Mayhem Problems

Proposals and solutions may be sent to **Mathematical Mayhem, 2191 Saturn Crescent, Orleans, Ontario, K4A 3T6** or emailed to

`mayhem-editors@cms.math.ca`

Please include in all correspondence your name, school, grade, city, province or state and country. We are especially looking for solutions from high school students. Please send your solutions to the problems in this edition by *1 October 2003*. Solutions received after this time will be considered only if there is time before publication of the solutions.

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5 and 7, English will precede French, and in issues 2, 4, 6 and 8, French will precede English.

The editor thanks Jean-Marc Terrier and Hidemitsu Saeki of the University of Montreal for translations of the problems.

M88. *Proposed by the Mayhem Staff.*

A set S consists of six numbers. When we take all possible subsets of S containing 5 elements, the sums of the elements of these subsets are 87, 92, 98, 99, 104, and 110, respectively. Determine the six numbers in S.

. . . .

Un ensemble S contient six nombres. Si l'on prend tous les sous-ensembles de S ne contenant que 5 éléments, les sommes des éléments de ces sous-ensembles sont respectivement 87, 92, 98, 99, 104, et 110. Déterminer les six nombres dans S.

M89. *Proposed by the Mayhem Staff.*

Find all positive integers x for which $x(x+60)$ is a perfect square.

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Trouver tous les entiers positifs x pour lesquels $x(x+60)$ est un carré parfait.

M90. *Proposed by the Mayhem Staff.*

Determine the largest positive integer n for which 2002^n is a factor of $2002!$. What happens if 2002 is replaced by 2003 or 2004?

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Trouver le plus grand entier positif n tel que 2002^n soit un facteur de $2002!$. Qu'arrive-t-il si l'on remplace 2002 par 2003 ou 2004?

M91. *Proposed by Robert Morewood, Burnaby South Secondary School, Burnaby, BC.*

Let k be a four-digit integer. Determine all possible values of k for which k^{2003} ends in the four digits 2003. What happens if 2003 is replaced by 2002 or 2004?

. . . .

Soit k un nombre de quatre chiffres. Trouver toutes les valeurs possibles de k pour lesquelles le nombre k^{2003} se termine par 2003. Qu'arrive-t-il si l'on remplace 2003 par 2002 ou 2004 ?

M92. *Proposed by the Mayhem Staff.*

A 3×3 magic square consists of nine distinct values, such that each of the rows, columns, and diagonals have a constant sum. Below is an example of a 3×3 magic square.

Suppose that a 3×3 magic square has a constant sum of T. Let the middle entry of this square be E. Prove that $T = 3E$.

2	9	4
7	5	3
6	1	8

Un carré magique de 3×3 est formé de neuf valeurs distinctes, telles que la somme des éléments de chacune des lignes, des colonnes et des diagonales donne la même constante. Voir l'exemple ci-dessus.

Soit T la somme constante d'un carré magique 3×3. Si l'on désigne l'élément du centre par E, montrer que $T = 3E$.

M93. *Proposed by José Luis Díaz-Barrero, Universitat Politècnica de Catalunya, Barcelona, Spain.*

In triangle ABC, suppose that $\tan A$, $\tan B$, $\tan C$ are in harmonic progression. Show that a^2, b^2, c^2 form an arithmetic progression.

[*Note:* x, y, z are in harmonic progression if $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$ form an arithmetic progression.]

. . . .

Dans un triangle ABC, on suppose que $\tan A$, $\tan B$, $\tan C$ sont en progression harmonique. Montrer que a^2, b^2, c^2 forment une progression arithmétique .

[*Note :* x, y, z sont en progression harmonique si $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$ forment une progression arithmétique.]