5. Let q be a real number such that $\frac{1+\sqrt{5}}{2} < q < 2$. When we represent a positive integer n in binary expansion as

$$n = 2^k + a_{k-1} \cdot 2^{k-1} + \cdots + a_1 \cdot 2 + a_0$$

(here $a_i = 0$ or 1), we define p_n by

$$p_n = q^k + a_{k-1} q^{k-1} + \cdots + a_1 q + a_0 .$$

Prove that there exist infinitely many positive integers k which satisfy the following condition: There exists no positive integer l such that $p_{2k} < p_l < p_{2k+1}$.

Solution by Mohammed Aassila, Strasbourg, France.

By induction on n, we can prove that $k = q_n$ satisfies the required condition, where q_n is defined by

$$\begin{cases} q_{2m} & = \sum_{k=0}^{m} 2^{2k} \\ q_{2m+1} & = \sum_{k=0}^{m} 2^{2k+1} . \end{cases}$$

That completes the *Corner* for this issue of ***CRUX with MAYHEM***. We are entering Olympiad season. Send me your nice solutions and generalizations as well as Olympiad Contests.

Professor Toshio Seimiya

Regular readers of this section will be aware of the many beautiful geometry problems that have been proposed by Professor Toshio Seimiya. We dedicated some to him in March 2001 [2001 : 114], in celebration of his 90^{th} birthday. Unfortunately, we did not have a photograph available then. We do now! See page 100.