BOOK REVIEW

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The Inquisitive Problem Solver
by Paul Vaderlind, Richard Guy, and Loren Larson, published by the Mathematical Association of America (MAA Problem Books Series), 2002
ISBN 0-88385-806-1, softcover, 327 + xv pages, US$34.95
Reviewed by ***Stan Wagon****, Macalester College, St. Paul, MN, USA.*

This welcome addition to the problem literature consists of 256 main problems (and much more), most of which come from a Swedish collection published in 1996 by Paul Vaderlind. Loren Larson came across the volume on a trip to Sweden and decided (correctly) that translation would be a worthwhile project. He and Richard Guy then turned it into a distinctive problem book by carefully working through the material, adding some problems of their own, and, most notably, adding much more material in the way of additional questions and remarks (and solutions to the additional questions).

The collection is noteworthy for the consistency and accessibility of the main problems. They are meant to appeal to a wide audience, with no mathematical prerequisites. The authors use the word "miniatures" to describe many problems in the collection.

Here is an example of a simple, but nonetheless inviting, problem (#202). Who has a winning strategy when the usual tic-tac-toe game is changed by adding a place at the right end of the first row? An extension to the problem asks for the smallest board for which the first player has a winning strategy.

Here is another nice one (#169), by no means difficult, yet with a compelling and surprising statement. Let W_k and L_k be the numbers of wins and losses for each of n players in a round-robin tournament. True or false: the sum of the squares of the W_k equals the sum of the squares of the L_k? There are many more such as these. Even though I am quite familiar with the problem literature, many were new to me.

An exceptional feature of the book is the variety of extensions and generalizations of the problems. Here is an example. At first we are presented with a not-too-difficult problem (#178): Start with $\{1, 2, \ldots, 2001\}$. Select a and b from the list, with $a \geq b$, remove them, and insert $a^2 - b^2$. Repeat until one number is left. Can it be 0? This problem has an easy parity-based solution (the answer is NO), but it has a surprising generalization. Define $S(n)$ to be the smallest possible number one can get by reducing the set $\{1, 2, 3, \ldots, n\}$ to a single number as before. For $n \geq 8$, the S-sequence is periodic, with period 12. For example, $S(10^{100})$ is 4. And the globally largest value of S is $S(6) = 63$.

These surprising results sent me to the website

http://www.math.uwaterloo.ca/JIS/HICKERSON/hickerson.html

to look up the paper in question. It is noteworthy that the discoverers of this result were motivated by the question by Guy, who posed it while working on the book under review.

There is one general point that applies to a few problems in this book, as well as occasional problems printed in ***Crux Mathematicorum*** or given on math contests. Students are usually as computer literate as their teachers, or more so. I find a problem such as #168 (If you add up the numbers 9, 99, 999, $\ldots$, $9999\ldots9$ with 99 digits, will the answer contain 99 ones?) a little silly as stated, since it is so easy to just add up the numbers in question and see that, yes, there are 99 ones. The same philosophy we have applied to calculus instruction for the past ten or more years, that one should not spend a lot of time on algebraic differentiation and integration when machines can do it far better than any human, should apply to problemists as well. If the method of solution is inherently interesting, find a way to pose it that makes solution by machine irrelevant.

There are a few errors. The solution given to problem #98 is incorrect, the solution to problem #209 does not match the given problem, and I found a few minor typos. Back to the plus side. The graphics are plentiful and superbly rendered, there is a 26-page section of hints, and there is a very valuable 48-page "Treasury" containing miscellaneous definitions, results, and techniques. The Treasury will be very valuable for students.

Overall the comments by Ron Graham on the back cover are right on the mark: "I don't know of a better book for introducing students (of all ages!) to the basic principles for attacking such problems". And the translators deserve extra credit for taking an inquisitive view of these elementary problems and adding much valuable material in the solutions and generalizations.

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