}$, there is at most one position of $\mathcal{S}$ accessible from v.

The figure below shows how the winning strategy for the blocking game differs from that of the ordinary game.

All moves except one



At least two moves

When denoting a position by (a, b, c), we will generally require $a \leq b \leq c$. In the proof, however, we do not always adhere to this convention because the arithmetic makes it difficult to compare the sizes of the piles once a move has been made. Using this notation, there are two terminal positions: $(0, 0, 0)$ and $(0, 0, 1)$. The position $(0, 0, 1)$ is terminal because the move $(0, 0, 1) \longmapsto (0, 0, 0)$ must be blocked.

Theorem 1 . Let $\mathcal{S}$ denote the set of all positions of the form (a, a, a), where $a \geq 0$, together with the positions (a, b, c) such that $a + b + 1 = c$, and let $\mathcal{U} = \mathcal{P} - \mathcal{S}$. Then the partition $(\mathcal{S}, \mathcal{U})$ of $\mathcal{P}$ satisfies conditions (a), (b), and (c) above.

Proof: We can write $\mathcal{S}$ as the union of three sets:

$$\begin{aligned} \mathcal{S} \;=\; & \{(a, a, a) |\ a \in N\} \cup \{(a, a, c) |\ 2a + 1 = c\} \\ & \cup \{(a, b, c) |\ a < b \text{ and } a + b + 1 = c\}\,. \end{aligned}$$

We can write $\mathcal{U}$ as:

$$\begin{aligned} \mathcal{U} \;=\; & \{(a, b, c) |\ a = b < c \text{ and } c \neq 2a + 1\} \\ & \cup \{(a, b, c) |\ a < b \leq c \text{ and } a + b + 1 \neq c\}\,. \end{aligned}$$

To see condition (a), note that $(0, 0, 0)$ and $(0, 0, 1)$ both belong to $\mathcal{S}$.

Let us show next that property (b) holds. Suppose that (a, b, c) belongs to $\mathcal{U}$. If $a = b < c$ and $c \neq 2a + 1$, then we have two cases to consider: either (i) $2a + 1 > c$ or (ii) $2a + 1 < c$. In case (i), there are two moves to $(c - a - 1, a, c)$, which is a member of $\mathcal{S}$. That is, either of the piles with a counters can be reduced to $c - a - 1$ counters where $c - a - 1 \geq 0$ since $a < c$. In case (ii), there are two moves to positions in $\mathcal{S}$, namely $(a, a, c) \longmapsto (a, a, 2a + 1)$ and $(a, a, c) \longmapsto (a, a, a)$.

On the other hand, if $a < b \leq c$ and $c \neq a + b + 1$, we again consider two cases: (i) $a+b+1 > c$ and (ii) $a+b+1 < c$. If $c > a+b+1$, there are two members of $\mathcal{S}$ we could move to, $(a, b, a+b+1)$ and $(a, b-a-1, b)$. The latter position is available because $b - a \geq 1$. In case (ii), the move $(a, b, c) \longmapsto (c-a-1, a, c) \in \mathcal{S}$ is always possible because $0 \leq c-a-1 < b$. Also, the move $(a, b, c) \longmapsto (c-b-1, b, c) \in \mathcal{S}$ is possible when $b < c$ because $0 \leq c - b - 1 < a$. When $b = c$, there are always two moves $(a, b, b) \longmapsto (a, b-a-1, b) \in \mathcal{S}$, since $a < b$, and we can reduce either pile with b counters to $b - a - 1$.

To prove property (c), let (a, b, c) belong to $\mathcal{S}$. If $a = b = c$, there is no move to another member of $\mathcal{S}$. If the position is of the form (a, a, b) with $2a + 1 = b$, there is only one move to another position of $\mathcal{S}$, namely (a, a, a), because any reduction in a pile of size a results in a position (e, a, b) that does not satisfy $e + a + 1 = b$. And finally, if (a, b, c) satisfies $a < b$ and $a + b + 1 = c$, then there is no move to a position of the form (a, a, a). There is at most one move to a position for which the sum of the first two smaller pile sizes is 1 less than the third. It would involve taking counters from the largest of the three piles. ■

4. Open Questions

We do not know how to extend this result to games with more than three piles or to games in which the blocking player can block more than one move. There is another version in which instead of blocking a single move, the blocking player is allowed to block a single position. Thus, for example the move from $(2, 2, 2)$ to $(1, 2, 2)$ could be prohibited. We can solve the three-pile game but we cannot extend this result.

5. References

[1] Berlekamp, Conway, and Guy, *Winning Ways*, Academic Press, New York, 1982.

[2] Richard K. Guy, *Fair Game*, 2nd. ed., COMAP, New York, 1989.

[3] A. Holshouser, H. Reiter, and J. Rudzinski, *(Last Move) Dynamic One-Pile Nim*, to appear.

[4] A. Holshouser, H. Reiter, and J. Rudzinski, *Dynamic One-Pile Nim*, to appear in The Fibonacci Quarterly.

[5] A. Holshouser and H. Reiter, *Dynamic One-Pile Blocking Nim*, to appear.

Arthur Holshouser
3600 Bullard Street
Charlotte, NC 28208
USA

Harold Reiter
Department of Mathematics
University of North Carolina Charlotte
Charlotte, NC 28223
USA
hbreiter@email.uncc.edu