PROBLEMS

Problem proposals and solutions should be sent to Jim Totten, Department of Mathematics and Statistics, University College of the Cariboo, Kamloops, BC, Canada. V2C 4Z9. Proposals should be accompanied by a solution, together with references and other insights which are likely to be of help to the editor. When a proposal is submitted without a solution, the proposer must include sufficient information on why a solution is likely. An asterisk ($\star$) after a number indicates that a problem was proposed without a solution.

In particular, original problems are solicited. However, other interesting problems may also be acceptable provided that they are not too well known, and references are given as to their provenance. Ordinarily, if the originator of a problem can be located, it should not be submitted without the originator's permission.

To facilitate their consideration, please send your proposals and solutions on signed and separate standard $8\frac{1}{2}''\times 11''$ or A4 sheets of paper. These may be typewritten or neatly hand-written, and should be mailed to the Editor-in-Chief, to arrive no later than **1 March 2003**. *They may also be sent by email to* **crux-editors@cms.math.ca**. (*It would be appreciated if email proposals and solutions were written in* LaTeX). *Graphics files should be in epic format, or encapsulated postscript. Solutions received after the above date will also be considered if there is sufficient time before the date of publication. Please note that we do not accept submissions sent by FAX.*

Each problem is given in English and French, the official languages of Canada. In issues 1, 3, 5 and 7, English will precede French, and in issues 2, 4, 6 and 8, French will precede English.

In the solutions section, the problem will be given in the language of the primary featured solution.

5 problems dedicated to Professor Jordi Dou, belatedly for his 90$^{\text{th}}$ birthday.

5 problèmes dédiés tardivement au Professeur Jordi Dou, pour son 90-ième anniversaire.

2751. *Proposed by Bill Sands, University of Calgary, Calgary, Alberta.*

On each side of $\triangle ABC$, draw squares outwards to create six new points, D, E, F, G, H and I. Characterise those triangles such that the points D, E, F, G, H and I are concyclic.

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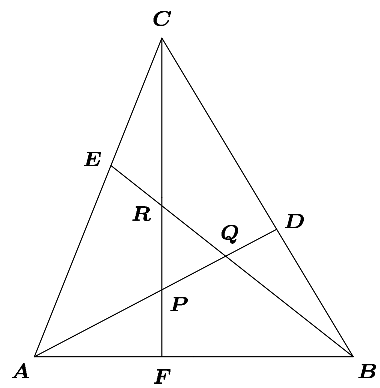
Sur chaque côté d'un triangle ABC, on dessine vers l'extérieur des carrés créant ainsi six nouveaux points D, E, F, G, H et I. Caractériser les triangles tels que les points D, E, F, G, H et I soient sur un même cercle.

2752. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, Newfoundland and Labrador.*

A generalization of Putnam 2001, question A4.

Suppose that $\frac{AF}{FB} = i$, $\frac{BD}{DC} = g$ and $\frac{CE}{EA} = h$. Determine the area of $\triangle PQR$ as a proportion of the area of $\triangle ABC$.



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Une généralisation du Putnam 2001, question A4.

Supposons que $\frac{AF}{FB} = i$, $\frac{BD}{DC} = g$ et $\frac{CE}{EA} = h$. Determiner l'aire de $\triangle PQR$, une proportion de l'aire de $\triangle ABC$.

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2753. *Proposed by Mikhail Kotchetov, Memorial University of Newfoundland, St. John's, Newfoundland and Labrador.*

Consider two circles, Γ_1 and Γ_2, centres O_1 and O_2, respectively, of different radii.

The two common tangents, t_1 and t_2, that do not intersect the line segment O_1O_2 meet at Q. A common tangent, t_c that does intersect the line segment O_1O_2 meets the tangents t_1 and t_2 at E_1 and E_2, respectively.

Let P be the mid-point of the line segment O_1O_2.

Prove that P, Q, E_1 and E_2 are concyclic.

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Considérons deux cercles Γ_1 et Γ_2, de centres respectifs O_1 and O_2 et de rayons différents.

Soit Q l'intersection des deux tangentes communes t_1 et t_2 qui ne coupent pas le segment O_1O_2. Désignons par t_c une tangente commune coupant O_1O_2, et par E_1 et E_2 les points d'intersection de t_c avec t_1 et t_2, respectivement.

Soit P le point milieu du segment O_1O_2.

Montrer que P, Q, E_1 et E_2 sont sur un même cercle.

2754. *Proposed by Juan-Bosco Romero Márquez, Universidad de Valladolid, Valladolid, Spain.*

Divide a triangle into five concyclic quadrilaterals.

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Diviser un triangle en cinq quadrilatères, chacun inscriptible dans un cercle.

2755. *Proposed by José Luis Díaz-Barrero, Universitat Politècnica de Catalunya, Terrassa, Spain.*

Evaluate

$$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{f_{n+1}^2}{1+f_n f_{n+1}^2 f_{n+2}}\right)$$

where f_n is the n^{th} Fibonacci number (that is, $f_0 = 0$, $f_1 = 1$ and, for $n \geq 2$, $f_n = f_{n-1} + f_{n-2}$).

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Evaluer

$$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{f_{n+1}^2}{1+f_n f_{n+1}^2 f_{n+2}}\right)$$

où f_n est le n-ième nombre de Fibonacci (c'est-à-dire, $f_0 = 0$, $f_1 = 1$ et, pour $n \geq 2$, $f_n = f_{n-1} + f_{n-2}$).

2756. *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*

Circles $\Gamma_1(O,R)$ and $\Gamma_2(I,r)$ touch line t at D, where $R > r$ and O and I lie on the same side of t. The point A is any point on Γ_1. The tangents to Γ_2 through A intersect t at B and C, respectively. Denote the inradii of $\triangle ABD$ and $\triangle ACD$ by r_1 and r_2, respectively.

Show that $r_1 + r_2$ is constant as A varies on Γ_1.

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Les cercles $\Gamma_1(O,R)$ et $\Gamma_2(I,r)$ touchent la droite t en D, $R > r$, O et I sont du même côté de t.

Le point A est quelconque sur Γ_1. Les tangentes à Γ_2 passant par A coupent t en B et C, respectivement. Notons r_1 et r_2 les rayons respectifs des cercles inscrits aux triangles $\triangle ABD$ et $\triangle ACD$.

Montrer que $r_1 + r_2$ reste constant lorsque A parcourt Γ_1.

2757★. *Proposed by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.*

Let A, B and C be the angles of a triangle. Show that

$$\sum_{\text{cyclic}} \frac{1}{\tan\left(\frac{A}{2}\right) + 8\tan\left(\frac{\pi - A}{4}\right)^3} \;\leq\; \frac{9\sqrt{3}}{11}\,.$$

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Soit A, B et C les angles d'un triangle. Montrer que

$$\sum_{\text{cyclique}} \frac{1}{\tan\left(\frac{A}{2}\right) + 8\tan\left(\frac{\pi - A}{4}\right)^3} \;\leq\; \frac{9\sqrt{3}}{11}\,.$$

2758. *José Luis Díaz and Juan José Egozcue, Universitat Politècnica de Catalunya, Terrassa, Barcelona. Spain.*

If $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} \neq 2$, determine all real numbers x, y, z such that

$$\begin{aligned} 0 \;&=\; (1+2a^2)x^2 + (1+2b^2)y^2 + (1+2c^2)z^2 \\ &\quad + 2xy(ab-a-b) + 2yz(bc-b-c) + 2zx(ca-c-a)\,. \end{aligned}$$

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Si $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} \neq 2$, déterminer tous les nombres réels x, y, z tels que

$$\begin{aligned} 0 \;&=\; (1+2a^2)x^2 + (1+2b^2)y^2 + (1+2c^2)z^2 \\ &\quad + 2xy(ab-a-b) + 2yz(bc-b-c) + 2zx(ca-c-a)\,. \end{aligned}$$

2759. *Proposed by Michel Bataille, Rouen, France.*

On the line segment AB, let C, D be such that $\frac{AC}{CB} = \frac{BD}{DA} = \frac{1}{3}$. Distinct points M_1, M_2, M_3 lie on a circle passing through B and C and are such that $\angle M_1BC = 2\angle M_1CB$, $\angle M_2BC = 2\angle M_2CB$, and $\angle M_3AD = 2\angle M_3DA$. Show that $\triangle M_1M_2M_3$ is equilateral.

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Sur le segment de droite AB, soit C, D tels que $\frac{AC}{CB} = \frac{BD}{DA} = \frac{1}{3}$. Des points distincts M_1, M_2, M_3 situés sur un cercle passant par B and C sont tels que $\angle M_1BC = 2\angle M_1CB$, $\angle M_2BC = 2\angle M_2CB$, et $\angle M_3AD = 2\angle M_3DA$. Montrer que le triangle $M_1M_2M_3$ est équilatéral.

2760. *Proposed by Michel Bataille, Rouen, France.*

Suppose that A, B, C are the angles of a triangle. Prove that

$$\begin{aligned} 8(\cos A + \cos B + \cos C) &\leq 9 + \cos(A+B) + \cos(B+C) + \cos(C+A) \\ &\leq \csc^2(A/2) + \csc^2(B/2) + \csc^2(C/2)\,. \end{aligned}$$

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Soit A, B, C les angles d'un triangle. Montrer que

$$\begin{aligned} 8(\cos A + \cos B + \cos C) &\leq 9 + \cos(A+B) + \cos(B+C) + \cos(C+A) \\ &\leq \csc^2(A/2) + \csc^2(B/2) + \csc^2(C/2)\,. \end{aligned}$$

2761★. *Proposed by Edgar G. Goodaire, Memorial University, St. John's, Newfoundland and Labrador.*

Give a proof by vectors that the medians of a triangle have a common point of intersection: a proof, however, ***which does not presuppose the answer***.

The vector proofs of this result with which I am familiar answer the question posed this way:

> Prove that the medians of $\triangle ABC$ intersect at $\frac{1}{3}\left(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}\right)$, where O is the origin.

The proof, of course, then amounts simply to showing that this point is on each median.

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En utilisant les vecteurs, montrer que les médianes d'un triangle se coupent en un point, ***sans toutefois présupposer***.

Les démonstrations vectorielles avec lesquelles je suis familier répondent à cette question posée de la manière suivant :

> Montrer que les médianes du triangle ABC se coupent en $\frac{1}{3}\left(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}\right)$, où O est l'origine.

La démonstration, bien entendu, se résume alors à montrer que ce point est sur chacune des médianes.

2762. *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*

Quadrilateral $ABCD$ is inscribed in circle Γ. The tangents at A, B, C, D to Γ are t_A, t_B, t_C, t_D, respectively. Given that BD, t_A and t_C are concurrent, prove that AC, t_B and t_D are concurrent.

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Soit $ABCD$ un quadrilatère inscrit dans un cercle Γ. Soit t_A, t_B, t_C, t_D les tangentes respectives à Γ. Sachant que BD, t_A et t_C sont concourantes, montrer que AC, t_B et t_D le sont aussi.